$$\Rightarrow \boxed{\mathcal{L}_{\substack{\text{matter}\\ \text{mass}}} \equiv -\bar{u}_L M^u u_R - \bar{d}_L M^d d_R - \bar{e}_L M^e e_R + \text{h.c.}}$$

where

$$M^u \equiv -\Gamma^u \frac{v_u}{\sqrt{2}} = -\frac{1}{\sqrt{2}}\Gamma^u v \sin\beta$$

$$M^d \equiv +\Gamma^d \frac{v_d}{\sqrt{2}} = +\frac{1}{\sqrt{2}}\Gamma^d v \cos\beta$$

$$M^e \equiv +\Gamma^e \frac{v_d}{\sqrt{2}} = +\frac{1}{\sqrt{2}}\Gamma^e v \cos\beta$$

As in the SM these matrices can be diagonalized with the mixing given by the CKM matrix.

So far we have found the matter fermion masses — as in the SM & the gauge $W^\pm, Z$ masses and the massless photon A_μ & gluons G_μ^m as SM usual.

Let's turn next to the scalar Higgs particle masses. There are $H_u^+, H_u^0, H_d^-, H_d^0$ — 4 complex scalar fields — 8 independent real fields. Recall the Higgs potential V_H pages -55- to -56-

with $H_u = \begin{bmatrix} H_u^+ \\ H_u^0 + \frac{v_u}{\sqrt{2}} \end{bmatrix}$; $H_d = \begin{bmatrix} H_d^0 + \frac{v_d}{\sqrt{2}} \\ H_d^- \end{bmatrix}$

we have

$$\boxed{\begin{aligned} V_H = {} & M_{H_u}^2 \left[H_u^+ H_u^- + \left| H_u^0 + \frac{v_u}{\sqrt{2}} \right|^2 \right] \\ & + M_{H_d}^2 \left[H_d^+ H_d^- + \left| H_d^0 + \frac{v_d}{\sqrt{2}} \right|^2 \right] \\ & - b \left[H_u^+ H_d^- - \left(H_u^0 + \frac{v_u}{\sqrt{2}} \right)\left(H_d^0 + \frac{v_d}{\sqrt{2}} \right) \right. \\ & \qquad \left. + H_u^- H_d^+ - \left(H_u^{0\dagger} + \frac{v_u}{\sqrt{2}} \right)\left(H_d^{0\dagger} + \frac{v_d}{\sqrt{2}} \right) \right] \\ & + \frac{g_2^2 + g_1^2}{4 \cdot 2} \left[\left(|H_u^+|^2 + \left| H_u^0 + \frac{v_u}{\sqrt{2}} \right|^2 \right)^2 \right. \\ & \qquad \left. + \left(|H_d^-|^2 + \left| H_d^0 + \frac{v_d}{\sqrt{2}} \right|^2 \right)^2 \right] \\ & + \frac{g_2^2 - g_1^2}{2 \cdot 2} \left[|H_u^+|^2 + \left| H_u^0 + \frac{v_u}{\sqrt{2}} \right|^2 \right] \left[|H_d^-|^2 + \left| H_d^0 + \frac{v_d}{\sqrt{2}} \right|^2 \right] \\ & - \frac{g_2^2}{\sqrt{2}} \left| H_u^+ H_d^- - \left(H_u^0 + \frac{v_u}{\sqrt{2}} \right)\left(H_d^0 + \frac{v_d}{\sqrt{2}} \right) \right|^2 \end{aligned}}$$

Expanding the potential about the vev's

$$V_H = V_H\Big|_{vev} + \frac{\partial V_H}{\partial H^i}\Big|_{vev} H^i + \frac{1}{2} \frac{\partial^2 V_H}{\partial H^i \partial H^j}\Big|_{vev} (H^i H^j) + \frac{1}{3!} \frac{\partial^3 V_H}{\partial H^i \partial H^j \partial H^k}\Big|_{vev} (H^i H^j H^k) + \cdots$$

shifted fields

Now we minimize the potential so

$$\left.\frac{\partial V_H}{\partial H^i}\right|_{vev} = 0 \quad .$$

The eq.'s on p. -63- and finally p. -70-

The curvature of the potential evaluated at the minimum gives the $(\text{masses})^2$ for the different scalar fields —

$$M^2_{ij} = \left.\frac{\partial^2 V_H}{\partial H^i \partial H^j}\right|_{vev}$$

Charged Scalar fields:

$$\mathcal{L}_{\substack{mass\\charged}} = \frac{1}{2}\begin{bmatrix} H_u^+ & H_d^- & H_u^- & H_d^+ \end{bmatrix}\begin{bmatrix} \frac{\partial^2 V_H}{\partial H_u^+ \partial H_u^-} & \frac{\partial^2 V_H}{\partial H_u^+ \partial H_d^+} & \cdots & \cdots \\ \frac{\partial^2 V_H}{\partial H_d^- \partial H_u^-} & \frac{\partial^2 V_H}{\partial H_d^- \partial H_d^+} & & \vdots \\ \vdots & & \ddots & \vdots \\ \frac{\partial^2 V_H}{\partial H_d^+ \partial H_u^-} & \cdots & \cdots & \frac{\partial^2 V_H}{\partial H_d^+ \partial H_d^-} \end{bmatrix}\begin{bmatrix} H_u^- \\ H_d^+ \\ H_u^+ \\ H_d^- \end{bmatrix}$$

gives 2 times charged mass matrix

$$= \begin{bmatrix} H_u^+ & H_d^+ \end{bmatrix}\begin{bmatrix} \frac{\partial^2 V_H}{\partial H_u^+ \partial H_u^-} & \frac{\partial^2 V_H}{\partial H_u^+ \partial H_d^-} \\ \frac{\partial^2 V_H}{\partial H_d^+ \partial H_u^-} & \frac{\partial^2 V_H}{\partial H_d^+ \partial H_d^-} \end{bmatrix}\begin{bmatrix} H_u^- \\ H_d^- \end{bmatrix}$$

