III) Magnetic Field:

A) The magnetic field $\vec{B}$ is defi ed by the force law

$$\vec{F} = q\vec{v} \times \vec{B}$$

1) SI units: $[\vec{B}]$ = tesla (T)

$$= 1 \frac{\text{newton}}{\text{Ampere-meter}}$$

(non-SI units: gauss(G) $= 10^{-4}$ T)

2) $\vec{F} \perp \vec{v}$: magnetic force does no work only change direction of $\vec{v}$, not magnitude.

3) Lorentz Force Law: total force on charge q in $\vec{E}$- and $\vec{B}$-fields

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

(This defines $\vec{B}$ and $\vec{E}$)

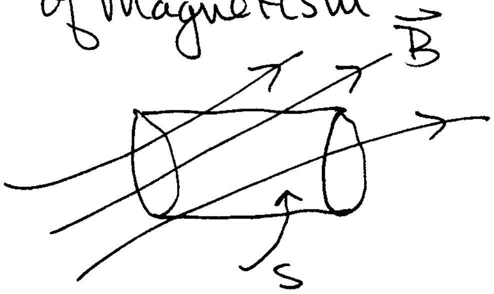
4) No Magnetic Charges: experimental result

Lines of $\vec{B}$ are continuous

III A4) Hence Flux of $\vec{B}$ through any closed surface is zero

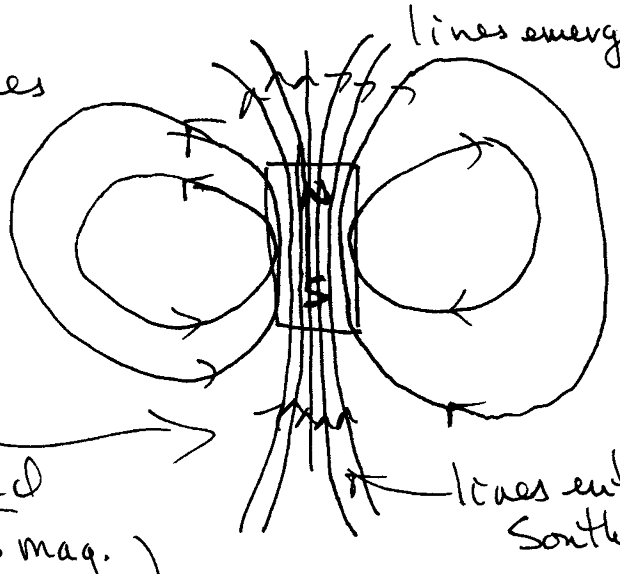
Fundamental Law of Magnetism

$$\boxed{\oint_S \vec{B}\cdot d\vec{A} = 0}$$



5) $\vec{B}$-lines

lines emerge from North pole

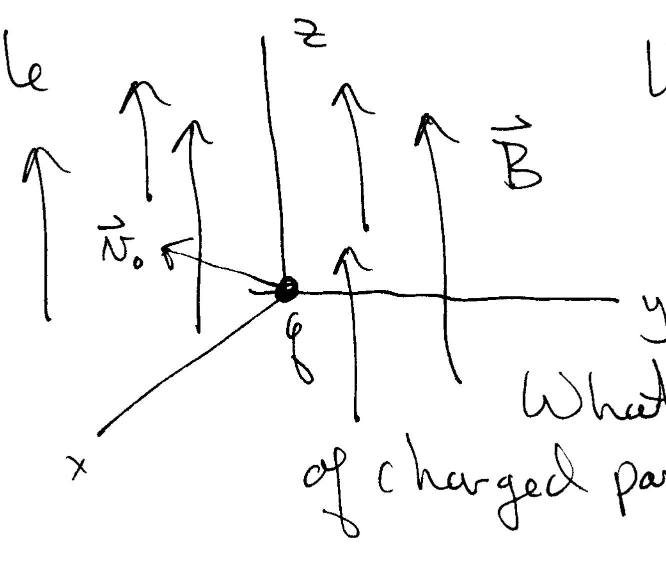


Opposite poles attract
Like poles repel

$\vec{B}$-lines form closed loops (no mag. charges)

lines enter South pole

B) Example



Uniform $\vec{B}$-field in z-direction
$\vec{B} = B\hat{k}$

What is trajectory of charged particle?

