VI) Magnetic Properties of Matter

A) Material placed in a magnetic field is either

1) Diamagnetic: when placed in an external magnetic field $\vec{B}_0$, it tends to lessen the resultant field $\vec{B}$. It is repelled from a region of greater magnetic field to lesser magnetic field.

2) Paramagnetic: when placed in an external magnetic field $\vec{B}_0$, it tends to increase the resultant $\vec{B}$-field. It is attracted toward a region of greater magnetic field from lesser magnetic field.

a) Ferromagnetic: when placed in an external magnetic field $\vec{B}_0$, it greatly increases ($\sim 10^2$) the resultant magnetic field $\vec{B}$.

B) Magnetic properties are due to atomic motion of electron clouds swirling about nucleus and electrons having intrinsic spin. This spin and orbital motion of the electrons results in atomic magnetic moments ("atomic currents")

1) No net magnetic moment results in diamagnetic properties — in an external $\vec{B}$-field a magnetic moment is induced in the opposite direction to $\vec{B}_0$.

VI) B2) Net magnetic moment results in paramagnetic material – in an external $\vec{B}$-field the magnetic moments tend to aligne along the direction of $\vec{B}_o$.

C) Magnetization, $\vec{M}$, is the dipole moment per unit volume at each macroscopic point of the material — characterizes all macroscopic magnetic properties of the material!

$$\vec{M} \equiv \lim_{\Delta V \to 0}^{\text{macro}} \frac{\langle \vec{\mu} \rangle}{\Delta V}$$

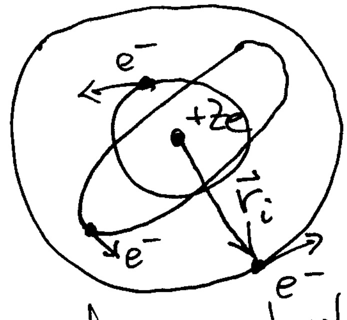
where $\langle \vec{\mu} \rangle$ is the average over the magnetic dipole moments of all the many atoms in the macroscopically small, but microscopically large, volume element ΔV of material

$$\langle \vec{\mu} \rangle = \sum_{\substack{a \\ \text{atoms} \\ \text{in } \Delta V}} \vec{\mu}_a .$$

If each $\vec{\mu}_a \simeq \vec{\mu}$, then $\vec{M} = N\vec{\mu}$ where $N = \frac{\#\text{ of atoms}}{\text{volume}}$.

1) SI units of $\vec{M}$ are $\frac{\text{Ampere}}{\text{meter}}$

VI D) Microscopic View of Magnetic Materials:
Use simplified planetary model of atom

1) 

circular orbits

$\vec{v}_i = \vec{\omega}_i \times \vec{r}_i$

i^{th} electron at radius $\vec{r}_i$ with velocity $\vec{v}_i$ $\Rightarrow$ atomic current

$$i = \frac{e v_i}{2\pi r_i} = \frac{e\omega_i}{2\pi} = \frac{dq}{dt}$$

and $\tau = \frac{2\pi}{\omega}$ = time to make one orbit.

This current has an associated magnetic dipole moment

magnitude : $\mu_i = iA = i\pi r_i^2$

$$= \frac{e\omega_i}{2\pi}\pi r_i^2$$

$$= \frac{1}{2}\left(\frac{e}{m}\right)\underbrace{(m\omega_i r_i^2)}_{=L_i}$$

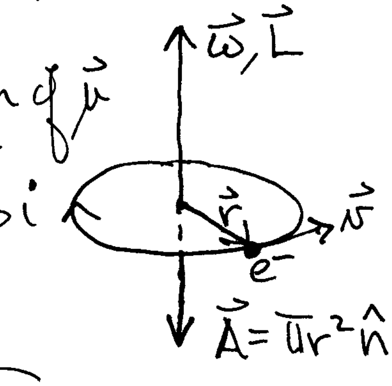
$$\mu_i = \frac{1}{2}\frac{e}{m}L_i$$

$\vec{L}_i$ = orbital $\measuredangle$ momentum of i^{th} electron

$$\left(\vec{L} = \vec{r}\times\vec{p} = m\vec{r}\times\vec{v} = m\vec{r}\times(\vec{\omega}\times\vec{r}) = mr^2\vec{\omega}\right)$$

direction of $\vec{\mu}$ opposite $\vec{L}$

direction of positive charge flow!



