Of course we eventually will build (or try to) models which predict suppression of FCNC & CP violating processes.

III.D.) Renormalized Perturbation Theory and Running coupling constants and masses

The soft SUSY breaking in the MSSM maintains the stability of the dimensionful parameters as they only receive logarithmic divergent radiative corrections. If the scale μ is the renormalization scale of the theory, then quantum corrections to the various parameters of the MSSM will receive contributions like $\lambda^2 \ln\left(\frac{\mu}{m}\right)$, m the SUSY parameters in the calculation and λ^2 some coupling or mass. Although potentially large if μ is the scale of unification M_{GUT} these log's can be summed by RG techniques to give perturbation theory in terms of running couplings and masses that depend on the energy scale μ.

Recall the running of the gauge coupling constants in the SM.

Lack of Unification

3.) Consider the running of the gauge coupling constants for $SU(3)$, $SU(2)$ & $U(1)$. For each of the gauge coupling constants, g_1, g_2, g_3 we have the renormalization group running determined by $\beta_{1,2,3}$:

$$\mu \frac{d}{d\mu} \bar{g}_i(\mu) = \beta_i(\bar{g}_i(\mu)) .$$

As we saw for $\underline{SU(3)}$ $\beta_3(\bar{g}_3) = -\frac{\bar{g}_3^3}{(4\pi)^2}\left[\frac{11}{3}C_2(8) - \frac{4}{3}T_F\right]$

where $\delta_{ij} T_F = \frac{1}{2}Tr[T^i T^j]$ where we sum over all fermi-quark-representations $\Rightarrow$

$N_F = 6 = \#$ of flavors $\qquad T_F = \frac{1}{2}\#$ of flavours $= \overset{\#\text{ of Families}}{F}$
$= \#$ of Families $= N_F/2$

Now in general we find for a gauge theory

$$\beta = -\frac{g^3}{16\pi^2}\left[\frac{11}{3}C_2(G) - \frac{4}{3}T_F - \frac{1}{3}T_S\right]$$

where $C_2(G)$ is the quadratic Casimir operator for gauge group G

$$C_2(SU(N)) = N \quad , \quad C_2(U(1)) = 0$$

and

$$T_F \delta_{ij} = \frac{1}{2}Tr[T^i T^j] \quad \text{where we}$$

sum over all fermi representations [if Dirac

fermions sum over both Left & Right handed representations, while if Weyl (Majorana) fermions only count ψ_L (ψ_α) or ψ_L^c ($\bar{\psi}_{\dot\alpha}$) once, not both. [these are Majorana fields)].
Likewise

$$T_S \delta_{ij} = Tr\{T^i T^j\}$$ where we sum over all scalar field representations.

So for SU(3) we have as previously — The Higgs field is an SU(3) color singlet ($T^i = 0$)

$$\beta_3 = -\frac{g_3^3}{(4\pi)^2}\left[\frac{11}{3}C_2(SU(3)) - \frac{4}{3}\cdot\frac{1}{2}N_F\right]$$ # of flavors

$$\boxed{\beta_3 = -\frac{g_3^3}{(4\pi)^2}\left[11 - \frac{4}{3}F\right]}$$

$\boxed{\frac{1}{2}N_F = F}$

F = # of Families

For SU(2) we also have $T_F = \frac{1}{2}N_F$ and one Higgs doublet $T_S = \frac{1}{2}$ i.e. $T_S\delta_{ij} = Tr\left[\frac{\sigma^i}{2}\frac{\sigma^j}{2}\right] = \frac{1}{2}\delta^{ij}$ and

$$C_2(SU(2)) = 2 \quad \text{so}$$

$$\boxed{\beta_2 = -\frac{g_2^3}{(4\pi)^2}\left[\frac{22}{3} - \frac{4}{3}F - \frac{1}{6}H\right]}$$

H = # of Higgs doublets = 1 for SM

Finally the hypercharge coupling constant β_1 has

$$C_2(U(1)) = 0$$

$$T_F^{(U(1))} = \frac{1}{2}\sum_{\text{fields}}' y^2_{\text{fields}} = \frac{1}{2}\sum_L' y_L^2 + \frac{1}{2}\sum_R' y_R^2$$

$$= \frac{5}{3}F \cdot \quad = \frac{1}{2}\Big[\frac{1}{4} + \frac{1}{4} + 3\cdot\frac{1}{36} + 3\cdot\frac{1}{36} + 1 + \frac{4}{9}\cdot 3 + \frac{1}{9}\cdot 3\Big]$$