The relevant terms can be found by setting $H_u^0, H_d^0 \to 0$

$$
\begin{aligned}
V_H\Big|_{H_u^0=0=H_d^0} &= M_{H_u}^2 H_u^+ H_u^- + M_{H_d}^2 H_d^+ H_d^- \\
&\quad - b\left[H_u^+ H_d^- + H_u^- H_d^+\right] \\
&\quad + \frac{g_2^2+g_1^2}{4\cdot 2}\left[v_u^2 H_u^+ H_u^- + v_d^2 H_d^+ H_d^-\right] \\
&\quad + \frac{g_2^2-g_1^2}{2\cdot 2}\left[\tfrac{1}{2} v_d^2 H_u^+ H_u^- + \tfrac{1}{2} v_u^2 H_d^+ H_d^-\right] \\
&\quad + \frac{g_2^2}{2\cdot 2} v_u v_d \left(H_u^+ H_d^- + H_u^- H_d^+\right)
\end{aligned}
$$

$\Rightarrow$

$$
= \begin{bmatrix} H_u^+ & H_d^+ \end{bmatrix}
\begin{bmatrix}
\left(M_{H_u}^2 + \frac{1}{8} g_2^2 v^2 - \frac{1}{8} g_1^2 v^2 \cos 2\beta\right) & \left(-b + \frac{g_2^2 v^2}{8} \sin 2\beta\right) \\
\left(-b + \frac{g_2^2 v^2}{8} \sin 2\beta\right) & \left(M_{H_d}^2 + \frac{g_2^2 v^2}{8} + \frac{g_1^2 v^2}{8} \cos 2\beta\right)
\end{bmatrix}
\times
\begin{bmatrix} H_u^- \\ H_d^- \end{bmatrix}
$$

$$
= \begin{bmatrix} H_u^+ & H_d^+ \end{bmatrix} M_{H^\pm}^2 \begin{bmatrix} H_u^- \\ H_d^- \end{bmatrix}
$$

Now we can use the minimum conditions in order to eliminate $M^2_{H_{u,d}}$ (p. -69-)

$$M^2_{Hu} = -b\cot\beta - \frac{M_Z^2}{2}\cos 2\beta$$

$$M^2_{Hd} = -b\tan\beta + \frac{M_Z^2}{2}\cos 2\beta$$

So

$$M^2_{H^\pm} = \begin{bmatrix} \left(-b\cot\beta - \frac{M_Z^2}{2}\cos 2\beta + \frac{M_W^2}{2} - \frac{M_W^2}{2}\tan^2\theta_W\cos 2\beta\right) & \left(-b + \frac{M_W^2}{2}\sin 2\beta\right) \\ \left(-b + \frac{M_W^2}{2}\sin 2\beta\right) & \left(-b\tan\beta + \frac{M_Z^2}{2}\cos 2\beta + \frac{M_W^2}{2} + \frac{M_W^2}{2}\tan^2\theta_W \cdot \cos 2\beta\right) \end{bmatrix}$$

$$= \begin{bmatrix} \left(-b\cot\beta + M_W^2\cos^2\beta\right) & \left(-b + \frac{M_W^2}{2}\sin 2\beta\right) \\ \left(-b + \frac{M_W^2}{2}\sin 2\beta\right) & \left(-b\tan\beta + M_W^2\sin^2\beta\right) \end{bmatrix}$$

Now we can diagonalize this with a similarity transformation

$$M^2_{H^\pm}\begin{bmatrix} H_u^- \\ H_d^- \end{bmatrix} = \underset{\substack{\uparrow \\ \text{diagonal}}}{m^2_{H^\pm}}\begin{bmatrix} H_u^- \\ H_d^- \end{bmatrix}$$

So

$$S M^2_{H^\pm} S^T S \begin{bmatrix} H_u^- \\ H_d^- \end{bmatrix} = m^2_{H^\pm} S \begin{bmatrix} H_u^- \\ H_d^- \end{bmatrix}$$

with $S = \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$

$$\Rightarrow \boxed{\begin{aligned} m^2_{H^\pm} &= S M^2_{H^\pm} S^T \\ &= \begin{bmatrix} -\frac{2b}{\sin 2\beta} + M_W^2 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}}$$

and $$\boxed{S \begin{bmatrix} H_u^- \\ H_d^- \end{bmatrix} \equiv \begin{bmatrix} H^- \\ \pi^- \end{bmatrix} = \begin{bmatrix} H_u^- \cos\beta + H_d^- \sin\beta \\ H_d^- \cos\beta - H_u^- \sin\beta \end{bmatrix}}$$

where π^- (and $\pi^{-\dagger} = \pi^+$) are the charged Goldstone bosons of EWSB which will be eaten by the $W^\pm$ gauge bosons to given them mass. $\pi^\pm$ are massless as Goldstone particles must be.

H^- (and $H^{-\dagger} = H^+$) is just a charge scalar particle in the theory with $(\text{mass})^2$ $\boxed{m^2_{H^\pm} = -\frac{2b}{\sin 2\beta} + M_W^2}$