As a start let's ignore the extrinsic variables & just focus on the spin dynamics:

As an illustration of the use of the spin basis, consider the interaction of a spin $\frac{1}{2}$ particle with an external magnetic field. Classically, a spinning charge has a magnetic moment $\vec{\mu}$ which interacts with the external magnetic field $\vec{B}$. The classical Hamiltonian describing the interaction is simply $H = -\vec{\mu}\cdot\vec{B}$. If the spinning charge has angular momentum $\vec{S}$ due to its spin then the magnetic moment is proportional to it, $\vec{\mu} = \gamma\vec{S}$ where the constant of proportionality γ, the gyromagnetic ratio, depends on the make up of the spinning charge.

Quantum mechanically, a particle with spin $\vec{S}$ has magnetic moment

$$\vec{\mu} = \gamma\vec{S} = \frac{q}{e}g\,\frac{e\hbar}{2m}\,\frac{1}{\hbar}\vec{S}$$

$$\equiv g\left(\frac{q}{e}\right)\mu_B\frac{1}{\hbar}\vec{S}$$

where $\mu_B \equiv \frac{e\hbar}{2m}$ is the Bohr magneton

and e is the magnitude of the electron charge, q is the electric charge of the particle, g depends on the particle being considered! (For neutral particles like neutron $\gamma \equiv g\,\frac{e\hbar}{2m} = g\mu_B$.) g is called the Landé g-factor

The interaction of the spin $\vec{S}$ with the external magnetic field $\vec{B}$ is again described by the Hamiltonian

$$H = -\vec{\mu}\cdot\vec{B} = -\gamma\vec{S}\cdot\vec{B}$$

$$= -g\frac{q}{e}\frac{\mu_B}{\hbar}\vec{S}\cdot\vec{B}.$$

For $\vec{B} = B\hat{z}$ this yields

$$H = -g\frac{q}{e}\frac{\mu_B B}{\hbar}S_z$$

$$\equiv \omega_B S_z$$

with the Bohr frequency $\omega_B \equiv -g\frac{q}{e}\frac{\mu_B B}{\hbar}$.

The spin basis vectors are the eigenstates of H

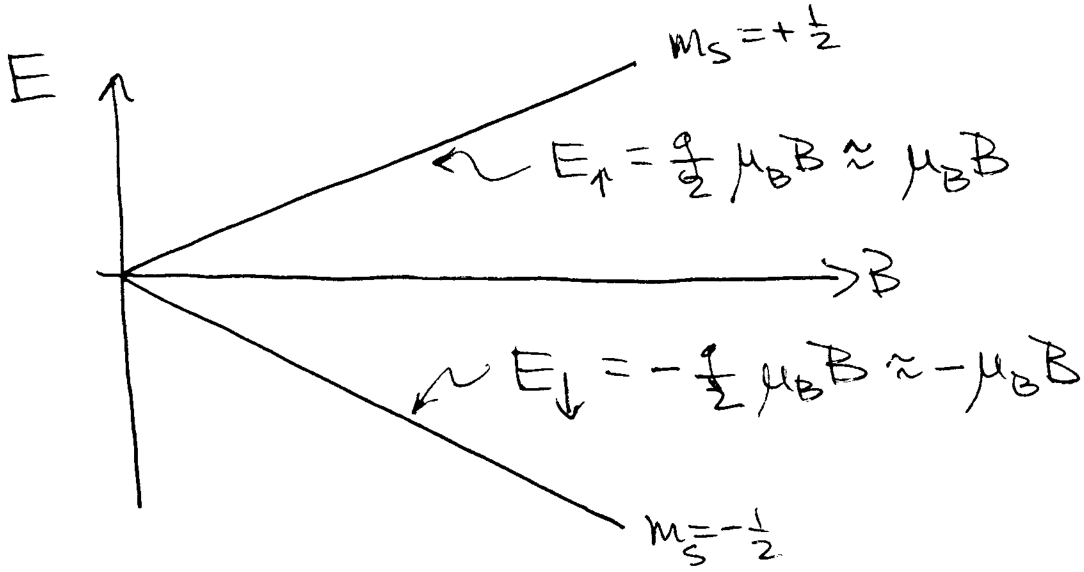
$$H|s=\tfrac{1}{2}, m_s\rangle = m_s\hbar\omega_B|s=\tfrac{1}{2}, m_s\rangle$$

