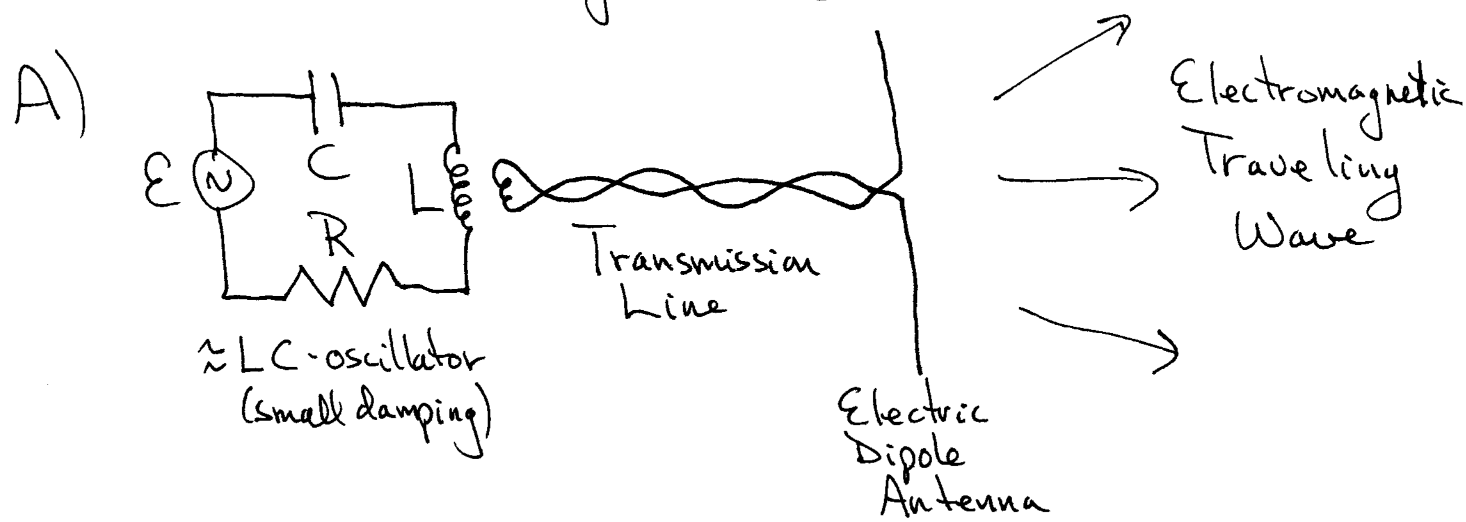
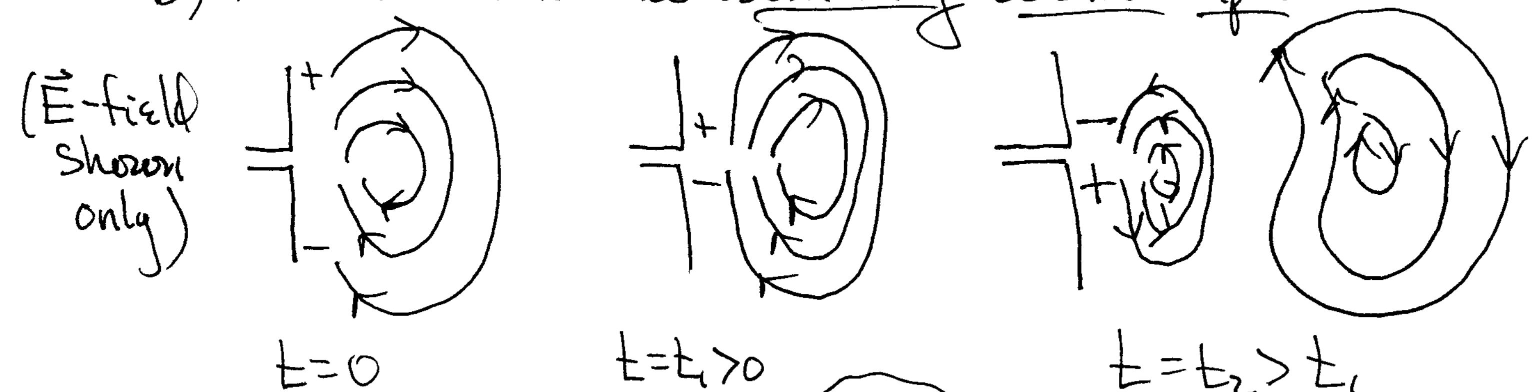
IX) Electromagnetic Radiation: Charges at rest or moving at constant velocity do not radiate.