$\vec{\mu} = i\vec{A}$

$$\boxed{\vec{\mu}_i = -\frac{e}{2m}\vec{L}_i}$$

VI.D1) Total magnetic moment due to orbital motion of electrons

$$\vec{\mu}_L = -\frac{e}{2m} \underbrace{\sum_i \vec{L}_i}_{\equiv \vec{L}}$$

$$\boxed{\vec{\mu}_L = -\frac{e}{2m} \vec{L}}$$

a) Bohr magneton: $\mu_B \equiv \frac{e\hbar}{2m} \left(= 9.27 \times 10^{-2} \frac{J}{T} \right)$

2) Electrons have intrinsic spin angular momentum $\vec{S}$ also which gives rise a spin magnetic dipole moment

$$\vec{\mu}_{s_i} = -\frac{eg}{2m} \vec{S}_i \qquad g \approx 2 \text{ for } e^- \quad \text{"g-factor"}$$

Total magnetic moment due to spin of electrons

$$\vec{\mu}_s = -\frac{eg}{2m} \underbrace{\sum_i \vec{S}_i}_{\equiv \vec{S}}$$

$$\boxed{\vec{\mu}_s = -\frac{eg}{2m} \vec{S}}$$

VI.) D2) Hence the <u>total atomic magnetic moment</u> is the vector sum of the orbital and spin moments

$$\vec{\mu} = -\frac{e}{2m}\underbrace{(\vec{L} + g\vec{S})}_{\neq\ \vec{J}\ =\ \text{total} \measuredangle\ \text{momentum of atom}\ =\ \vec{L}+\vec{S}}$$

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3) So if $\vec{L} = 0 = \vec{S} \Rightarrow \vec{\mu} = 0 \Rightarrow \vec{M} = 0$ hence there is no net magnetization of the material. This is necessary for a diamagnetic material ~~+~~ we must still show opposite **to** $\vec{B}_o$ direction for the induced $\vec{\mu}$.

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a) Lenz's Law: place atom in $\vec{B}$ field $\Rightarrow$ flux through atomic orbits <u>increases</u>, Lenz's law implies atomic motion will change to <u>oppose</u> this increase in flux (i.e. e^- speed up or slow down), the atomic dipole moments will change (induced) to oppose the external $\vec{B}$-field $\Rightarrow$ repulsion $\Rightarrow$ diamagnetic material

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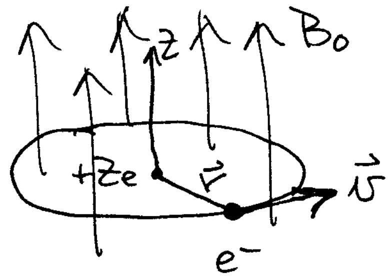
VI D3.b) Quantatative argument: Place atom in an external $\vec{B}$-field along the z-direction



Total force on electron

$$\vec{F} = \underbrace{-e\vec{v}\times\vec{B}_0}_{\substack{\text{magnetic}\\ \text{force}\\ \text{due to } \vec{B}_0}} - \underbrace{\frac{Ze^2\hat{r}}{4\pi\epsilon_0 r^2}}_{\substack{\text{Coulomb force}\\ \text{attraction of}\\ \text{nucleus}}}$$

circular motion ($\vec{v} = \vec{\omega}\times\vec{r}$)
$\Rightarrow$

attractive force

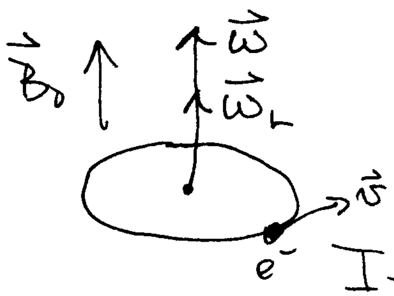
$$\vec{F} = -\left[e\omega r B_0 + \frac{Ze^2}{4\pi\epsilon_0 r^2}\right]\hat{r}$$

Newton's 2nd law $\Rightarrow$ $-mr\omega^2 = F = -\left[e\omega r B_0 + \frac{Ze^2}{4\pi\epsilon_0 r^2}\right]$

$\Rightarrow$ quadratic equation fore ω

$$\omega \approx \frac{e}{2m}B_0 \pm \underbrace{\frac{e}{2m}\sqrt{\frac{Zm}{\pi\epsilon_0 r^3}}}_{\equiv \omega_0}$$