with $m_s = \pm\frac{1}{2}$. Hence the energy eigenvalues are

$$E_{m_s} = m_s\hbar\omega_B = m_s\left(-g\frac{q}{e}\mu_B B\right)$$

For example for an electron $\frac{q}{e} = -1$ and $\hbar\omega_B = g\mu_B B$. Experimentally we find $(g-2) = 0.002319312$ (i.e. $g = 2(1.001159656)$) which is known as the anomalous magnetic moment of the electron.

Hence



Suppose at time $t=0$, the spin is in the ~~state~~

$$|\psi(0)\rangle = \cos\frac{\theta}{2}\, e^{-i\varphi/2}\, |\tfrac{1}{2},\tfrac{1}{2}\rangle + \sin\frac{\theta}{2}\, e^{+i\frac{\varphi}{2}}\, |\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

The state at time t is simply

$$|\psi(t)\rangle = \cos\frac{\theta}{2}\, e^{-i\frac{\varphi}{2}}\, e^{-\frac{i}{\hbar}E_{\frac{1}{2}}t}\, |\tfrac{1}{2},\tfrac{1}{2}\rangle + \sin\frac{\theta}{2}\, e^{+i\frac{\varphi}{2}}\, e^{-\frac{i}{\hbar}E_{-\frac{1}{2}}t}\, |\tfrac{1}{2},-\tfrac{1}{2}\rangle$$

since $|\frac{1}{2}, m_s\rangle$ are energy eigenstates and

$$i\hbar\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle.$$

So

$$|\psi(t)\rangle = \cos\frac{\theta}{2}\, e^{-i\frac{(\varphi+\omega_B t)}{2}} |\tfrac{1}{2},\tfrac{1}{2}\rangle$$
$$+ \sin\frac{\theta}{2}\, e^{i\frac{(\varphi+\omega_B t)}{2}} |\tfrac{1}{2},-\tfrac{1}{2}\rangle.$$

The magnetic field $\vec{B}$ produces a phase shift between the coefficients of the spin eigenstates that depends on time.

Since $[H, S_z]=0$, the observable S_z is a constant of the motion. The probability to measure spin $\pm\frac{1}{2}$ at t is

$$P_{\pm\frac{1}{2}} = |\langle\tfrac{1}{2},\pm\tfrac{1}{2}|\psi(t)\rangle|^2$$
$$= \begin{cases} \cos^2\frac{\theta}{2} & \text{for } +\frac{1}{2} \\ \sin^2\frac{\theta}{2} & \text{for } -\frac{1}{2} \end{cases}$$

independent of t. However $S_{x,y}$ do not commute with H, $[H, S_{x,y}] \neq 0$ and hence are not constants of the motion. Their expectation values are

$$\langle\psi(t)|\vec{S}\,|\psi(t)\rangle =$$

$$= \left(\cos\frac{\theta}{2} e^{+i\frac{(\varphi+\omega_B t)}{2}} \quad \sin\frac{\theta}{2} e^{-i\frac{(\varphi+\omega_B t)}{2}} \right) \frac{\hbar}{2}\vec{\sigma} \times \begin{pmatrix} \cos\frac{\theta}{2} e^{-i\frac{(\varphi+\omega_B t)}{2}} \\ \sin\frac{\theta}{2} e^{+i\frac{(\varphi+\omega_B t)}{2}} \end{pmatrix}$$

$\Rightarrow$

$$\langle \psi(t) | S_x | \psi(t) \rangle = \frac{\hbar}{2} \sin\theta \cos(\varphi + \omega_B t)$$

$$\langle \psi(t) | S_y | \psi(t) \rangle = \frac{\hbar}{2} \sin\theta \sin(\varphi + \omega_B t)$$

$$\langle \psi(t) | S_z | \psi(t) \rangle = \frac{\hbar}{2} \cos\theta \ .$$

The expectation values of $\vec{S}$ behave like classical angular momentum of magnitude $\frac{\hbar}{2}$ undergoing Larmor precession with angular velocity ω_B.