III B) Particle has charge q, mass m and at $t=0$ passes through origin with velocity $\vec{v}_0$

$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j} + z(t)\,\hat{k}$$

So initial conditions are
$$\left.\begin{aligned} \vec{r}(0) &= 0 \\ \frac{d\vec{r}}{dt}(0) &= \vec{v}_0 \end{aligned}\right\} \text{six conditions}$$

$$\Rightarrow \quad x(0) = 0 \qquad \frac{dx}{dt}(0) = v_{0x}$$
$$y(0) = 0 \qquad \frac{dy}{dt}(0) = v_{0y}$$
$$z(0) = 0 \qquad \frac{dz}{dt}(0) = v_{0z}$$

Newton's 2nd Law: $\vec{F} = m\vec{a}$

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = \frac{d^2x(t)}{dt^2}\,\hat{i} + \frac{d^2y(t)}{dt^2}\,\hat{j} + \frac{d^2z(t)}{dt^2}\,\hat{k}$$

and

$$\vec{F} = q\vec{v} \times \vec{B} = q \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{dx}{dt} & \frac{dy}{dt} & \frac{dz}{dt} \\ 0 & 0 & B \end{vmatrix}$$

$$= qB\left(\frac{dy}{dt}\,\hat{i} - \frac{dx}{dt}\,\hat{j}\right)$$

III B) Newton's Law $\Rightarrow$

$$\boxed{\begin{aligned} m\frac{d^2x(t)}{dt^2} &= qB\frac{dy(t)}{dt} \\ m\frac{d^2y(t)}{dt^2} &= -qB\frac{dx(t)}{dt} \\ m\frac{d^2z(t)}{dt^2} &= 0 \end{aligned}}$$

(see class notes)
Solve $\Rightarrow$

$$x(t) = \frac{v_{oy}}{\omega} + \frac{v_{ox}}{\omega}\sin\omega t - \frac{v_{oy}}{\omega}\cos\omega t$$

$$y(t) = -\frac{v_{ox}}{\omega} + \frac{v_{oy}}{\omega}\sin\omega t + \frac{v_{ox}}{\omega}\cos\omega t$$

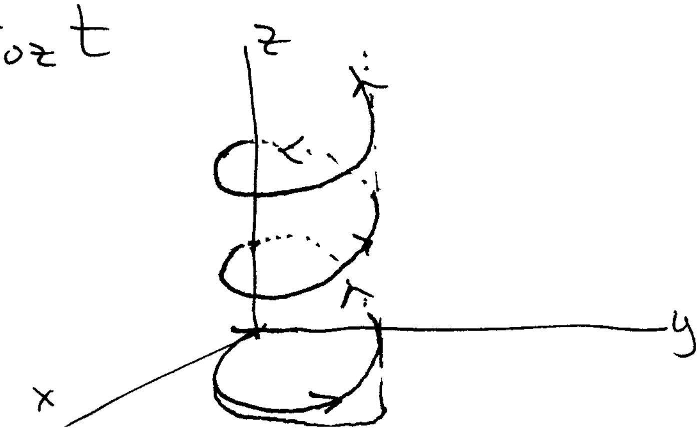
$$z(t) = v_{oz}t$$

where $\omega \equiv \frac{qB}{m}$

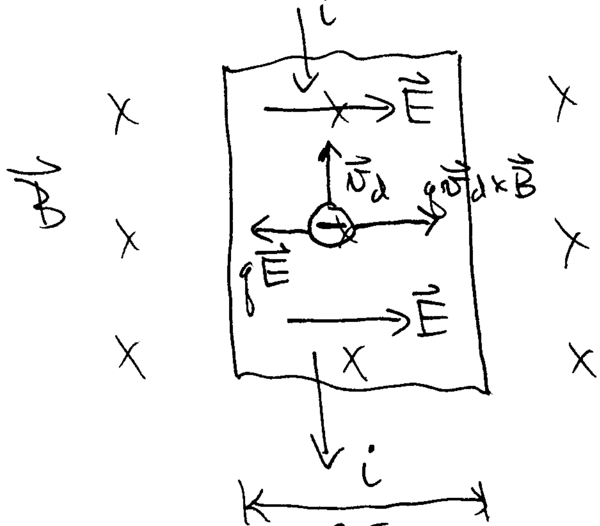
Trajectory is a helix:

$$\left(x(t) - \frac{v_{oy}}{\omega}\right)^2 + \left(y(t) + \frac{v_{ox}}{\omega}\right)^2 = \left[\frac{v_{ox}^2 + v_{oy}^2}{\omega^2}\right]$$

$$z(t) = v_{oz}t$$



III C) Hall Effect:



current carrying strip of thickness t

Potential difference across strip = Hall Voltage = $V = Ew$

At equilibrium

$$0 = q\vec{E} + q\vec{v}_d \times \vec{B}$$

$$\Rightarrow E = v_d B$$

Recall $j = nev_d = \frac{i}{A} = \frac{i}{wt}$

So

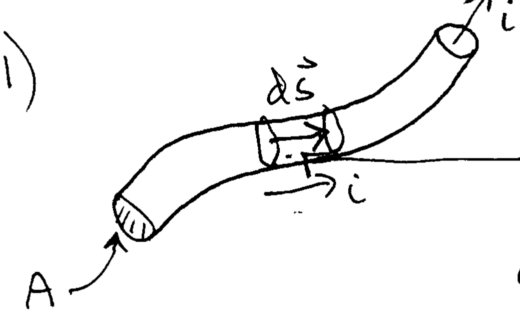
$$E = \frac{V}{w} = v_d B = \frac{j}{ne} B = \frac{iB}{newt}$$

$$\Rightarrow \boxed{n = \frac{iB}{etV}}$$ density of charge carriers

Measure Hall Voltage → if positive charge carriers are +'ve
if negative charge carriers are -'ve.

III D) Magnetic Force on a Current

1)



charge $dq = nA|d\vec{s}|q$

$$d\vec{F} = dq\, \vec{v}_d \times \vec{B}$$
$$= nAq\, |d\vec{s}|\, \vec{v}_d \times \vec{B}$$

But $d\vec{s} \parallel \vec{v}_d \Rightarrow$

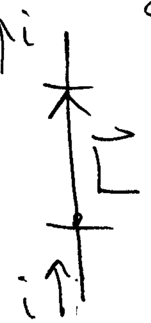
$$d\vec{F} = nA|\vec{v}_d|q\, d\vec{s} \times \vec{B}$$

Recall $i = nq|\vec{v}_d|A \Rightarrow$

$$\boxed{d\vec{F} = i\, d\vec{s} \times \vec{B}}$$ the

magnetic force exerted on wire segment $d\vec{s}$

2) Straight wire segment of length $\vec{L}$ in uniform $\vec{B}$



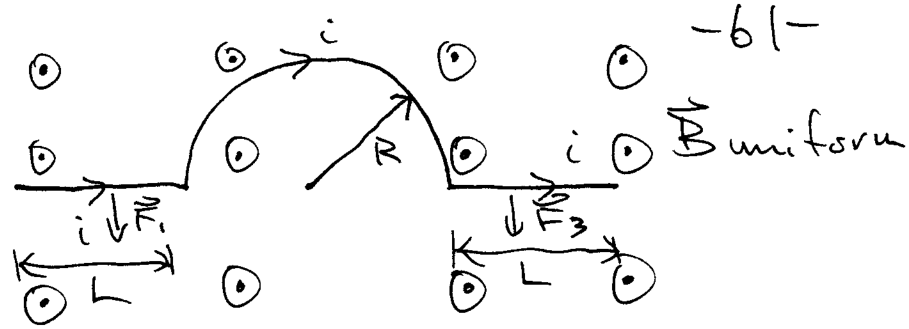
$$\vec{F} = \int d\vec{F} = i\left(\int_0^L d\vec{s}\right) \times \vec{B}$$
$$= i\vec{L} \times \vec{B}$$ a sideways force.