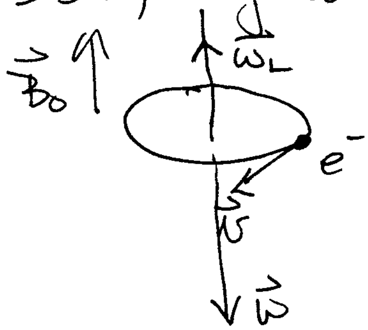
$\pm$ Cases: 1) Positive Root:



$$\omega = \omega_0 + \underbrace{\frac{e}{2m}B_0}_{\equiv \omega_L = \text{Larmor } \cancel{4} \text{ frequency}}$$

If $\vec{B}_0 \parallel \vec{\omega}_0$ then <u>ω increases</u> to $\vec{\omega} = (\omega_0 + \omega_L)\hat{k}$ $\Rightarrow$ <u>$\vec{L}$ increases</u> since $\vec{L} = mr^2\vec{\omega}$ <u>BUT</u> $\vec{\mu}_i = -\frac{e}{2m}\vec{L}_i$ so <u>$\vec{\mu}_i$ decreases</u> $\Rightarrow$ <u>$\vec{\mu}_{induced}$ opposite $\vec{B}_0$</u>

VI D3b2) Negative Root: $\vec{\omega} = -\omega_0 \hat{k} + \omega_L \hat{k}$



$\omega = |\vec{\omega}| = |\omega_0 - \omega_L|$ <u>decreases</u>

Since $\vec{\omega}_{induced} = \omega_L \hat{k}$

$\Rightarrow$ induced $\vec{L}_{induced} = mr^2 \vec{\omega}_{induced}$ is along $\vec{B}_0$

$\Rightarrow$ <u>induced magnetic moment opposite $\vec{B}_0$</u>

Both cases $\Rightarrow$

$$\boxed{\vec{\mu}_{induced}} = -\frac{e}{2m}\vec{L}_{induced}$$

$$= -\frac{e^2 r^2}{4m}\vec{B}_0$$

Now Z electrons with different orbits - we must average for total induced moment — need QM results is

$$\boxed{\vec{\mu} = \langle \vec{\mu}_{induced} \rangle_{QM} = -\frac{Ze^2}{6m} r_0^2 \vec{B}_0}$$

with r_0^2 = mean square radius of atom

Hence

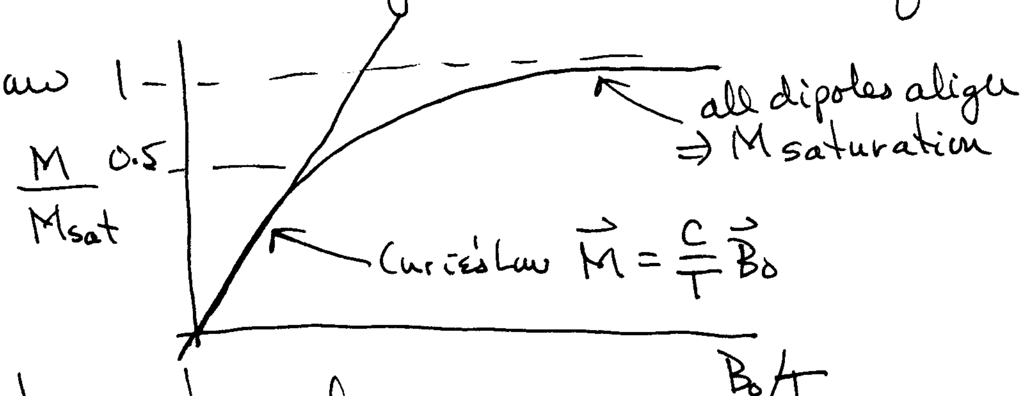
$$\boxed{\vec{M} = -\left[\frac{NZe^2 r_0^2}{6m}\right]\vec{B}_0}$$

$\vec{M}$ is opposite direction of $\vec{B}_0 \Rightarrow$ <u>decreases resultant field</u>

VI D 3b) So this is diamagnetic material.

