PHYSICS 271
ELECTRICITY AND MAGNETISM
FINAL EXAMINATION

16 December 1999

INSTRUCTIONS: Answer all questions on the answer sheet provided, it will be the only paper that is collected. This is a closed book exam.

1. Positive charge Q is distributed uniformly throughout an insulating sphere of radius R. The magnitude of the electric field at a point $R/2$ from the center is:

a) $\dfrac{Q}{4\pi\varepsilon_0 R^2}$

b) $\dfrac{Q}{8\pi\varepsilon_0 R^2}$

c) $\dfrac{Q}{16\pi\varepsilon_0 R^2}$

d) $\dfrac{3Q}{16\pi\varepsilon_0 R^2}$

2. Positive charge Q is distributed uniformly throughout a non-conducting sphere. The highest electric potential occurs

a) far from the sphere

b) at the surface

c) halfway between the center and surface

d) at the center

3. For the sphere in problem **2**, the highest potential is

a) $\dfrac{Q}{4\pi\varepsilon_0 R}$

b) $\dfrac{3Q}{8\pi\varepsilon_0 R}$

c) $\dfrac{-Q}{4\pi\varepsilon_0 R}$

d) $\dfrac{-3Q}{8\pi\varepsilon_0 R}$

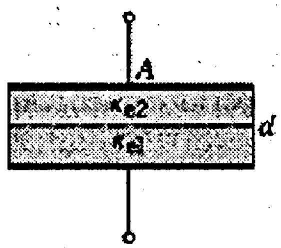
4. A parallel plate capacitor is filled with two dielectrics as shown below. The capacitance is given by

a) $C = \frac{\varepsilon_0 A}{d}\left(\frac{\kappa_{e1} + \kappa_{e2}}{2}\right)$

b) $C = \frac{2\varepsilon_0 A}{d}\left(\frac{\kappa_{e1}\kappa_{e2}}{\kappa_{e1} + \kappa_{e2}}\right)$

c) $C = \frac{\varepsilon_0 A}{d}\left(\frac{2}{\kappa_{e1} + \kappa_{e2}}\right)$

d) $C = \frac{\varepsilon_0 A}{d}\frac{1}{2}\left(\frac{1}{\kappa_{e1}} + \frac{1}{\kappa_{e2}}\right)$



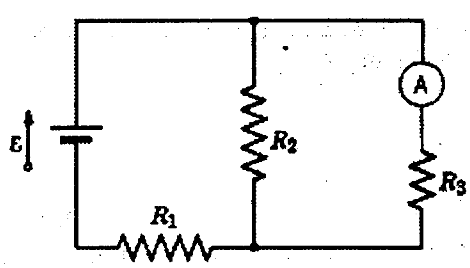
5. In the figure below imagine an ammeter inserted in the branch containing R_3. What will it read, assuming $\mathcal{E}=5.0V$, $R_1=2.0\ \Omega$, $R_2=4.0\ \Omega$, and $R_3=6.0\ \Omega$?

a) $\frac{3}{22}A$

b) $\frac{5}{22}A$

c) $\frac{5}{11}A$

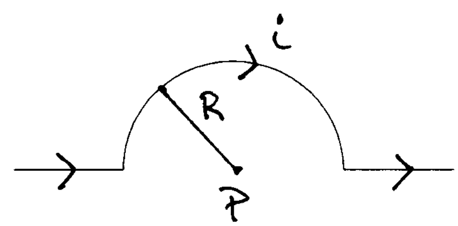
d) $\frac{1}{11}A$



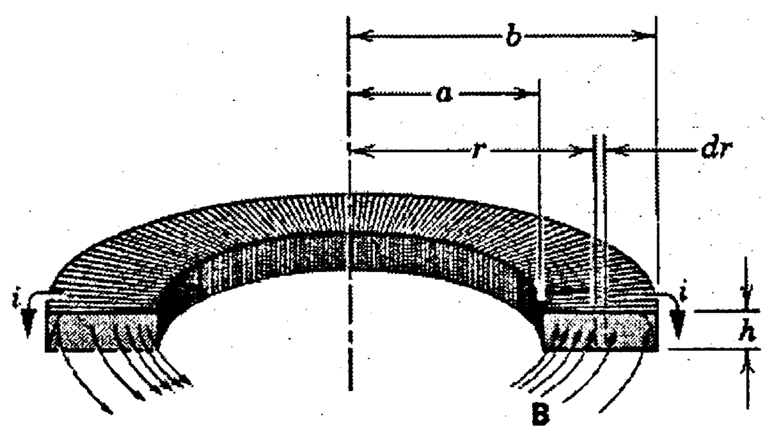
6. The magnitude of the magnetic field at point P, at the center of the semi-circle shown below is

