Initially let's consider such separable systems. $H = H_o + H_s$

Further, since we have worked out many "spinless" problems — let's ignore the orbital motion $\Psi_o(\vec{r},t)$ Schrödinger eq.

Eg. Suppose consider the Hydrogen atom. For the pure Coulomb interaction we have $H = H_o$

$$= \frac{\vec{p}^2}{2m} + V(\vec{r})$$

$$V(\vec{r}) = -\frac{e^2}{4\pi\epsilon_0}\frac{1}{r}$$

Hence this is a separable (trivial) case since the interaction is completely independent of the (e) spin. Hence

$$|\Psi\rangle = |\Psi_o\rangle \otimes |\chi_s\rangle$$

& $\quad i\hbar\frac{\partial}{\partial t}|\chi_s\rangle = 0 \quad \Rightarrow \quad |\chi_s\rangle$ is indep of time

The wavefunctions are then just products of orbital & spin functions.

Choosing $\{H, \vec{L}^2, L_z; \vec{S}^2 S_z\}$ as our CSCO, then the eigenfunctions are simply $\langle \vec{r}; \overset{1/2}{s}, \overset{1/2}{m_s} | n, l, m; \overset{1/2}{s}, \overset{\uparrow\downarrow}{m_s} \rangle$))

$$|n, l, m; s, m_s\rangle \longrightarrow \psi_{nlm}(\vec{r})\, e^{\uparrow\downarrow}$$

That is the wavefunctions are just products of the Coulomb wavefunctions $\psi_{nlm}(\vec{r})$ and the S_z eigenspinors $\begin{pmatrix}1\\0\end{pmatrix} = e^{\uparrow}$, $e^{\downarrow} = \begin{pmatrix}0\\1\end{pmatrix}$.

Since $H = H_0$; the energy levels remain unchanged — But we now have doubled the # of states ~~with~~ for each energy level corresponding to the e^- spin up or down along the z-axis!.

Next ~~put~~ the H-atom in a weak $\vec{B}$-field $\vec{B} = B\hat{k}$. The proton and e^- will interact with the $\vec{B}$-field through their orbital & spin magnetic moments. Since $\frac{m_e}{m_p} \ll 1$ $\left(\approx \frac{1}{2000}\right)$ we will neglect the proton's magnetic moment's interaction energies (NMR!) compared to the electron.

Now the dipole energy is $H_{dipole} = -\vec{\mu}\cdot\vec{B}$

for the e^- $\vec{\mu}$ has 2-terms (more on this later)

$$\vec{\mu} = \frac{q}{2m}\vec{L} + \gamma\vec{S} = \frac{-e}{2m}\vec{L} - \frac{e}{m}\vec{S}$$

$$\Rightarrow \quad H_{dipole} = \frac{eB}{2m}\left(L_z + 2S_z\right)$$

And the H-atom Hamiltonian becomes

$$H = H_{coulomb} + \frac{eB}{2m}\left(L_z + 2S_z\right)$$

$$= H_{coul} + H_{dipole}.$$

H is function of H_{coul}, L_z, S_z + so $|n,l,m;s,m_s\rangle$ still eigenstate.

Since H still commutes with H_{coul}, $\vec{L}^2$, L_z, $\vec{S}^2$, S_z, it is diagonalized by the same eigenstates!

$$|n,l,m;s,m_s\rangle = |n,l,m;\tfrac{1}{2},m_s\rangle$$

Since $s=\frac{1}{2}$ we drop it from list to save writing

$$\rightarrow \quad |n,l,m,m_s\rangle$$ Hence

$$H|n,l,m,m_s\rangle = \left\{-\left[\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\right]\frac{1}{n^2} + \frac{eB\hbar}{2m}(m+2m_s)\right\}|n,l,m,m_s\rangle$$

$$= \left[-\frac{mc^2}{2}\alpha^2\frac{1}{n^2} + \mu_B B(m+2m_s)\right]|n,l,m,m_s\rangle$$

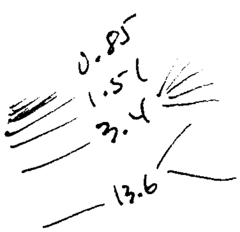
Bohr magneton $= 5.788\times10^{-5}\ \frac{eV}{T}$

Hence E_n now depends on m, m_s also $\Rightarrow$ we have removed some of the degeneracy of the energy levels.

ex. The ground state was 2-fold degenerate due to spin ↑ ↓ of e^-
now it's "split" into 2 non-deg. states

$n=1 \Rightarrow l=0=m$ but $m_s = \pm 1/2$

$$E_{n=1} = -13.6\,eV \pm \mu_B B$$



spin down is groundstate
agrees with intuition
$H_{dipole} = -\vec{\mu}\cdot\vec{B}$ & $\vec{\mu}$ is opposite spin (neg. charge)
So lower energy $\vec{\mu}$ aligns with $\vec{B} \Rightarrow$
$\vec{S}$ anti-parallel with $\vec{B}$.

For the first excited state we had $n=2$, $l=0$, $m=0$, $m_s=\pm\frac{1}{2}$
$l=1$, $m=-1,0,+1$, $m_s=\pm\frac{1}{2}$

8 degenerate states for Hcoul.

Now

$$E_{n=2} = -\frac{13.6\,eV}{4} + \mu_B B \begin{array}{c} \\ \left[\begin{array}{c} 2 \\ 1 \\ 0 \\ -1 \\ -2 \end{array}\right. \end{array} \begin{array}{ccc} m & m_s & l \\ 1 & \frac{1}{2} & 1 \\ 0 & \frac{1}{2} & 0 \text{ or } 1 \\ 1 \text{ or } -1 & -\frac{1}{2} \text{ or } \frac{1}{2} & 1 \\ 0 & -\frac{1}{2} & 0 \text{ or } 1 \\ -1 & -\frac{1}{2} & 1 \end{array} \begin{array}{c} \\ \left.\begin{array}{c} \\ \\ \\ \\ \\ \end{array}\right] \end{array}$$

So now we "split" the 8 degenerate states into 5 states, 3 of which are still doubly degenerate. etc.

In multi-electron atoms – add contributions from all e^- → the # of spectral lines increase and the spacing may be varied by varying B.

This is called the (weak field) Zeeman Effect. More on this later (strong field).

Since we have worked a lot of "spinless" problems let's focus just on some pure spin dynamics.

i.e. $$i\hbar \frac{\partial}{\partial t}\left(\chi_s(t)\right)_{m_s} = \left(H_s\right)_{m_s m_s'} \left(\chi_s(t)\right)_{m_s'}$$