4) Paramagnetic material: atomic magnetic moments do not sum to zero ⇒ molecules have a permanent magnetic dipole moment — these tend to align in external $\vec{B}$-field

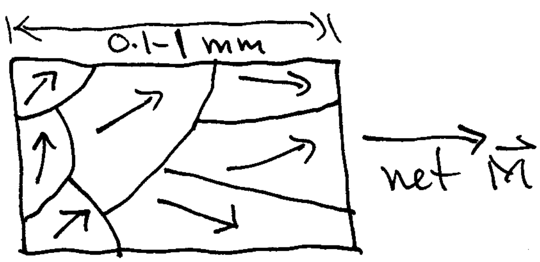
Curie's Law



Paramagnetic materials

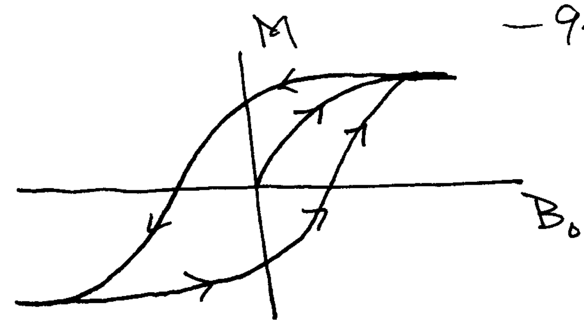
$$\vec{M} = \left[\frac{C}{T} - \frac{NZe^2 r_0^2}{6m}\right]\vec{B}_0 \approx \frac{C}{T}\vec{B}_0$$

5) Ferromagnetic material: Large paramagnetic effects — electron spin magnetic moment and strong interactions between neighboring atoms ⇒ alignment of dipoles even after external $\vec{B}$-field is removed



Macroscopic regions of aligned moments called domains

VI D5) Hysteresis Curve

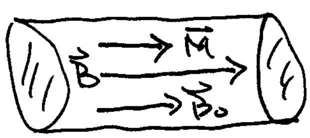


E) Macroscopic Fields & the Magnetization Current

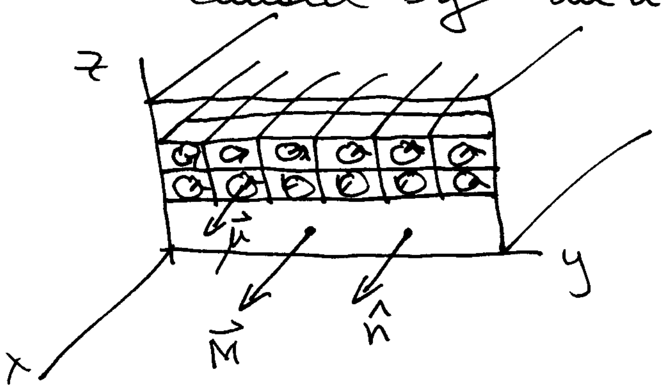
$$\vec{B} = \vec{B}_0 + \vec{B}_M$$

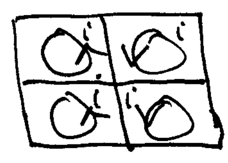
Total — external — due to Magnetization $\vec{M}$ of material

1) Consider solenoid containing magnetic material



$\vec{M}\Delta V = \vec{\mu}$ each magnetic moment $\vec{\mu}$ is viewed as caused by an atomic current loop





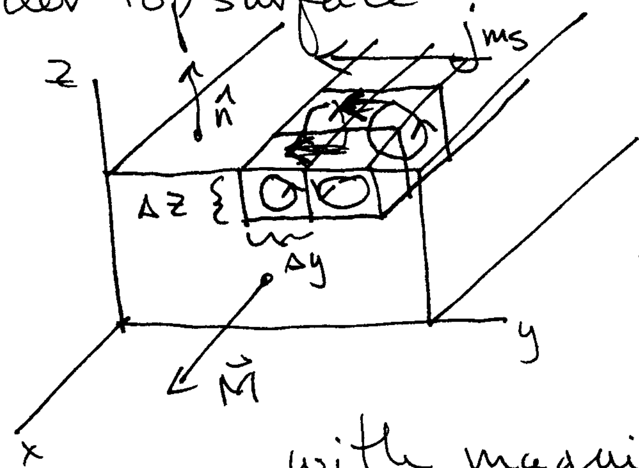
adjacent current loops flow in opposite direction — no net flow