3 colours

and likewise $T_S = \frac{1}{2}\sum_{\substack{\text{real}\\ \text{scalars}}}' y_s^2$

$$= \frac{1}{2}\cdot 4 \times \frac{1}{4} = \frac{1}{2}$$

So

$$\boxed{\beta_1 = +\frac{g_1^3}{(4\pi)^2}\left[\frac{20}{9}F + \frac{1}{6}H\right]}$$

In general $\beta_i = g_i^3 b_i$ hence the running coupling constants obey the DE

$$\mu\frac{d}{d\mu}\bar{g}_i(\mu) = b_i \bar{g}_i^3(\mu)$$

$\Rightarrow$

$$\mu\frac{d}{d\mu}\frac{\bar{g}_i^2}{4\pi} = 8\pi b_i\left(\frac{\bar{g}_i^2}{4\pi}\right)^2$$

Fine Structure constant $\boxed{\alpha_i(\mu) \equiv \frac{\bar{g}_i^2(\mu)}{4\pi}}$

$\Rightarrow$

$$\mu \frac{d}{d\mu} \alpha_i = 8\pi b_i \alpha_i^2$$

$$\Rightarrow \int_{\alpha_i(\mu_1)}^{\alpha_i(\mu_2)} \frac{d\alpha_i}{\alpha_i^2} = 8\pi b_i \int_{\mu_1}^{\mu_2} \frac{d\mu}{\mu}$$

$$-\frac{1}{\alpha_i(\mu_2)} + \frac{1}{\alpha_i(\mu_1)} = 8\pi b_i \ln\left(\frac{\mu_2}{\mu_1}\right)$$

$$\boxed{\frac{1}{\alpha_i(\mu_2)} = \frac{1}{\alpha_i(\mu_1)} - 8\pi b_i \ln\left(\frac{\mu_2}{\mu_1}\right)}$$

&

$$\boxed{\alpha_i(\mu_2) = \frac{\alpha_i(\mu_1)}{1 - 8\pi b_i \alpha_i(\mu_1) \ln\left(\frac{\mu_2}{\mu_1}\right)}}$$

where we have

$$b_1 = \frac{1}{(4\pi)^2}\left[\frac{20}{9}F + \frac{1}{6}H\right] = \frac{41/6}{(4\pi)^2}$$

$$b_2 = \frac{-1}{(4\pi)^2}\left\{\frac{22}{3} - \frac{4}{3}F - \frac{1}{6}H\right] = \frac{-19/6}{(4\pi)^2}$$

$$b_3 = \frac{-1}{(4\pi)}\left[11 - \frac{4}{3}F\right] = \frac{-7}{(4\pi)^2}$$

Further suppose $\mu_1 = M_Z$; $\mu_2 = e^t M_Z$

$$\ln\left(\frac{\mu_2}{\mu_1}\right) = t \qquad \left[\left(\mu \frac{d}{d\mu} = \frac{d}{dt}\right)\left(\mu = e^t M_Z\right)\right]$$

$g_1^2(M_Z) = 0.128$ $g_3^2(M_Z) = 1.479$
$g_2^2(M_Z) = 0.423$

So we can plot

$$\boxed{\frac{1}{\alpha_i(t)} = \frac{1}{\alpha_i(M_Z)} - 8\pi b_i t}$$

Now $\alpha_3(M_Z) = 0.1176$ $\quad M_Z = 91.1874$ GeV $\quad (g_3 = 1.216)$

$\alpha_2(M_Z) = 0.0336$ $\quad (g_2 = 0.65)$ $\quad (g_2^2 \approx 0.423)$

$\alpha_1(M_Z) = 0.0102$ $\quad (g_1 = 0.358)$

are the known initial conditions.