a) $\frac{\mu_0 i}{R^2}$ d) $\frac{\mu_0 i}{4\pi R}$ e) $\frac{\mu_0 i}{4R}$

b) $\frac{\mu_0 i}{2\pi R}$ c) $\frac{\mu_0 i}{2R}$



For questions **7-12** a rectangular toroid, whose cross section is shown below, is considered. It has inner radius a and outer radius b with a thickness h with a total of N turns of wire carrying current i.



7. A toroid with a rectangular cross section carries current i. The magnetic field has its largest magnitude

a) at the center of the hole

b) just inside the toroid at its inner surface

c) just inside the toroid at its outer surface

d) at any point inside (the field is uniform)

8. The value of the magnitude of the magnetic field at radius r inside the toroid is

a) $\frac{\mu_0 iN}{h}$

b) $\frac{\mu_0 iN}{2\pi r}$

c) $\frac{\mu_0 iN}{4\pi h}\frac{b}{a}$

d) $\frac{\mu_0 iN}{\pi r^2}$

9. The inductance of the toroid is given by

a) $L = \mu_0 n^2 h \pi \left(b^2 - a^2\right)$

b) $L = \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right)$

c) $L = \frac{\mu_0 N^2 ab}{2\pi h}$

d) none of the above

10. If the toroid is filled with magnetic material of permeability μ, its inductance L as compared to its inductance L_0 without the material is

a) $L = \mu L_0$

b) $L = \frac{\mu}{\mu_0} L_0$

c) $L = \frac{1}{\mu} L_0$

d) $L = \chi_M L_0$

11. The energy density as a function of radial distance r for the unfilled (vacuum) toroid is

a) $u_B = \dfrac{\mu_0 i^2 N^2}{8\pi^2 r^2}$

c) $u_B = \left(\dfrac{\mu_0 iNh}{8\pi}\right)^2$

b) $u_B = \dfrac{\mu_0 iN}{4\pi}\ln\left(\dfrac{r}{h}\right)$

d) $u_B = (\mu_0 in)^2$

12. The total energy stored in the magnetic field of the unfilled (vacuum) toroid is

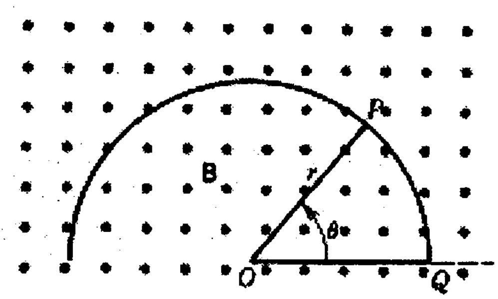
a) $U_B = i^2 Rh\pi(b^2 - a^2)$

c) $U_B = (\mu_0 in)^2 h\pi(b^2 - a^2)$

b) $U_B = \dfrac{1}{2}Li^2$

d) none of the above

13. A wire with cross sectional area A and resistivity ρ is bent into a circular arc of radius r as shown below. An additional straight length of this wire, OP, is free to pivot about O and makes sliding contact with the arc at P. Finally, another straight length of this wire, OQ, completes the circuit. The entire arrangement is located in a magnetic field of magnitude B directed out of the plane of the figure. The straight wire OP starts from rest with $\theta = 0$ and has a constant angular acceleration of α.



The resistance of the loop $OPQO$ as a function of θ is given by

a) R

c) $\dfrac{\rho r\theta}{A}$

b) $\dfrac{\rho r}{A}(2+\theta)$

d) $\dfrac{2\rho r}{A}$

14. Referring to problem **13** above, the magnetic flux through the loop as a function of θ is

a) $\Phi_B = \frac{1}{2} B r^2 \theta$

b) $\Phi_B = BA$

c) $\Phi_B = rBr\theta$

d) $\Phi_B = \frac{1}{2} B \pi r^2$

15. Referring to problem **13** above, the induced current, *i*, in the loop is

a) $i = \frac{\mathcal{E}}{R}$

b) $i = R\alpha t$

c) $i = \frac{1}{\sqrt{2}} \frac{ABr}{\rho(2+\theta)} \sqrt{\alpha \theta}$

d) $i = \frac{1}{2} r^2 B \alpha t$

EXTRA CREDIT QUESTION 1 (5 POINTS): Referring to problem **13**, for what value of θ is the induced current in the loop a maximum? (Just state the result on the answer sheet.)