So the current density flowing in a macroscopically thin layer of this surface will be zero

$$\boxed{\vec{j}_{M_s} = 0 \text{ when } \vec{M} \parallel \hat{n}}$$

But

VII E) Consider top surface



A surface current density appears to flow across top surface in the direction $\vec{M} \times \hat{n}$

with magnitude

$$j_{Ms} = \frac{i}{\Delta x}$$

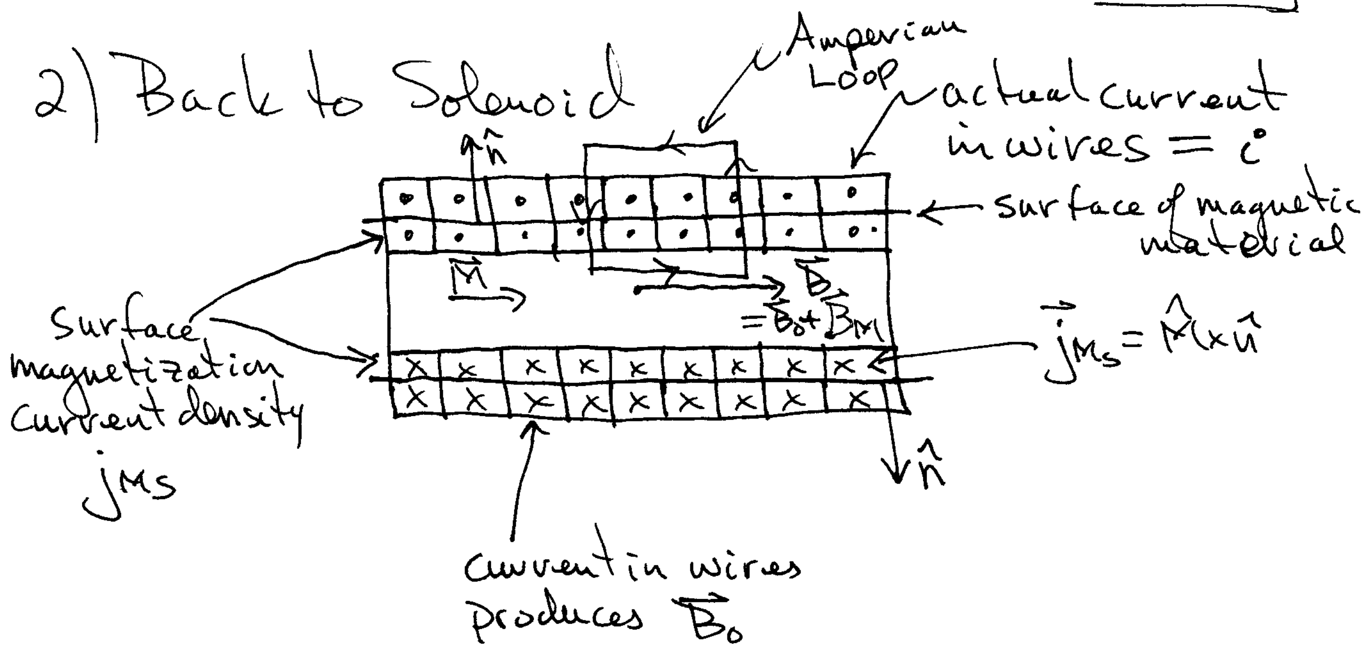
But $\mu = iA = i\Delta y \Delta z = M\Delta V = M\Delta x \Delta y \Delta z$

$$\Rightarrow M = \frac{i}{\Delta x} = j_{Ms}$$

$$\Rightarrow \boxed{\vec{j}_{Ms} = \vec{M} \times \hat{n}}$$

<u>The magnetization current density</u>

2) Back to Solenoid



VII) E2) Ampere's Law: $\oint_C \vec{B}\cdot d\vec{s} = \mu_0 i_{enclosed}$

$$\oint_C \vec{B}\cdot d\vec{s} = \mu_0 i_{enclosed}$$
$$\parallel$$
$$Bh = \mu_0\left[\frac{Nh}{L} i + j_{Ms} h\right]$$
$$= \mu_0 \frac{Nh}{L} i + \mu_0 M h$$

$$\Rightarrow \quad \boxed{B = \underbrace{\mu_0 \frac{N}{L} i}_{=B_0} + \underbrace{\mu_0 M}_{=B_M}} \qquad \text{i.e. } \vec{B}_M = \mu_0 \vec{M}$$

