4) Charmed hadron decay, non-leptonic hyperon & kaon decays, $\Delta I = \frac{1}{2}$, CP-violation, $K_L - K_S$ mass difference — all seem to be compatible with SM.

Let's discuss some <u>not</u> so satisfactory issues. The first 2 concern the Higgs sector the last the gauge coupling running.

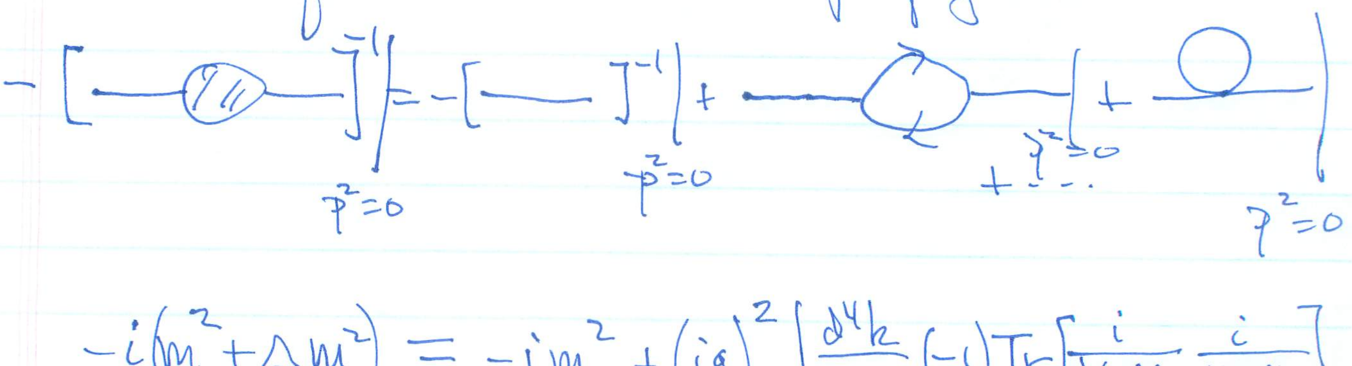
<u>Fine-Tuning Problem</u>

1) Highly sensitive Higgs Potential to heavy, high scale physics. — consider scalar field coupled to fermions & self.

$$\mathcal{L} = \bar{\psi} i \not{\partial} \psi - M\bar{\psi}\psi + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + g\phi\bar{\psi}\psi$$

$$- \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

The radiative corrections to the scalar mass is found from inverse propagator



$$-i(m^2 + \Delta m^2) = -im^2 + (ig)^2 \int \frac{d^4k}{(2\pi)^4} (-1) \mathrm{Tr}\left[\frac{i}{\not{k} - M} \frac{i}{\not{k} - M}\right]$$

$$+ \frac{(-i\lambda)}{2} \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2}$$

$$= k^2 + M^2 + 2kM$$
$$= \not{k}\not{k} + 2\not{k}M + M^2$$

$\Rightarrow$

$$-i\Delta m^2 = -g^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\frac{\overbrace{(\not{k}+M)(\not{k}+M)}}{[k^2-M^2]^2}\right]$$
$$+ \frac{\lambda}{2}\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2-m^2}$$
$$= -g^2 \int \frac{d^4k}{(2\pi)^4} 4 \frac{k^2+M^2}{[k^2-M^2]^2} + \frac{\lambda}{2}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m^2}$$

Wick rotate

$$= -ig^2 \int \frac{d^4k_E}{(2\pi)^4} 4 \frac{-k_E^2+M^2}{[k_E^2+M^2]^2} - i\frac{\lambda}{2}\int \frac{d^4k_E}{(2\pi)^4}\frac{1}{k_E^2+m^2}$$

Now $d^4k_E = d\Omega_4\, k^3 dk = \frac{1}{2} d\Omega_4\, k^2 dk^2$

& $\int d\Omega_4 = \Omega_4 = 2\pi^2$

$\Rightarrow$

$$\Delta m^2 = -4g^2 \frac{(2\pi^2)}{(2\pi)^4}\frac{1}{2}\int_0^{\Lambda^2} dk^2\, k^2 \frac{k^2 - M^2 + M^2 - M^2}{(k^2+M^2)^2}$$
$$+ \frac{\lambda}{2}\frac{(2\pi^2)}{(2\pi)^4}\frac{1}{2}\int_0^{\Lambda^2} dk^2 \frac{k^2 + m^2 - m^2}{k^2+m^2}$$