16. In a LCR series circuit, which is connected to a source $E_M \cos(\omega t)$ the current lags the voltage by 45° if

a) $R = \frac{1}{\omega C} - \omega L$

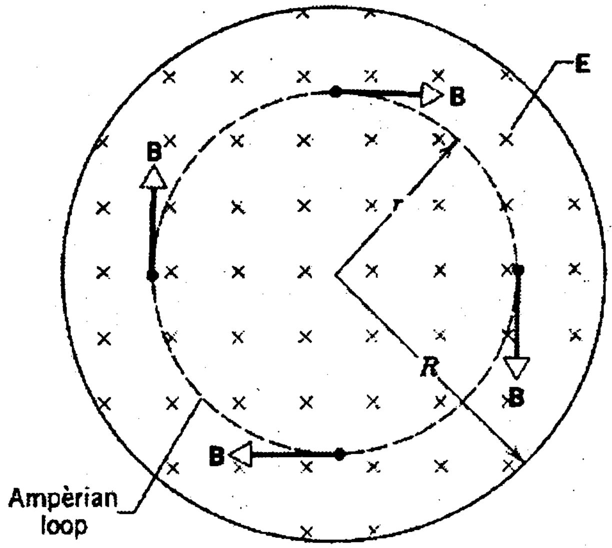
b) $R = \frac{1}{\omega L} - \omega C$

c) $R = \omega L - \frac{1}{\omega C}$

d) $R = \omega C - \frac{1}{\omega L}$

e) $\omega L = \frac{1}{\omega C}$

17-18. A parallel plate capacitor with circular plates is being charged. Determine the induced magnetic field at various radii r.



17. In the region between the plates, $r \le R$, the magnetic field is

a) $B = \frac{1}{2}\varepsilon_0 \mu_0 r \frac{dE}{dt}$

c) $B = \frac{\mu_0 i}{8\pi^2 r}$

b) $B = \frac{\mu_0 i}{2\pi r}$

d) $B = -\frac{1}{2\pi r}\frac{dE}{dt}$

18. In the region outside the plates, $r \ge R$, the magnetic field is

a) $B = \frac{\varepsilon_0 \mu_0 R^2}{2r}\frac{dE}{dt}$

c) $B = \frac{\mu_0 i}{8\pi^2 R}$

b) $B = \frac{\mu_0 i}{2\pi R}$

d) $B = -\frac{1}{2\pi R}\frac{dE}{dt}$

19. For the electromagnetic wave pictured below, in which direction is the wave traveling:

a) $+x$ direction

c) $+y$ direction

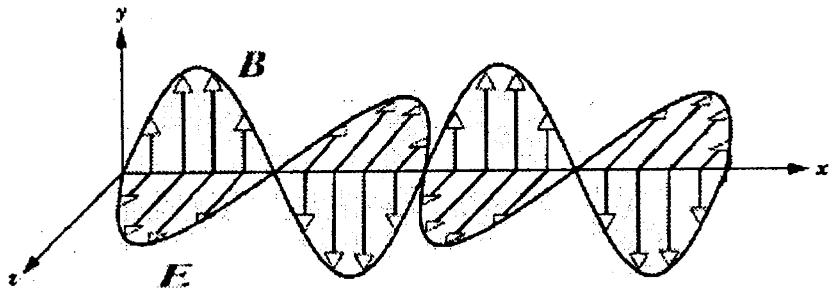
e) $+z$ direction

b) $-x$ direction

d) $-y$ direction

f) $-z$ direction

g) it is a standing wave



20. An electromagnetic wave is travelling in the positive z-direction with its electric field along the x-axis and its magnetic field along the y-axis, the fields are related by

a) $\frac{\partial E}{\partial z} = \varepsilon_0 \mu_0 \frac{\partial B}{\partial t}$

c) $\frac{\partial B}{\partial z} = \varepsilon_0 \mu_0 \frac{\partial E}{\partial x}$

b) $\frac{\partial E}{\partial z} = \varepsilon_0 \mu_0 \frac{\partial B}{\partial x}$

d) $\frac{\partial B}{\partial z} = -\varepsilon_0 \mu_0 \frac{\partial E}{\partial t}$

e) $\frac{\partial B}{\partial z} = \varepsilon_0 \mu_0 \frac{\partial E}{\partial t}$

