I-A) 4) Note: 1) Schrödinger eq. is linear and homogeneous in Ψ. Therefore it satisfies the principle of (linear) superposition. If Ψ_1 and Ψ_2 individually satisfy the Schrödinger equation, then so does

$$\Psi = \Psi_1 + \lambda \Psi_2 \quad \text{with } \lambda \in \mathbb{C}.$$

2) Equation is first order in $\frac{\partial}{\partial t}$. Thus knowledge of the system at t_0, $\Psi(\vec{r}, t_0)$ for $\forall \vec{r}$, determines the state of the system, $\Psi(\vec{r}, t)$, at all other times via Schrödinger's Equation.

3) Since $|\Psi|^2 = |\Psi_1 + \lambda \Psi_2|^2$ has the interpretation of position probability density, wave-like effects such as interference can occur from the cross-terms.

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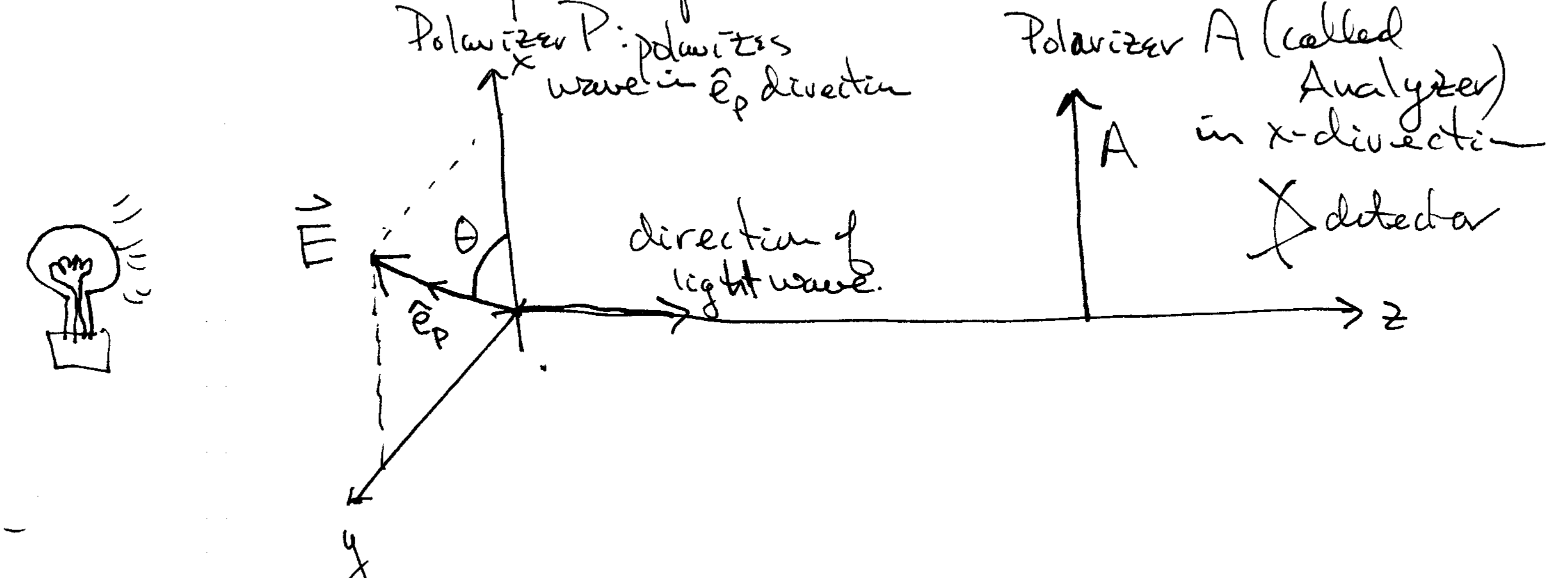
4) Hermitian operators have real eigenvalues.

A) Clarification of the principle of spectral decomposition:

Consider the optics experiment of plane polarized monochromatic light impinging on the plane polarizer A.