So in general Ampere's law can be written

$$\oint_C \vec{B}\cdot d\vec{s} = \mu_0 i_{free} + \mu_0 \underbrace{\oint_C \vec{M}\cdot d\vec{s}}_{=i_M}$$

conduction current

magnetization current

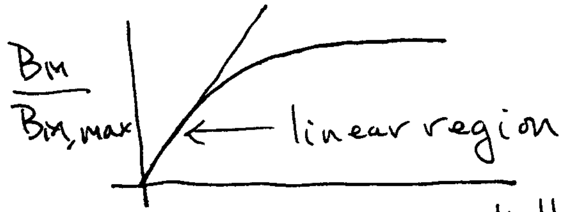
That is defining the magnetic intensity field

$$\vec{H} \equiv \frac{1}{\mu_0}(\vec{B} - \mu_0 \vec{M}) \quad \left(= \frac{1}{\mu_0}\vec{B}_0\right)$$

Ampere's law can be written as

$$\boxed{\oint_C \vec{H}\cdot d\vec{s} = i_{\substack{conduction \\ (free)}}}$$

VI) F) Constitutive Equation: For paramagnetic and diamagnetic materials that are isotropic and linear



$$\boxed{\vec{M} = \chi_M \vec{H}}$$ linear, isotropic magnetic material.

$\chi_M \equiv$ magnetic susceptibility

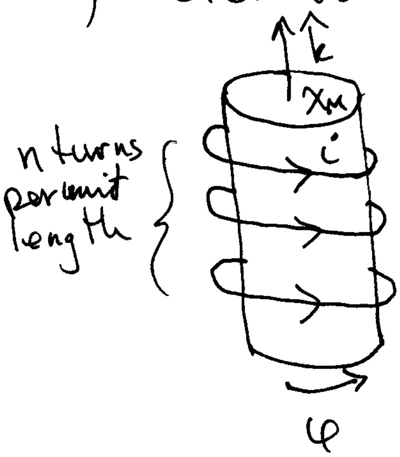
But
$$\vec{B} = \vec{B}_0 + \mu_0 \vec{M} = \mu_0 \vec{H} + \mu_0 \vec{M}$$
$$= \mu_0 (1 + \chi_M) \vec{H} \equiv \boxed{\mu \vec{H} = \vec{B}}$$
for linear, isotropic magnetic material.

$\mu \equiv \mu_0 (1 + \chi_m) =$ permeability of material

(Other notation: $K_M = \mu_r \equiv \frac{\mu}{\mu_0}$ = relative permeability)

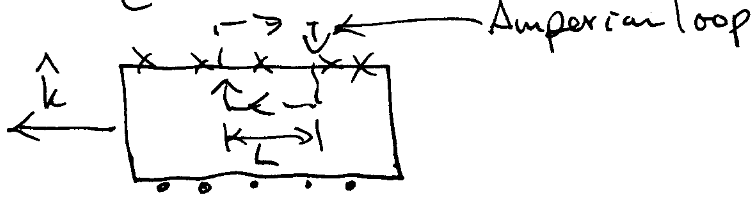
VI G) Examples

1) Solenoid filled with magnetic material



Ampere's law

$$\oint_C \vec{H} \cdot d\vec{s} = i_{free}$$



$$\Rightarrow \quad HL = inL \quad \Rightarrow \quad \vec{H} = in\,\hat{k}$$

$$\Rightarrow \quad \boxed{\begin{aligned} \vec{B} &= \mu_0(1+\chi_m)\vec{H} \\ &= \mu_0(1+\chi_m)\, in\,\hat{k} \end{aligned}}$$

a) If material is <u>paramagnetic</u> $\chi_M > 0 \Rightarrow B > B_0$
if <u>diamagnetic</u> $\chi_M < 0 \Rightarrow B < B_0$.

b) The magnetization current density runs around cylinder's surface

$$\vec{j}_{Ms} = \vec{M} \times \hat{n} = \chi_M \vec{H} \times \hat{n}$$

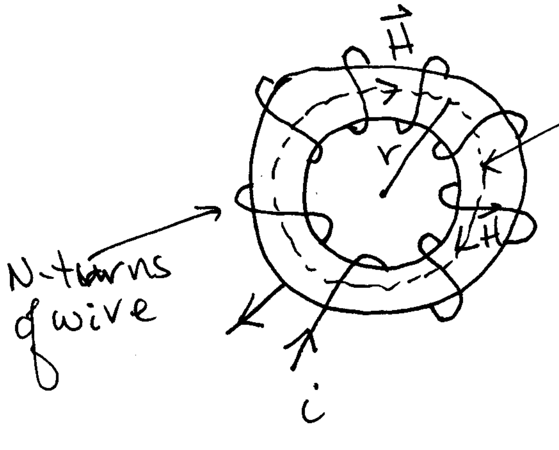
$$= \chi_M\, in\, \underbrace{\hat{k} \times \hat{n}}_{= \hat{\varphi}}$$