The running is displayed in a Mathcad program:

GUT Normalization: For a $SU(5)$ or $SO(10)$ GUT g_1 is normalized as

$$g_1(GUT) = \sqrt{\tfrac{5}{3}}\, g_1 \Rightarrow \alpha_1(t)(GUT) = \tfrac{5}{3}\alpha_1(t)$$

$$\Rightarrow \alpha_1(GUT)(M_Z) = \tfrac{5}{3}(0.0102)$$

and $$\frac{1}{\alpha_1(GUT)(t)} = \frac{1}{\alpha_1(GUT)(M_Z)} - \left(\frac{3}{5}\right)8\pi b_1 t$$

$$\Rightarrow \boxed{b_1(GUT) = \frac{41/10}{(4\pi)^2} = \left(\frac{3}{5}\right)b_1}$$

Running of the Standard Model Gauge Coupling Constants

Initial values of the fine structure constants are given at $M_Z := 91.1874\text{GeV}$

There the inverse fine structure constants for U(1), SU(2) and SU(3) gauge coupling constants are

$$\alpha\text{inverseM}_Z := \begin{pmatrix} \dfrac{1}{0.0102} \\ \dfrac{1}{0.0336} \\ \dfrac{1}{0.1176} \end{pmatrix}$$

The coefficients for their β functions are given by

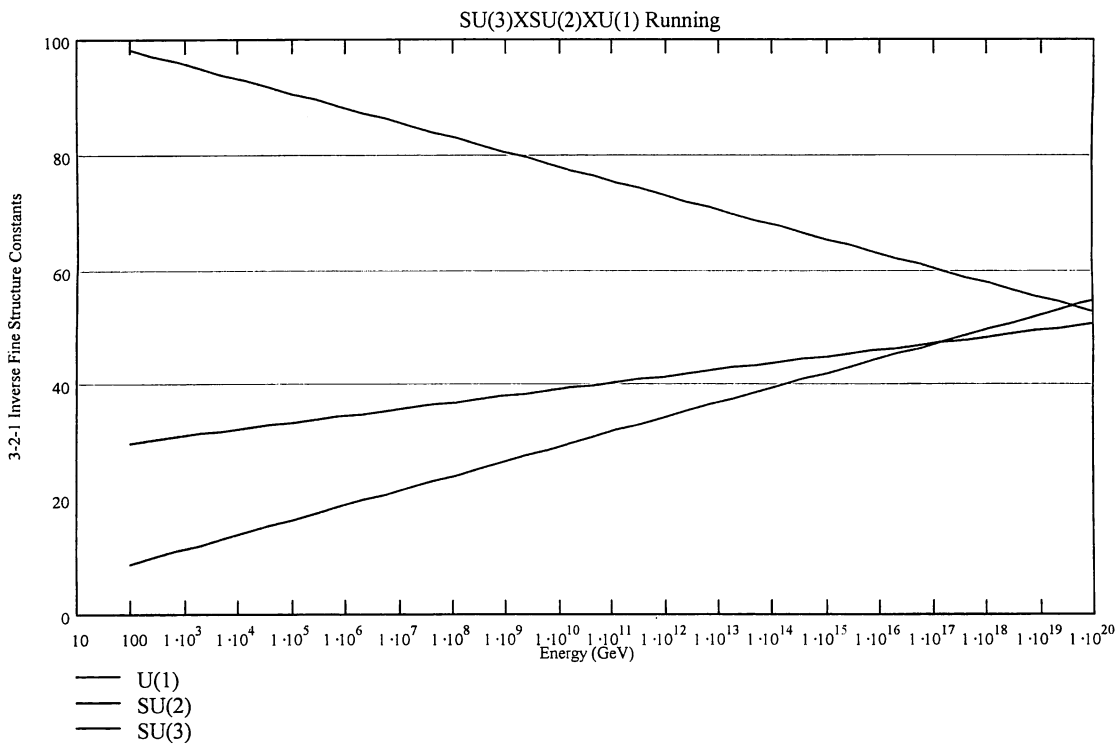
$$b := \begin{pmatrix} \dfrac{41}{6} \\ \dfrac{-19}{6} \\ -7 \end{pmatrix} \cdot \frac{1}{16\cdot\pi^2}$$

The renormalization group running of the inverse fine structure constants is described by the RGE equation

$$\alpha\text{inverse}(t) := \overrightarrow{\left(\overrightarrow{\alpha\text{inverseM}_Z} - \overrightarrow{\left(8\cdot\pi\cdot\vec{b}\cdot t\right)}\right)}$$

where the energy scale of the effective coupling constant is given by $Q(t) := M_Z\cdot e^t$

$t := 0, 1 .. 42$



Now if the SU(3)XSU(2)XU(1) groups are embedded in a SU(5) or SO(10) GUT it is conventional to normalize the U(1) coupling to be $g_1(\mathrm{GUT}) = (5/3)^{(1/2)}\, g_1$. Hence the running for such a normalization is a bit different. The U(1) intial fine structure constant becomes