---

3) Closed Loop in a constant $\vec{B}$-field

$$\vec{F} = \oint_C d\vec{F} = i\left(\oint_C d\vec{s}\right) \times \vec{B} = 0.$$

III D) 4.) Example:



$F_1 = F_3 = iLB$ pointing down

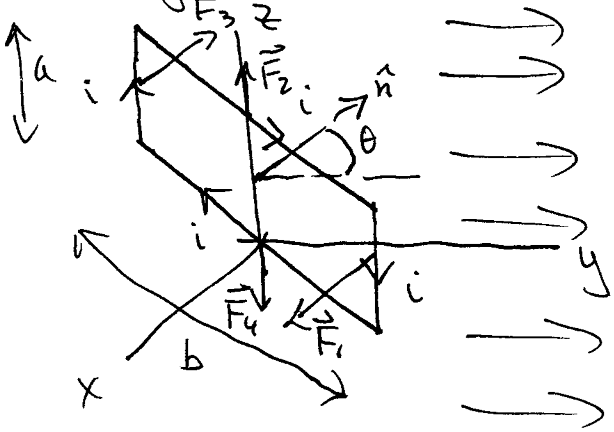
$$F_2 = \int_{\theta=0}^{\pi} dF \sin\theta = iBR \int_{\theta=0}^{\pi} \sin\theta \, d\theta$$

$$= 2iBR \text{ pointing down}$$

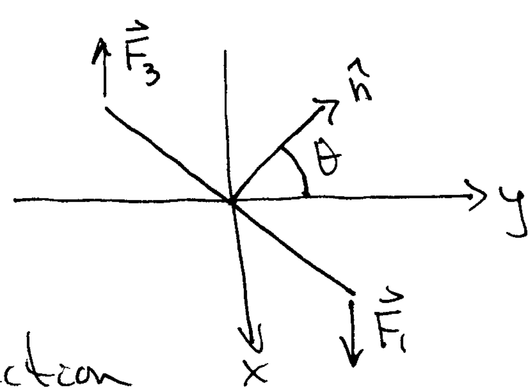
Total force $F = (2L + 2R)iB$ pointing down

E) Torque Due to Magnetic Field

1) Rectangular Current loop in uniform $\vec{B}$-field



$\vec{B} = B\hat{j}$



$\vec{F}_2 = ibB\cos\theta\,\hat{k} = -\vec{F}_4$

no torque since same line of action

III.E.1) $\vec{F}_1 = iaB\,\hat{i} = -\vec{F}_3$ produce torque since they act as a couple.

$$\vec{\tau} = \frac{b}{2} F_1 \sin\theta\,(-\hat{k}) + \frac{b}{2} F_3 \sin\theta\,(-\hat{k})$$

$$\vec{\tau} = iabB\sin\theta\,(-\hat{k})$$

$$\vec{\tau} = iAB\sin\theta\,(-\hat{k})$$ $A = ab =$ area of loop

For N-turns of wire

$$\vec{\tau} = NiAB\sin\theta\,(-\hat{k})$$ this result is true for any shaped planar area A.

The normal $\hat{n}$ to loop points in direction determined by right hand rule applied to direction of current flow about perimeter

Define Magnetic Dipole Moment

$$\boxed{\vec{\mu} = NiA\,\hat{n}}$$

Then

$$\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$$

III. E.2.) Potential energy of magnetic dipole in magnetic field $\vec{B}$

$$U(\theta) = -\vec{\mu}\cdot\vec{B}\ .$$

IV) Magnetostatics: (steady currents)

A) Biot - Savart Law

$$d\vec{B} = \frac{\mu_0}{4\pi}\,\frac{i\,d\vec{s}\times\vec{r}}{r^3}$$

Magnetic Field at point P due to $i\,d\vec{s}$ current element

(SI: $\mu_0 \equiv 4\pi\times10^{-7}\ \frac{T\cdot m}{A}$

μ_0 = permeability of free space)

$$\vec{B} = \int_C d\vec{B} = \frac{\mu_0}{4\pi}\int_C \frac{i\,d\vec{s}\times\vec{r}}{r^3}$$

Magnetic Field at point P due to current carrying wire C.