Physics 460: Quantum Mechanics I

I Schrödinger Wave Mechanics

a) Planck & Einstein: Electromagnetic wave ($\omega = 2\pi\nu$) is associated with a collection of photons each of energy E: $E = \hbar\omega = h\nu$ (Planck-Einstein Relation)

$\hbar = \frac{h}{2\pi}$, h = Planck's constant

(SI units: $\hbar = 1.05457 \times 10^{-34}$ J·s)

b) deBroglie: All free particles have an associated wave of frequency $\omega = E/\hbar$

<u>and</u> wavelength $\lambda \equiv \frac{2\pi}{k} = \frac{2\pi\hbar}{p}$ (deBroglie Wavelength)

That is the wave number $k = p/\hbar$ with p the momentum of the particle.

Non-relativistic particle of mass m: $E = \frac{\vec{p}^{\,2}}{2m}$

So $E = \hbar\omega = \frac{\vec{p}^{\,2}}{2m} = \frac{\hbar^2 \vec{k}^2}{2m}$

(dispersion relation) $\Rightarrow$ $\omega = \frac{\hbar k^2}{2m}$ $\Rightarrow$ group velocity of the matter wave $v_g = \frac{d\omega}{dk} = \frac{\hbar k}{m} = \frac{p}{m}$,

the velocity of the non-relativistic particle.

Ib) deBroglie used matter waves to derive Sommerfeld-Bohr quantization:

Bohr: $\oint_{orbit} \frac{ds}{\lambda} = n$, $n = 1, 2, \ldots$ So waves are in phase.

Circular Bohr orbit $ds = r\,d\theta$ and $\frac{1}{\lambda} = \frac{p}{h}$

So $\oint_{orbit} p r\,d\theta = nh = \boxed{\oint_{orbit} p_\theta\,d\theta = nh}$

i.e. Sommerfeld Quantization Rule $\oint p_k\,dq_k = n_k h$; $n_k = 1, 2, \ldots$

More simply for the circular orbit $n = \frac{2\pi r}{\lambda} = kr$

$\Rightarrow pr = n\hbar$, action is quantized

Ultimately incorrect

c) Schrödinger: Associate a function of space-time, the wavefunction, with a free particle $(E = \frac{\vec{p}^2}{2m})$

$$\psi(\vec{r}, t) = A e^{\frac{i}{\hbar}(\vec{p}\cdot\vec{r} - Et)}$$

where A = constant, $\vec{p}$ is the momentum and E the energy of the free, non-relativistic particle of mass m, $E = \vec{p}^2/2m$

- Ic) Since $i\hbar \frac{\partial \psi}{\partial t} = E\psi$ and

$-\frac{\hbar^2}{2m}\nabla^2\psi = \frac{\vec{p}^{\,2}}{2m}\psi$, the wavefunction for a free particle obeys the partial differential equation

$$i\hbar \frac{\partial}{\partial t}\psi(\vec{r},t) = -\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r},t) \, .$$

If the particle moves under the influence of a potential $V(\vec{r})$, Schrödinger replaced the free equation with

$$E\,\psi(\vec{r},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r})\right]\psi(\vec{r},t)$$

He identified KE : $T = \frac{\vec{p}^{\,2}}{2m} \rightarrow -\frac{\hbar^2}{2m}\nabla^2$

PE : $V = V(\vec{r}) \rightarrow V(\vec{r})$

Total Energy : $E = T + V \rightarrow i\hbar\frac{\partial}{\partial t}$

Further if $V = V(\vec{r},t)$, Schrödinger hypothesized that the wavefunction of a particle of mass m moving under its influence obeys the partial differential equation — Schrödinger's Equation:

I c.)

$$\boxed{i\hbar \frac{\partial}{\partial t} \Psi(\vec{r},t) = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r},t) \right] \Psi(\vec{r},t)}$$

And quantum (wave) mechanics was Born!