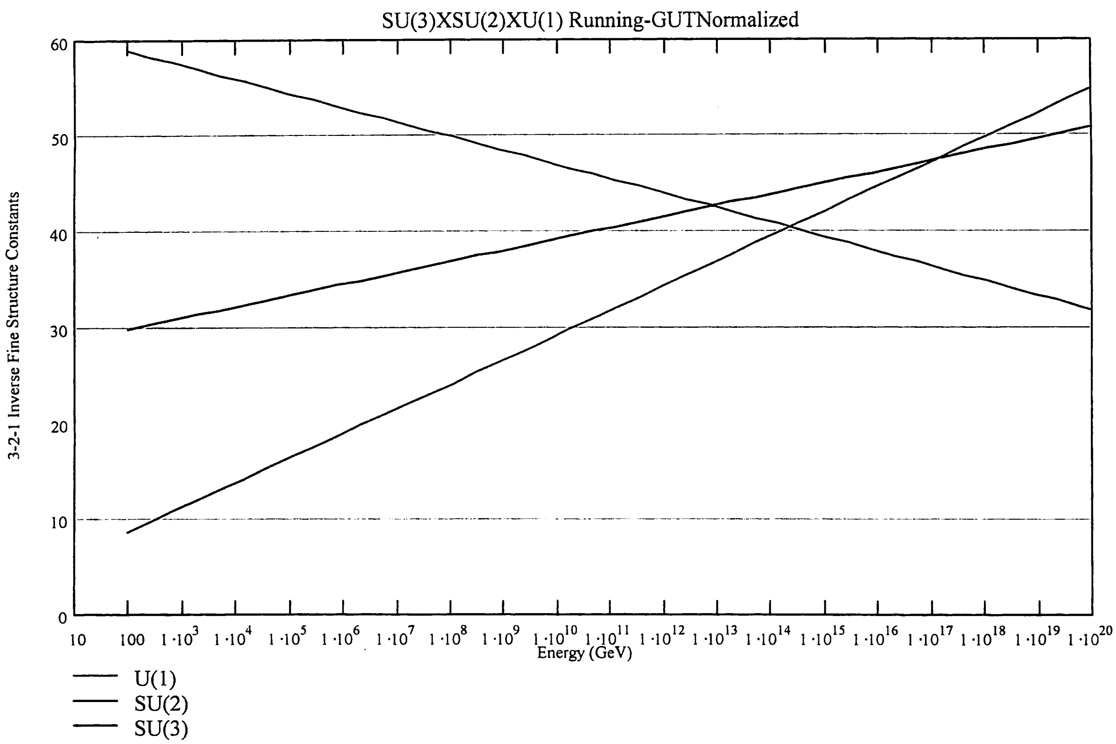
$$\alpha\mathrm{inverseGUTM}_Z := \begin{pmatrix} \dfrac{\frac{3}{5}}{0.0102} \\ \dfrac{1}{0.0336} \\ \dfrac{1}{0.1176} \end{pmatrix}$$

and the normalization changes the β function to be

$$\mathrm{bGUT} := \begin{pmatrix} \dfrac{41}{10} \\ \dfrac{-19}{6} \\ -7 \end{pmatrix} \cdot \frac{1}{16\cdot\pi^2}$$

The RGE running is now given by

$$\alpha\mathrm{inverseGUT}(t) := \overrightarrow{\left(\overrightarrow{\alpha\mathrm{inverseGUTM}_Z} - \overrightarrow{\left(8\cdot\pi\cdot\overrightarrow{\mathrm{bGUT}}\cdot t\right)}\right)}$$



The SUSY puts fermions & bosons on equal footing so we will apply the general 1-loop formula for β that includes fermion & scaler loops

$$\beta = -\frac{g^3}{16\pi^2}\left[\frac{11}{3}C_2(G) - \frac{4}{3}T_F - \frac{1}{3}T_S\right]$$

Now $C_2(SU(N)) = N$. $C_2(U(1)) = 0$ and for each super Yang-Mills field we have an ordinary Y-M field as well as a corresponding partner gaugino that is a Weyl fermion in the adjoint representation. So the Y-M field + gaugino contribute to β according to

$$\frac{11}{3}C_2(G) - \frac{4}{3}\frac{1}{2}C_2(G) = 3C_2(G)$$

since $T_F = \frac{1}{2}C_2(G)$ for the adjoint representation Weyl fermions. i.e. $\delta^{ij}T_F^{adj} = \frac{1}{2}\mathrm{Tr}\left[T^i_{adj}T^j_{adj}\right]$

$$\left(\begin{array}{l}\text{recall } (T^i_R)_{jk}(T^i_R)_{kl} = \delta_{jl}C_2(R) \\ (T^i_R)_{kl}(T^j_R)_{lk} = T(R)\delta^{ij}\end{array}\right) \qquad \begin{array}{l} = -\frac{1}{2}f_{kil}f_{ljk} \\ = \frac{1}{2}C_2(G)\delta^{ij}\end{array}$$