Light wave propagates in z-direction and is polarized along direction $\hat{e}_p$. The polarizing filter A transmits light polarized parallel to the x-axis and absorbs light polarized parallel to the y-axis.

Classically the electric field is given by

$$\vec{E}(\vec{r},t) = E_0 \hat{e}_p e^{i(kz-\omega t)}, \text{ with}$$

E_0 a constant and $\hat{e}_p = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$ and $\omega/k = c$. The light intensity $I \propto |E_0|^2$.

I A) After the light passes through the polarizing filter A, The plane wave is polarized along the x' axis

$$\vec{E}'(\vec{r},t) = E_0' \hat{i}\, e^{i(kz-\omega t)}.$$

Its intensity is $I' \propto |E_0'|^2$ and

$$I' = I \cos^2\theta \quad \text{(Malus' law)}.$$

Einstein-Planck light ↔ photons

Now at the quantum level when I is weak enough photons reach the analyzer one by one. The photon detector just after the analyzer never detects a fraction of a photon — either the photon goes through the analyzer or it is entirely absorbed (stopped) by it. Except for special cases described below, we cannot predict with certainty whether a given incident photon will pass through or be absorbed by A. We only know the probability to pass through or be absorbed.

Finally if we send out a large number N of photons one after the other, the result will correspond to the classical law about $N\cos^2\theta$ photons will be detected to pass through.

-I A) From this example we see the corresponding concepts that are used in the principle of spectral decomposition.

1) The measurement device (the analyzer) can only give certain specific results, called eigen-results (values). In this case the 2 possible results are a) the photon passes through the analyzer or b) the photon is stopped. There is a "quantization" of the result of the measurement in contrast to the classical case, $I' = I\cos^2\theta$, I'/I can vary continuously (between 0 & 1) depending on θ.

2) To each of these eigen results corresponds an eigenstate: $\hat{e}_p = \hat{i}$ or $\hat{e}_p = \hat{j}$.

If $\hat{e}_p = \hat{i}$, we know with certainty that the photon will pass through the analyzer. If $\hat{e}_p = \hat{j}$ we know with certainty it will be stopped. Thus if the particle (γ) is in one of the eigenstates before the measurement, the result of this measurement is certain — it is the eigen result.

- I A) 3) When the state before the measurement is arbitrary, only probabilities of obtaining the different eigen results can be predicted. To find the probabilities, one decomposes the state into a linear combination of the various eigenstates: $\hat{e}_p = \hat{i}\cos\theta + \hat{j}\sin\theta$

The probability of obtaining a given eigenresult is proportional to the square of the absolute value of the coefficient of the corresponding eigenstate. The proportionality factor is determined by the condition that the sum of all these probabilities must $= 1$.

So this tells us that each photon has a probability $\cos^2\theta$ of passing through A and a probability $\sin^2\theta$ of being absorbed by A (and $\sin^2\theta + \cos^2\theta = 1$).

This is the principle of spectral decomposition. The decomposition performed depends on the type of measurement device being considered - since it is its eigenstates that are used (in our example $\hat{e}_p = \hat{i}\cos\theta + \hat{j}\sin\theta$, it is the analyzer A that fixes the choice of x- and y- axes).

I A) 4) Finally, after passing through the analyzer, the light is now completely polarized along $\hat{i}$. Placing a second analyzer A' after A having the same axis - all photons which traversed A will also traverse A'. By 2) this means that after passing through A the state of the photon is the eigenstate given by $\hat{e}_p = \hat{i}$.

There has been an abrupt change of state of the particles. Before the measurement the state was defined by $\vec{E}(\vec{r},t)$ which was along with $\hat{e}_p$. After measurement we have additional information that the photon passed through A and that is incorporated by describing the state by a different vector: $\hat{e}_p = \hat{i}$. And that state is to be normalized to one since ~~there~~ is Prob=1 ~~to~~ find this somewhere in space.

According to deBroglie's hypothesis and Schrödinger's generalizations we apply these above photonic observations to all matter as Postulate 3.