Accelerated charges radiate

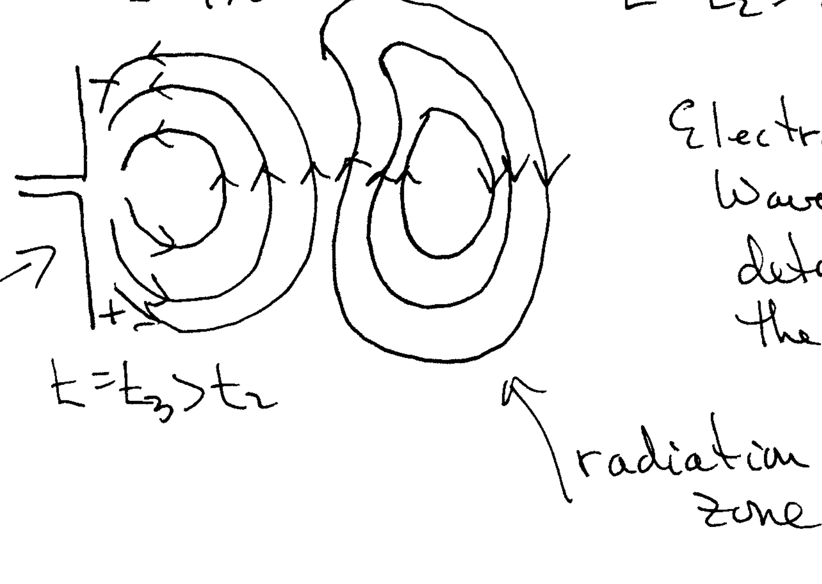
A)



B) Antenna acts like oscillating electric dipole moment

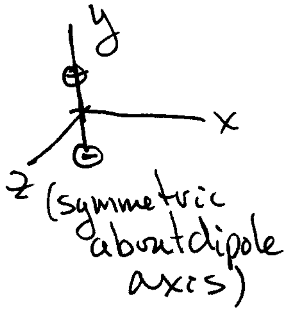


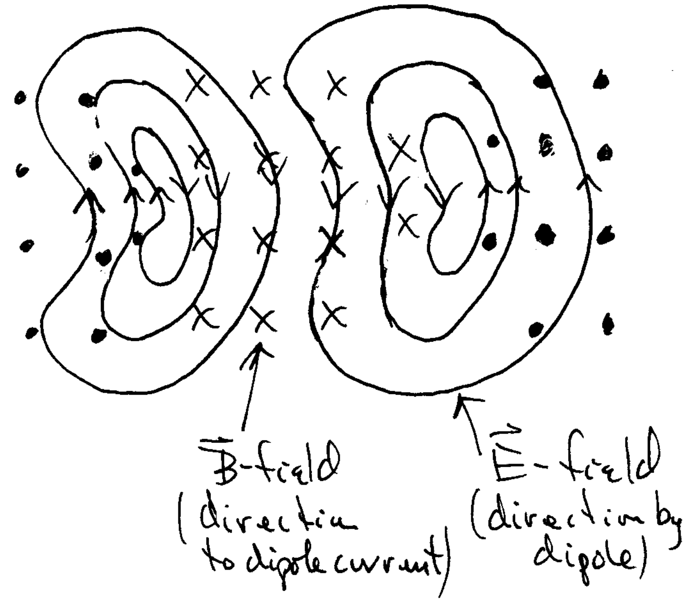
$$\left(\begin{array}{l} q = q_0 \cos\omega t \\ p = qd \\ i = \dot{q} = \frac{\dot{p}}{d} \end{array} \right)$$



Electromagnetic Wave becomes detached from the charges

IX) C) Closer look at wave:





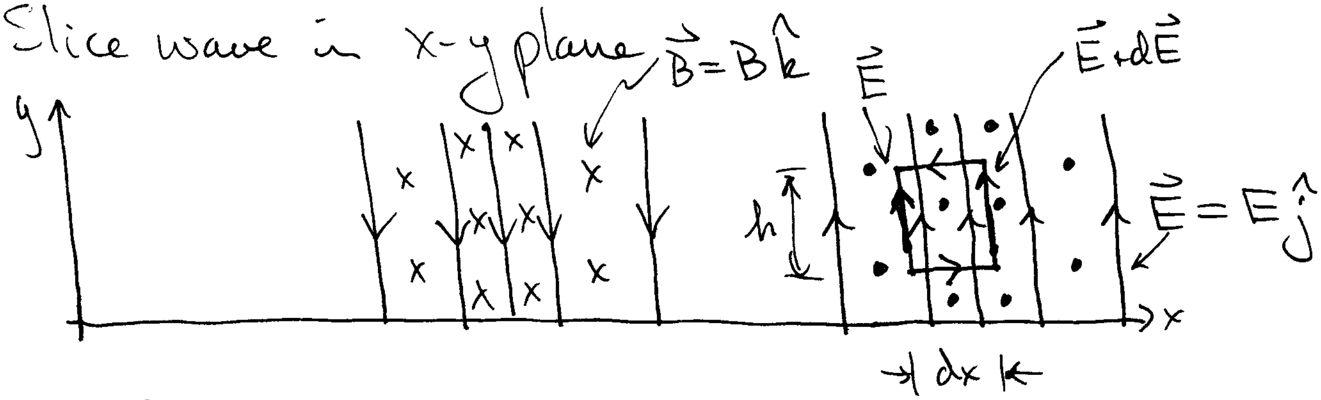
D) Maxwell's Equations applied to wave in Rad. zone

$$\oint_C \vec{E}\cdot d\vec{s} = -\int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{A} \quad : \text{Faraday's law}$$

$$\oint_C \vec{B}\cdot d\vec{s} = \epsilon_0 \mu_0 \int_S \frac{\partial \vec{E}}{\partial t}\cdot d\vec{A} \quad : \text{Ampere's law}$$

(far from dipole in Radiation Zone)

1) Slice wave in x-y plane



Faraday's Law

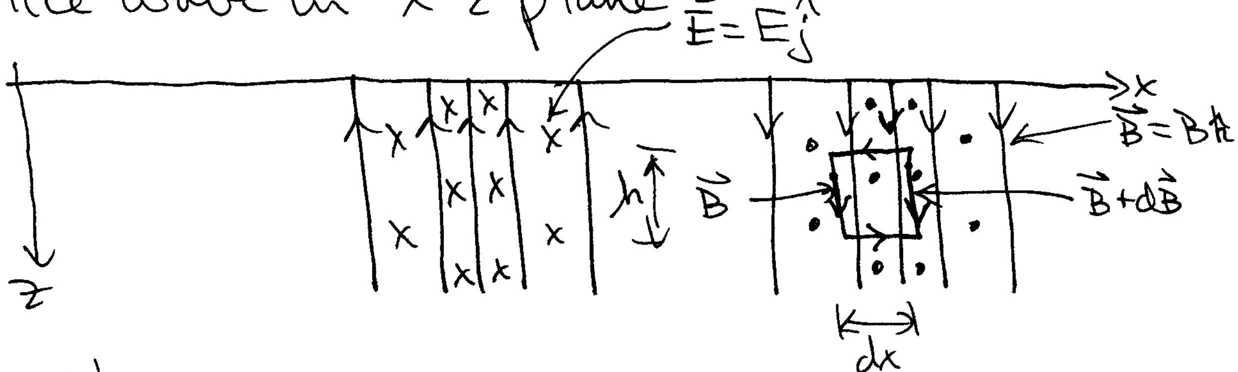
$$\oint_C \vec{E}\cdot d\vec{s} = (E+dE)h - (E)h = dE\,h$$

$$= -\frac{\partial B}{\partial t}(h\,dx)$$

IX. D)1) ⟹

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

2) Slice wave in x-z plane



Ampere's Law:

$$\oint_C \vec{B}\cdot d\vec{s} = -(B+dB)h + Bh = -h\,dB$$

$$= \epsilon_0\mu_0 \frac{\partial E}{\partial t}(h\,dx)$$

$$\Rightarrow \quad \frac{\partial B}{\partial x} = -\epsilon_0\mu_0 \frac{\partial E}{\partial t}$$