The each chiral superfield we have a Weyl fermion and a complex scalar in the same group representation. Therefore for each super matter and super Higgs multiplet we find a contribution to β of

$$\left[-\frac{4}{3}\frac{1}{2}T_S - \frac{1}{3}T_S\right] = -T_S = -\frac{1}{d}\mathrm{Tr}\left[T^i T^i\right]$$

where d = the dimension of the group and the trace is summed over the chiral superfields i.e. for quarks and leptons this is twice the number of flavors ($=4F$)

So for supersymmetric theories we find

$$\boxed{\begin{aligned}\beta &= -\frac{g^3}{16\pi^2}\left[3C_2(G) - \sum_{\substack{\text{chiral}\\ \text{super-}\\ \text{fields}}}' \frac{1}{d}\mathrm{Tr}\left[T^i T^i\right]\right] \\ &= -\frac{g^3}{16\pi^2}\left[3C_2(G) - \sum_{\substack{\text{chiral}\\ \text{super-}\\ \text{fields}}}' T_S\right]\end{aligned}}$$

Now let's apply this to the $SU(3)\times SU(2)\times U(1)$ MSSM

Recall we have

$$\mu \frac{d}{d\mu} \bar{g}_i(\mu) = b_i \bar{g}_i^3(\mu)$$

with $\boxed{\beta_i \equiv g_i^3 b_i .}$ So for the fine structure constant for each group

$$\boxed{\alpha_i(\mu) \equiv \frac{\bar{g}_i^2(\mu)}{4\pi}}$$ we have

$$\mu \frac{d}{d\mu} \alpha_i(\mu) = 8\pi b_i \alpha_i^2(\mu)$$

So integrating between the scales $\mu_1 \leq \mu_2$ we have as usual

$$\boxed{\frac{1}{\alpha_i(\mu_2)} = \frac{1}{\alpha_i(\mu_1)} - 8\pi b_1 \ln\left(\frac{\mu_2}{\mu_1}\right)}$$

or

$$\boxed{\alpha_i(\mu_2) = \frac{\alpha_i(\mu_1)}{1 - 8\pi b_i \alpha_i(\mu_1) \ln\left(\frac{\mu_2}{\mu_1}\right)}}$$

for each $i = 1, 2, 3$.

From our SUSY β formula we have

$$b_3 = -\frac{1}{16\pi^2}\left[3C_2(SU(3)) - \sum_{\substack{\text{quark}\\\text{chiral}\\\text{super-}\\\text{fields}}} T_S\right]$$

$$\boxed{b_3 = -\frac{1}{16\pi^2}\left[9 - 2F\right]}$$

(F = # of families = # of generations)

i.e. For quark superfields

$$\delta^{ij} T_S = \mathrm{Tr}\left[\frac{\lambda^i}{2}\frac{\lambda^j}{2}\right] = \frac{1}{4}\mathrm{Tr}\{\lambda^i\lambda^j\}$$

$$= \frac{1}{4}\frac{1}{2}\mathrm{Tr}\left[\{\lambda^i, \lambda^j\}\right]$$

$$= \frac{1}{8}\mathrm{Tr}\left[\frac{4}{3}\delta^{ij}\mathbb{1}_{3\times3} + 2d_{ijk}\lambda^k\right]$$

$$= \delta^{ij}\frac{1}{2}$$ for each quark chiral superfield u_F, u^c_F, d_F, d^c_F, $F = 1, 2, 3$

$\Rightarrow 4 \cdot 3 = 4F$ = # of such contributions

For g_2:

$$b_2 = -\frac{1}{16\pi^2}\Big[3C_2(SU(2)) - \sum_{\substack{\text{Doublets}\\ \text{chiral}\\ \text{superfields}}} T_S\Big]$$

$$\boxed{b_2 = -\frac{1}{16\pi^2}\Big[6 - 2F - \tfrac{1}{2}H\Big]}$$

H = # of Higgs chiral superfield SU(2) Doublets

For SU(2) doublets $\delta^{ij} T_S = \mathrm{Tr}\Big[\frac{\sigma^i}{2}\frac{\sigma^j}{2}\Big] = \frac{1}{2}\delta^{ij}$