$k^2 = x$

$$= -\frac{2g^2}{8\pi^2}\int_0^{\Lambda^2} dx\; x\left[\frac{1}{x+M^2} - \frac{2M^2}{(x+M^2)^2}\right]$$
$$+ \frac{1}{4}\frac{\lambda}{8\pi^2}\int_0^{\Lambda^2} dx \left[1 - \frac{m^2}{(x+m^2)}\right]$$

$$\Delta m^2 = \frac{-2g^2}{8\pi^2}\int_0^{\Lambda^2} dx \left[\frac{x+M^2-M^2}{x+M^2} - 2M^2\frac{x+M^2-M^2}{(x+M^2)^2}\right]$$

$$+\frac{1}{4}\frac{\lambda}{8\pi^2}\left[\Lambda^2 - m^2\ln\left[\frac{\Lambda^2+m^2}{m^2}\right]\right]$$

$$= \frac{-2g^2}{8\pi^2}\int_0^{\Lambda} dx\left[1 - \frac{3M^2}{x+M^2} + \frac{2M^4}{(x+M^2)^2}\right]$$

$$+\frac{1}{4}\frac{\lambda}{8\pi^2}\left[\Lambda^2 - m^2\ln\left[\frac{\Lambda^2+m^2}{m^2}\right]\right]$$

$$\Delta m^2 = \frac{-2g^2}{8\pi^2}\left[\Lambda^2 - 3M^2\ln\left[\frac{\Lambda^2+M^2}{M^2}\right] - \frac{2M^4}{\Lambda^2+M^2} + 2M^2\right]$$

$$+\frac{1}{4}\frac{\lambda}{8\pi^2}\left[\Lambda^2 - m^2\ln\left[\frac{\Lambda^2+m^2}{m^2}\right]\right]$$

$\left(\frac{v}{\sqrt{2}} = \frac{2^{1/4}}{2\sqrt{G_F}} = 174\,\mathrm{GeV}\right)$

$G_F = 1.167\times10^{-5}\,\mathrm{GeV}^{-2}$

So we had that $\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\v\end{pmatrix}$ and from weak interaction properties $\Rightarrow \frac{v}{\sqrt{2}} = 174\,\mathrm{GeV}$

But $m_\eta^2 = 2\lambda v^2$ so we expect $m_\eta^2 \approx 100\,\mathrm{GeV}^2$. Now the corrections to m_η^2; Δm^2 grow quadratically with the cutoff.

$\Lambda \to$ largest scale (M_{pl}?)

$$\Delta m^2 \longrightarrow -\frac{2g^2}{8\pi^2}[\Lambda^2] + \frac{1}{4}\frac{\lambda}{8\pi^2}[\Lambda^2]$$

Even if we use dimensional regulator where the quadratic divergences we have that.

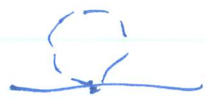
$$\Delta m^2 \sim \frac{-2g^2}{8\pi^2}\left[-3M^2 \ln\frac{\Lambda^2}{M^2}\right]$$

The mass of the Higgs grows like the heaviest fermion mass, which is greater than 100 GeV.

Also the quark, lepton & $W^\pm$, Z masses are given by v, hence the whole spectrum is indirectly related to radiative corrections and the cutoff Λ.

Also additional heavy scalars could affect Δm^2 add a scalar

$$\mathcal{L}_S = \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{\chi}{4}\phi^2 S^2 - \frac{1}{2}m_S S^2$$

$\Delta m^2 =$ 

$$= \frac{1}{4}\frac{\chi}{8\pi^2}\left[\Lambda - m_S^2 \ln\left[\frac{\Lambda^2 + m_S^2}{m_S^2}\right]\right]$$

Even with no direct coupling, but through gauge interactions this sensitivity to the largest scales occurs.

So we need a systematic way to cancel these large contributions to Δm^2. A Symmetry between the bosons & fermions of the model would do the job: For each fermion there is a partner scalar for each scalar there is a partner fermion such that the leading effects of the graphs cancel i.e.

$$\frac{1}{4}\lambda = 2g^2 \text{ (need complex scalar).}$$

Also the masses will have to be related for their dependence to be softened.

Of course one could just "tune" the parameters of the model to reduce the size of the corrections — choose a mass or coupling constant to eliminate Λ^2 dependence — but this will have to be done every order & what happens for new massive particles —. This is all known as the (technical) "fine-tuning problem".

(Hierarchy problem is why is $m_H \ll m_{pl}$ to begin with)