E) Wave Equation: uncouple the above by differentiating again ⟹

$$\left[\epsilon_0\mu_0 \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right] E(x,t) = 0$$

$$\left[\epsilon_0\mu_0 \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right] B(x,t) = 0$$

IX. E) General Solution to wave equation

$$E(x,t) = E(x - v t)$$
$$B(x,t) = B(x - v t)$$

$$\Rightarrow \quad v = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \equiv c$$

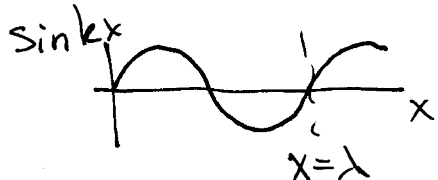
All electromagnetic waves travel at speed c!

and $\left[\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} \right] \begin{matrix} E \\ \text{or} \\ B \end{matrix} = 0$

F) Plane Waves: Simplest Solution

$$E(x,t) = E_m \sin(kx - \omega t)$$
$$B(x,t) = B_m \sin(kx - \omega t)$$

Wave Equation $\Rightarrow \frac{\omega}{k} = c$



k = wave number

$kx = 2\pi$ gives zeroes of $\sin kx$ ($x = \lambda$)

λ is called wave length $\quad k\lambda = 2\pi$

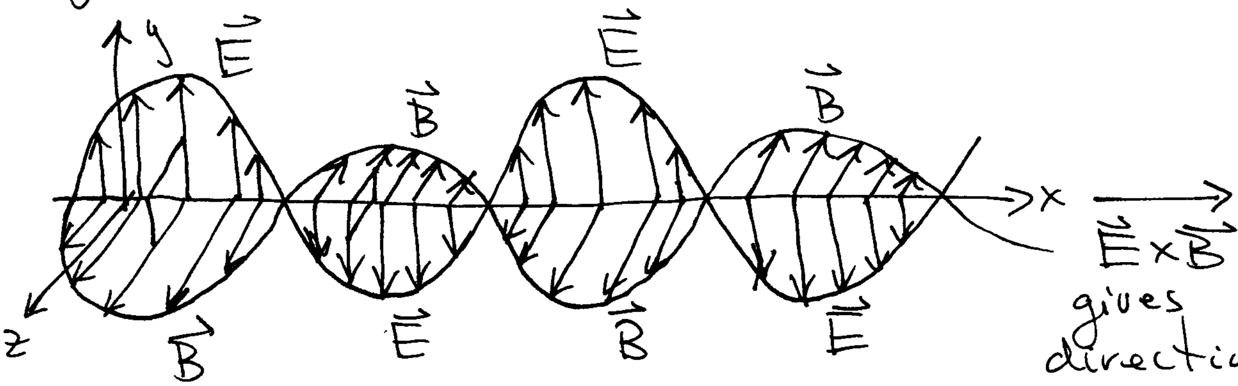
$$\Rightarrow \quad k = \frac{2\pi}{\lambda}$$

So $\quad \frac{\omega}{k} = c = \frac{\omega}{2\pi} \frac{2\pi}{k} = \nu \lambda$