$\Rightarrow T_S = \frac{1}{2}$ for each chiral superfield doublet

$\Rightarrow$

$\frac{1}{2}$ for Q^{a}_{F} ($a \leftarrow$ 3 colours, $F \leftarrow$ 3 families)
$+\ \frac{1}{2}$ for L_F ($F \leftarrow$ 3 families)

$\Big\} = \frac{1}{2}\cdot(3+1)\cdot F = 2F$ (3 colours of Q^a; L an Lepton)

$+\ \frac{1}{2}$ for H_u
$+\ \frac{1}{2}$ for H_d

$\Big\} = \frac{1}{2}\cdot H$ ← 2 doublet Higgs in MSSM

Note: For $F=3$; $b_2 > 0$ So α_2 is not asymptotically free for MSSM

Finally for g_1:

$$b_1 = -\frac{1}{16\pi^2}\Big[3\,\cancel{C_2}^{\to 0}(U(1)) - \sum_{\substack{\text{all}\\ \text{chiral}\\ \text{superfields}}}^{i} Y^2\Big]$$

Y = hypercharge for each chiral superfield

$$\boxed{b_1 = +\frac{1}{16\pi^2}\Big[\frac{10}{3}F + \frac{1}{2}H\Big]}$$

since $T_S\delta^{ij} = Tr[T^iT^j]$ for $U(1)$ is just The charge $T^i \to$ $U(1)$ charge

So

$$T_S = Y^2 \text{ for each chiral superfield}$$

$$\left.\begin{array}{llll} u: & Y = +1/6 & \times 3\,\text{colors} \times F & = \frac{1}{12}F \\ d: & Y = +1/6 & \times 3\,\text{colors} \times F & = \frac{1}{12}F \\ U^c: & Y = -\frac{2}{3} & \times 3\,\text{colors} \times F & = \frac{4}{3}F \\ D^c: & Y = +\frac{1}{3} & \times 3\,\text{colors} \times F & = \frac{1}{3}F \\ \nu: & Y = -\frac{1}{2} & \times F & = \frac{1}{4}F \\ e: & Y = -\frac{1}{2} & \times F & = \frac{1}{4}F \\ E^c: & Y = 1 & \times F & = 1F \end{array}\right\} \frac{10}{3}F$$

$$\left.\begin{array}{llll} H_u: & Y = +\frac{1}{2} \times 2 \leftarrow H_u^+,\ H_u^0 & & = \frac{1}{2} \\ H_d: & Y = -\frac{1}{2} \times 2 \leftarrow H_d^0,\ H_d^- & & = \frac{1}{2} \end{array}\right\} \frac{1}{2}H$$

Note that the SUSY slopes b_i are less than the SM slopes b_i^{SM} for SU(3) & SU(2) & $b_1 > b_1^{SM}$ for U(1).

$$b_3^{SM} = -\frac{1}{16\pi^2}\left[11 - \frac{4}{3}F\right]$$

$$b_2^{SM} = -\frac{1}{16\pi^2}\left[\frac{22}{3} - \frac{4}{3}F - \frac{1}{6}H\right].$$

$$b_1^{SM} = +\frac{1}{16\pi^2}\left[\frac{20}{9}F + \frac{1}{6}H\right]$$

So $\alpha_{3,2}$ will run more slowly but α_1 more quickly. Again we will normalize the U(1) coupling constant to its GUT normalization

$$g_1(GUT) = \sqrt{\tfrac{5}{3}}\, g_1 \Rightarrow \alpha_1(GUT) = \tfrac{5}{3}\alpha_1$$

So $b_1(GUT) = \frac{3}{5} b_1$.

Now we have our measured values of $\alpha_{1,2,3}$ at the $\mu_1 = M_Z$ scale. At this "low" energy SUSY is already broken. Hence we can evolve the coupling constants according to the SM β-functions until ~~we~~ reach the SUSY breaking scale M_{SUSY}. After which the SUSY partners propagate and are no longer decoupled from the running of the coupling constants. For simplicity we will assume all the susy partner's effects occur at the same scale M_{susy}. Hence above M_{susy} the couplings evolve according to the susy β-functions.