$$\boxed{\vec{j}_{Ms} = \chi_M\, in\, \hat{\varphi}}$$

$\chi_M > 0$: $\vec{j}_{Ms}$ is same direction as i

$\chi_M < 0$: $\vec{j}_{Ms}$ opposite i

-VII. G.) 2) Toroid with magnetic material



(Note: $H = 0 = B$ outside toroidal Volume)

Ampere's Law

$$\oint_C \vec{H} \cdot d\vec{s} = i_{free}$$

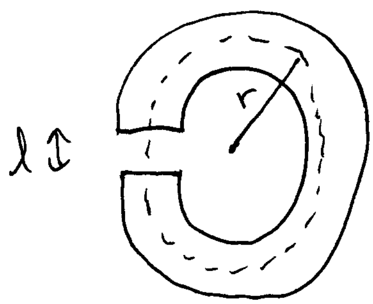
$$H 2\pi r = Ni$$

$$\Rightarrow H = \frac{Ni}{2\pi r}$$

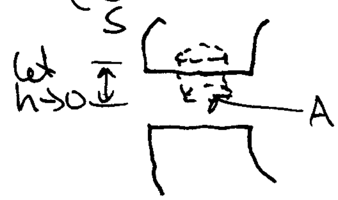
But

$$\vec{B} = \mu \vec{H} \Rightarrow B = \frac{\mu N i}{2\pi r} = (1 + \chi_m) \frac{\mu_0 N i}{2\pi r}$$

3) Torus with a gap



$B_{in} = B_{gap}$ $\left(\oint_S \vec{B} \cdot d\vec{A} = 0 = B_{in} A - B_{gap} A \right.$



$\left. \Rightarrow B_{in} = B_{gap} \right)$

So $B_{in} = B_{gap} \equiv B(r)$

But $H_{inside} = \frac{B}{\mu}$; $H_{gap} = \frac{B}{\mu_0}$

Now apply Ampere's Law

$$\oint_C \vec{H} \cdot d\vec{s} = i_{free} = Ni$$

$$\parallel$$

$$H_{in}(2\pi r - l) + H_{gap} l = \frac{B}{\mu}(2\pi r - l) + \frac{B}{\mu_0} l$$

- VI.G) 3) $\Rightarrow$ $$\boxed{B = \frac{\mu\mu_0 N i}{\mu l + \mu_0(2\pi r - l)}}$$

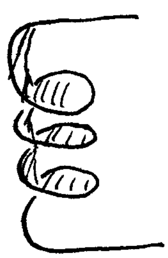
for ferromagnetic materials $\mu l >> 2\pi r$
$(\mu = \mu(H))$ $\quad B \simeq \frac{\mu_0 N i}{l} >> \frac{\mu_0 N i}{2\pi r}$

VII) Inductance:

A) Self inductance: Faraday's law $\mathcal{E}_L = -\frac{d\phi_B}{dt}$
$$= -L\frac{di}{dt}$$
$$L = \frac{d\phi_B}{di} = \frac{\phi_B}{i} \text{ for linear media.}$$

1) SI unit of inductance = henry $(H) \equiv 1 \frac{\text{Volt}\cdot\text{sec.}}{\text{Ampere}}$

2) Solenoid: a) flux linkages = total flux thru solenoid.



$$\phi_B = nl\,\phi_{B\,coil} = nl\,B\pi r^2$$
$$= \mu_0 n^2 l A i$$
$$\Rightarrow L = \mu_0 n^2 l A$$

b) Voltage across each coil $= \mathcal{E}_{coil}$

$$\mathcal{E}_{coil} = -\frac{d\phi_{B\,coil}}{dt} = -\mu_0 n\pi r^2\frac{di}{dt}$$

But there are nl-coils $\Rightarrow \mathcal{E}_L = nl\cdot\mathcal{E}_{coil}$
$$\Rightarrow L = \mu_0 n^2 l A$$