IX. G) Linearly Polarized: $\vec{E} = E\hat{j}$
$\vec{B} = B\hat{k}$

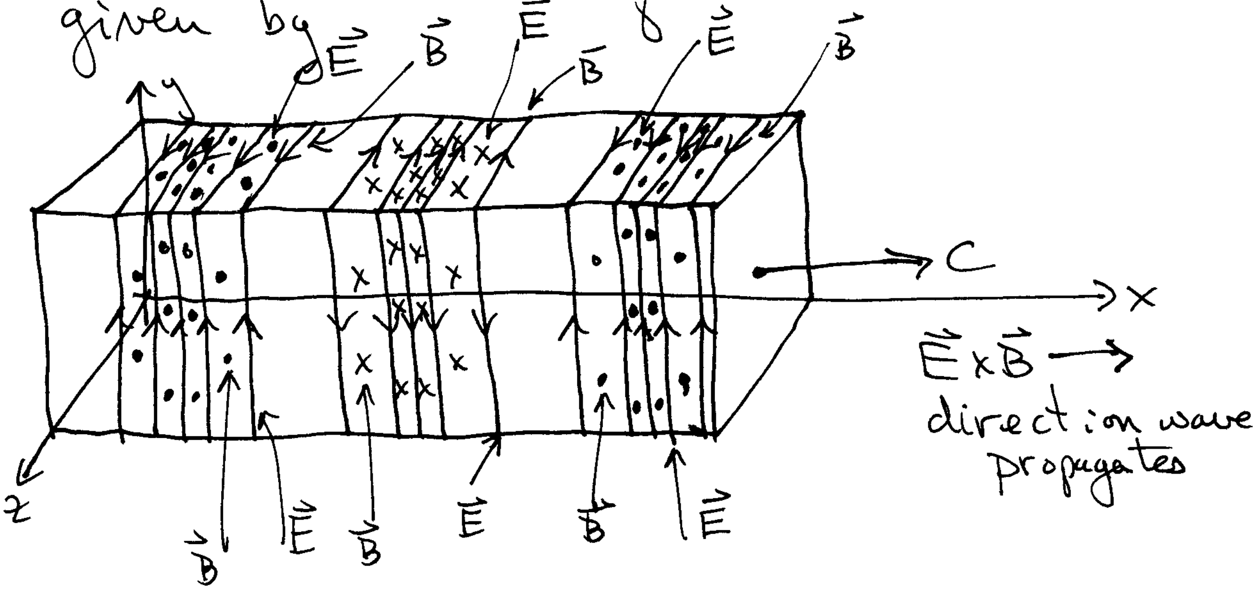
$\vec{E}$ oscillates along one direction only, as does $\vec{B}$.

For fixed t this looks like



$\vec{E}\times\vec{B}$ gives direction wave propagates.

Another representation of the wave is given by



IX. H) Maxwell's Equations: $\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$: Faraday's Law

$\frac{\partial B}{\partial x} = -\frac{1}{c^2}\frac{\partial E}{\partial t}$: Ampere's Law

⇒ 1) E & B must be in phase

2) $\boxed{E = cB}$

I) Energy Transport: The Poynting Vector

$$\vec{S} = \vec{E} \times \vec{H}$$

$$= \frac{1}{\mu_0} \vec{E} \times \vec{B} \quad \text{in vacuum}$$

$\vec{S}$ = rate of energy flow per unit area in the direction of $\vec{S}$

= electromagnetic power per unit area.

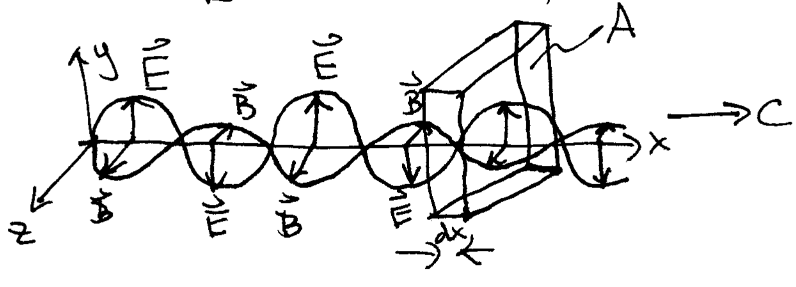
1) For above plane wave $\vec{S} = \frac{1}{\mu_0} EB\,\hat{i} \equiv S\,\hat{i}$

↑ gives direction of propagation

2) $dU = dU_E + dU_B = (u_E + u_B)(A\,dx)$

$u_E = \frac{1}{2}\epsilon_0 E^2$

$u_B = \frac{1}{2\mu_0} B^2$



IX.I.) 2) $dU = \frac{EBA}{\mu_0}\frac{dx}{c}$ but $dt = \frac{dx}{c}$

This is amount of energy that passes through the box in time dt, & the energy flow per unit time thru area A : $\frac{dU}{dt} = \frac{1}{\mu_0}EBA = S \cdot A$

So $S = \frac{dU}{dt\,A} = \frac{1}{\mu_0}EB$ is the energy flow per unit time per unit area.

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3) Intensity $I = \bar{S}$ = time average of S

$$I = \bar{S} = \frac{1}{2\mu_0}E_m B_m$$

$$= \frac{1}{\mu_0}E_{rms}B_{rms}$$