EXTRA CREDIT QUESTION 2 (5 POINTS): The dimensions of $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$ are:

a) J/m^2

c) W/s

e) J/m^3

b) J/s

d) W/m^2

Physics 271 Electricity and Magnetism
Final Examination: Equation Sheet

$$\vec{F}_q = q\vec{E} \tag{1}$$

$$\begin{aligned}\vec{E}(\vec{r}) &= \frac{1}{4\pi\epsilon_0}\left\{\sum_{i=1}^{N}\frac{\vec{r}-\vec{r}_i}{|\vec{r}-\vec{r}_i|^3}q_i + \int_V \frac{\vec{r}-\vec{r'}}{|\vec{r}-\vec{r'}|^3}\rho(\vec{r'})dV'\right.\\ &+ \left.\int_S \frac{\vec{r}-\vec{r'}}{|\vec{r}-\vec{r'}|^3}\sigma(\vec{r'})dA' + \int_C \frac{\vec{r}-\vec{r'}}{|\vec{r}-\vec{r'}|^3}\lambda(\vec{r'})ds'\right\}\end{aligned} \tag{2}$$

$$\vec{p} = qd\hat{k} \quad ; \quad \vec{\tau} = \vec{p}\times\vec{E} \quad ; \quad U = -\vec{p}\cdot\vec{E} \tag{3}$$

$$\epsilon_0 \oint_S \vec{E}\cdot d\vec{A} = Q_{\text{enclosed}} \tag{4}$$

$$W_{ab} = \int_a^b \vec{F}\cdot d\vec{s} \quad ; \quad \Delta U = U_b - U_a = -q\int_a^b \vec{E}\cdot d\vec{s} \tag{5}$$

$$U = \frac{q_1 q_2}{4\pi\epsilon_0|\vec{r_1}-\vec{r_2}|} \tag{6}$$

$$U = \frac{1}{2}\sum_{i=1}^{N} q_i V(\vec{r_i}) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1\neq i}^{N}\frac{q_i q_j}{4\pi\epsilon_0|\vec{r_i}-\vec{r_j}|} \tag{7}$$

$$\Delta V = V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{s} \quad ; \quad \Delta U = q\Delta V \tag{8}$$

$$\vec{E} = -\vec{\nabla}V \quad ; \quad V(\vec{r}) = \frac{q}{4\pi\epsilon_0}\frac{1}{r} \quad ; \quad V = \frac{\vec{p}\cdot\vec{r}}{4\pi\epsilon_0 r^3} \tag{9}$$

$$q = CV \quad ; \quad U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} \quad ; \quad u_E = \frac{1}{2}\epsilon_0|\vec{E}|^2 \tag{10}$$

$$\kappa_e \equiv \frac{C}{C_0} \quad ; \quad \kappa_e = \epsilon/\epsilon_0 = 1 + \chi_E \tag{11}$$

$$\sigma_P \Delta A = \vec{P}\cdot\Delta\vec{A} \tag{12}$$

$$\vec{P} = \chi_E\epsilon_0\vec{E} = (\kappa_e - 1)\epsilon_0\vec{E} \quad ; \quad \vec{D} \equiv \epsilon_0\vec{E} + \vec{P} = \kappa_e\epsilon_0\vec{E} \tag{13}$$

$$\oint_S \vec{D}\cdot d\vec{A} = q = \epsilon_0\oint_S \kappa_e\vec{E}\cdot d\vec{A} \tag{14}$$

$$u_E = \frac{1}{2}\vec{D}\cdot\vec{E} \quad ; \quad U_E = \frac{1}{2}\int_V \vec{D}\cdot\vec{E}dV \tag{15}$$

$$i = \frac{dq}{dt} \quad ; \quad \vec{j} = Nq\vec{v}_d \quad ; \quad i = \int_S \vec{j} \cdot d\vec{A} \tag{16}$$

$$\int_V \frac{\partial \rho}{\partial t} dV + \oint_S \vec{j} \cdot d\vec{A} = 0 \tag{17}$$

$$\vec{j} = \sigma \vec{E} \quad ; \quad V = iR \quad ; \quad R = \frac{\rho L}{A} \tag{18}$$

$$P = iV \quad ; \quad P = i^2 R = \frac{V^2}{R} \tag{19}$$

Kirchhoff's First Rule: At any junction, the sum of the currents leaving the junction equals the sum of the currents entering the junction.