Introducing the running parameter t

$$\mu_1 = M_Z \; ; \; \mu = e^t M_Z \; ; \; \mu_{susy} = e^{t_{susy}} M_Z \equiv M_{susy}$$

So $\ln\left(\frac{\mu}{\mu_1}\right) = t$ and as usual $\mu \frac{d}{d\mu} = \frac{d}{dt}$

Hence running from $M_Z \to M_{susy}$ with the SM β's $\Rightarrow$

$$\frac{1}{\alpha_i(t_{susy})} = \frac{1}{\alpha_i(M_Z)} - 8\pi b_i^{SM} t_{susy}$$

This gives us the new initial conditions at M_{susy} to integrate up to high energy

$$\boxed{\begin{aligned} \frac{1}{\alpha_i(t)} &= \frac{1}{\alpha_i(t_{susy})} - 8\pi b_i (t - t_{susy}) \\ &= \frac{1}{\alpha_i(M_Z)} - 8\pi b_i t - 8\pi b_i^{SM} t_{susy} \\ &\qquad + 8\pi b_i t_{susy} \end{aligned}}$$

Using GUT normalization for the U(1) coupling constant $g_1(GUT) = \sqrt{\frac{5}{3}}\, g_1$; $\alpha_1(GUT) = \frac{5}{3}\alpha_1$

and $b_1(GUT) = \frac{3}{5} b_1$, we can evolve the coupling constants from the initial values at $M_Z \simeq 91$ GeV

$$\alpha_3(M_Z) = 0.1176 \qquad M_Z = 91.1874 \text{ GeV}$$
$$\alpha_2(M_Z) = 0.0336$$
$$\alpha_1(GUT)(M_Z) = \tfrac{5}{3}(0.0102) \qquad (t_{M_Z} = 0)$$

So
$$\frac{1}{\alpha_i(t)} = \begin{cases} \dfrac{1}{\alpha_i(M_Z)} - 8\pi b_i^{SM} t \ , & 0 \le t \le t_{susy} \\[2ex] \dfrac{1}{\alpha_i(t_{susy})} - 8\pi b_i (t - t_{susy}) \ , & t_{susy} \le t \end{cases}$$

where we use the $\alpha_1(GUT)$ & $b_1(GUT)$.

Suppose $M_{susy} = 200$ GeV $\Rightarrow t_{susy} = \ln\left(\frac{M_{susy}}{M_Z}\right)$

and so on.

As seen in the graph — the SU(3), SU(2) and U(1) coupling constants unify at about $M_{GUT} = 2 \times 10^{16}$ GeV for $M_{susy} \sim 100 - 1000$ GeV!

Running of the MSSM Gauge Coupling Constants

Now if the SU(3)XSU(2)XU(1) groups are embedded in a SU(5) or SO(10) GUT it is conventional to normalize the U(1) coupling to be $g_1(GUT) = (5/3)^{(1/2)}\ g_1$.

Initial values of the fine structure constants are given at $M_Z := 91.1874$ GeV

There the inverse fine structure constants for U(1), SU(2) and SU(3) gauge coupling constants are

$$\alpha inverseM_Z := \begin{pmatrix} \dfrac{\frac{3}{5}}{0.0102} \\ \dfrac{1}{0.0336} \\ \dfrac{1}{0.1176} \end{pmatrix}$$

The coefficients for their β functions are given by the SM values below MSUSY

$$bSM := \begin{pmatrix} \dfrac{41}{10} \\ \dfrac{-19}{6} \\ -7 \end{pmatrix} \cdot \frac{1}{16\cdot\pi^2}$$

After SUSY is broken the β function coefficients are given by

$$bMSSM := \begin{pmatrix} \dfrac{33}{5} \\ 1 \\ -3 \end{pmatrix} \cdot \frac{1}{16\cdot\pi^2}$$

The SUSY breaking scale is

$MSUSY := 100$ GeV

Hence the running parameter tSUSY is

$$tSUSY := \ln\left(\frac{MSUSY}{M_Z}\right)$$

The renormalization group running of the inverse fine structure constants is described by the RGE equation

$$\alpha inverse(t) := \begin{cases} \overrightarrow{\left(\overrightarrow{\alpha inverseM_Z} - \overrightarrow{\left(8\cdot\pi\cdot\overrightarrow{bSM\cdot t}\right)}\right)} & \text{if } t \le tSUSY \\ \overrightarrow{\left(\overrightarrow{\left(\overrightarrow{\left(\overrightarrow{\alpha inverseM_Z} - \overrightarrow{\left[8\cdot\pi\cdot\left(\overrightarrow{bSM} - \overrightarrow{bMSSM}\right)\cdot tSUSY\right]}\right)} - \left(8\cdot\pi\cdot\overrightarrow{bMSSM\cdot t}\right)\right)}\right)} & \text{if } t \ge tSUSY \end{cases}$$