Kirchhoff's Second Rule: The algebraic sum of the changes in potential encountered in a complete traversal of any closed circuit is zero.

Parallel Resistors: $\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i}$

Series Resistors: $R_{eq} = \sum_{i=1}^{N} R_i$

Series Capacitors: $\frac{1}{C_{eq}} = \sum_{i=1}^{N} \frac{1}{C_i}$

Parallel Capacitors: $C_{eq} = \sum_{i=1}^{N} C_i$

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \quad ; \quad \oint_S \vec{B} \cdot d\vec{A} = 0 \tag{20}$$

$$d\vec{F} = i d\vec{s} \times \vec{B} \quad ; \quad \vec{F} = i\vec{L} \times \vec{B} \tag{21}$$

$$\vec{\mu} = NiA\hat{n} \quad ; \quad \vec{\tau} = \vec{\mu} \times \vec{B} \quad ; \quad U = -\vec{\mu} \cdot \vec{B} \tag{22}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \vec{r}}{r^3} \quad ; \quad \vec{B} = \frac{\mu_0}{4\pi} \int_C \frac{i d\vec{s} \times \vec{r}}{r^3} \tag{23}$$

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enclosed}} \quad ; \quad \mathcal{E} = -\frac{d\Phi_B}{dt} \tag{24}$$

$$\Phi_B = \int_S \vec{B} \cdot d\vec{A} \quad ; \quad \mathcal{E} = \oint_C \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{A} \tag{25}$$

$$\vec{\mu} = -\frac{e}{2m}\left(\vec{L}+\vec{S}\right) = -\frac{e}{2m}\vec{J} \quad ; \quad \vec{M} = \left[\frac{c}{T} - \frac{NZe^2r_0^2}{6m}\right]\vec{B}_0 \tag{26}$$

$$\vec{j}_{MS} = \vec{M}\times\hat{n} \quad ; \quad \vec{H} = \frac{1}{\mu_0}\left(\vec{B}-\mu_0\vec{M}\right) \tag{27}$$

$$\vec{M} = \chi_M\vec{H} \quad ; \quad \vec{B} = \mu\vec{H} \quad ; \quad \oint_C \vec{H}\cdot d\vec{s} = i_{\text{conduction}} \tag{28}$$

$$\mathcal{E}_L = -L\frac{di}{dt} \quad ; \quad L = \frac{\Phi_B}{i} \quad ; \quad U_B = \frac{1}{2}Li^2 = \frac{1}{2}i\phi_B \tag{29}$$

$$u_B = \frac{1}{2}\vec{B}\cdot\vec{H} \quad ; \quad U_B = \frac{1}{2}\int_V \vec{B}\cdot\vec{H}dV \tag{30}$$

$$\text{ELI the ICE man} \quad ; \quad X_L = \omega L \quad ; \quad X_C = \frac{1}{\omega C} \tag{31}$$

$$i_m = \frac{\mathcal{E}_m}{Z} \quad ; \quad Z = \sqrt{R^2+(X_L-X_C)^2} \quad ; \quad \tan\phi = \frac{X_L-X_C}{R} \tag{32}$$

$$\bar{P} = i_{\text{rms}}^2R \quad ; \quad \bar{P} = \mathcal{E}_{\text{rms}}i_{\text{rms}}\cos\phi \quad ; \quad \cos\phi = \frac{R}{Z} \tag{33}$$

$$\oint_C \vec{H}\cdot d\vec{s} = i + \frac{d}{dt}\int_S \vec{D}\cdot d\vec{A} \tag{34}$$

Maxwell's Equations In Vacuum

$$\begin{aligned} \oint_S \vec{E}\cdot d\vec{A} &= \frac{1}{\epsilon_0}q \\ \oint_S \vec{B}\cdot d\vec{A} &= 0 \\ \oint_C \vec{E}\cdot d\vec{s} &= -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{A} \\ \oint_C \vec{B}\cdot d\vec{s} &= \mu_0 i + \epsilon_0\mu_0\frac{d}{dt}\int_S \vec{E}\cdot d\vec{A} \end{aligned} \tag{35}$$

$$\vec{S} = \vec{E}\times\vec{H} \quad ; \quad I = \bar{S} \quad ; \quad I = \frac{1}{\mu_0}E_{\text{rms}}B_{\text{rms}} \tag{36}$$