where the energy scale of the effective coupling constant is given by

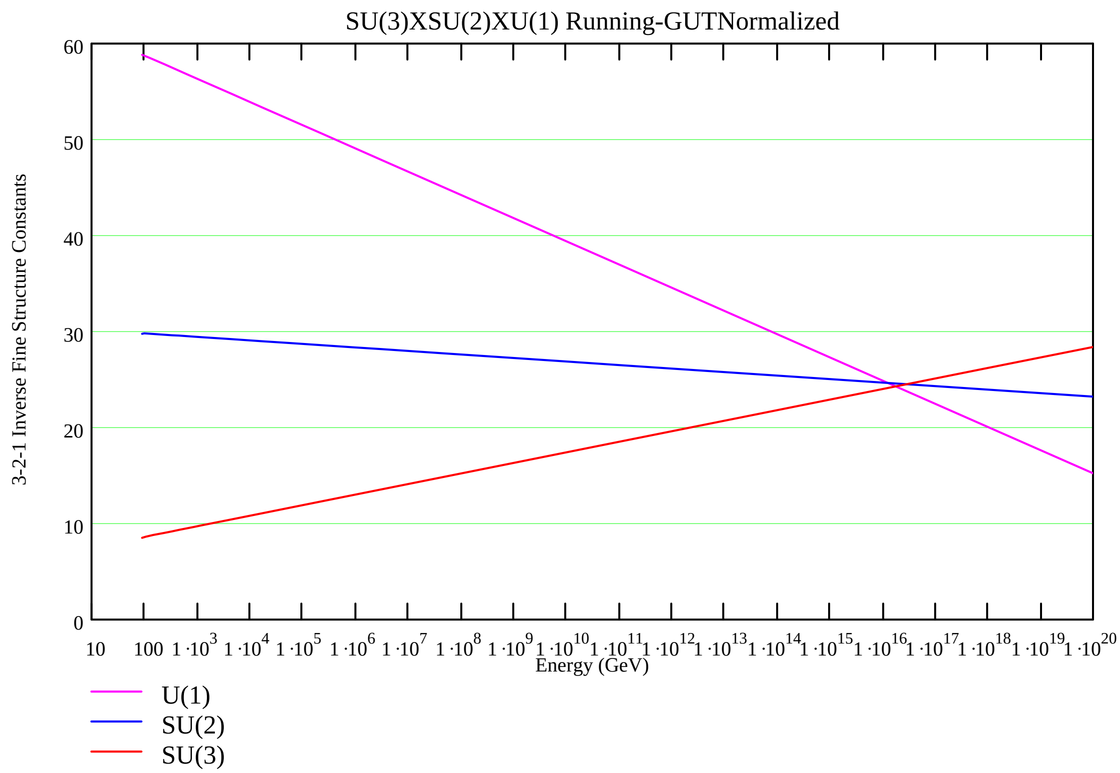
$t := 0, 0.1 .. 42$

$$Q(t) := M_Z\cdot e^t$$

$tSUSY = 0.092$

$Q(tSUSY) = 100$

The RGE running is now given by



$MSUSY2 := 1000 \qquad tSUSY2 := \ln\left(\frac{MSUSY2}{M_Z}\right)$

$tSUSY2 = 2.395$

$Q(tSUSY2) = 1 \times 10^3$

$$\alpha inverse(t) := \begin{cases} \overrightarrow{\left(\overrightarrow{\alpha inverseM_Z} - \overrightarrow{\left(8\cdot\pi\cdot\overrightarrow{bSM}\cdot t\right)}\right)} & \text{if } t \le tSUSY2 \\ \overrightarrow{\left(\overrightarrow{\left(\overrightarrow{\left(\overrightarrow{\alpha inverseM_Z} - \overrightarrow{\left[8\cdot\pi\cdot\left(\overrightarrow{bSM} - \overrightarrow{bMSSM}\right)\cdot tSUSY2\right]}\right)} - \overrightarrow{\left(8\cdot\pi\cdot\overrightarrow{bMSSM}\cdot t\right)}\right)}\right)} & \text{if } t \ge tSUSY2 \end{cases}$$