PHYSICS 271
ELECTRICITY AND MAGNETISM
FIRST EXAMINATION

15 October 1999

INSTRUCTIONS: Answer all questions on the answer sheet provided, it will be the only paper that is collected. This is a closed book exam.

1. a) A thin glass rod is bent into a semicircle of radius R. A charge $+q$ is uniformly distributed along the upper half and a charge $-q$ is uniformly distributed along the lower half, as shown below. What is the direction of the electric field at P, the center of the semicircle?

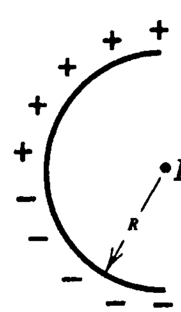
a) **E** points up

b) **E** points down

c) **E** points to the right

d) **E** points to the left

e) **E** is zero



1. b) The magnitude of the electric field at P in problem **1a** is:

a) $\dfrac{q}{4\pi\varepsilon_0 R^2}$

b) $\dfrac{q}{\pi^2\varepsilon_0 R^2}$

c) $\dfrac{q}{2\pi\varepsilon_0 R^2}$

d) $\dfrac{q}{2\pi^2\varepsilon_0 R^2}$

e) 0

2. a) Suppose a spherically shaped planet with radius R has an excess positive charge Q uniformly distributed over its surface. The electric field at radial distances $r > R$ from the center of the planet is:

a) $\vec{E} = \dfrac{Q}{4\pi\varepsilon_0 R^2}\hat{r}$

b) $\vec{E} = \dfrac{-Q}{4\pi\varepsilon_0 R^2}\hat{r}$

c) $\vec{E} = \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$

d) $\vec{E} = \dfrac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$

e) $\vec{E} = 0$

2. b) What is the electric field inside the planet?

a) $\vec{E} = \dfrac{Q}{4\pi\varepsilon_0 R^2}\hat{r}$

b) $\vec{E} = \dfrac{-Q}{4\pi\varepsilon_0 R^2}\hat{r}$

c) $\vec{E} = \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$

d) $\vec{E} = \dfrac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$

e) $\vec{E} = 0$

2. c) In addition, the planet is surrounded by a positively charged dust cloud containing total charge q and extending from the surface of the planet out to a radius R_D. The cloud has a spherically symmetric charge distribution with volume charge density given by $\rho(r) = \dfrac{q}{4\pi(R_D - R)r^2}$ for $R \le r \le R_D$, and zero otherwise. What is the electric field at radial distances $r > R_D$?

a) $\vec{E} = \dfrac{Q+q}{4\pi\varepsilon_0 R_D{}^2}\hat{r}$

b) $\vec{E} = \dfrac{-Q-q}{4\pi\varepsilon_0 R_D{}^2}\hat{r}$

c) $\vec{E} = \dfrac{Q+q}{4\pi\varepsilon_0 r^2}\hat{r}$

d) $\vec{E} = \dfrac{q}{4\pi\varepsilon_0 r^2}\hat{r}$

e) $\vec{E} = 0$

2. d) What is the electric field at radial distances $R \le r \le R_D$?

a) $\vec{E} = \dfrac{Q+q}{4\pi\varepsilon_0 r^2}\hat{r}$

b) $\vec{E} = \dfrac{Q+q}{4\pi\varepsilon_0 R^2}\hat{r}$

c) $\vec{E} = \dfrac{Q+q}{4\pi\varepsilon_0 r^2}\hat{r} - \dfrac{q}{4\pi\varepsilon_0 r^2}\dfrac{(R_D - r)}{(R_D - R)}\hat{r}$

d) $\vec{E} = \dfrac{q}{4\pi\varepsilon_0 r^2}\dfrac{(r - R)}{(R_D - R)}\hat{r}$

e) $\vec{E} = 0$

3. a) Two conducting spheres are far apart. The smaller sphere carries a total charge of q. The larger sphere has a radius that is twice that of the smaller and is neutral. After the two spheres are connected by a conducting wire, the charges on the smaller and larger spheres, respectively, are:

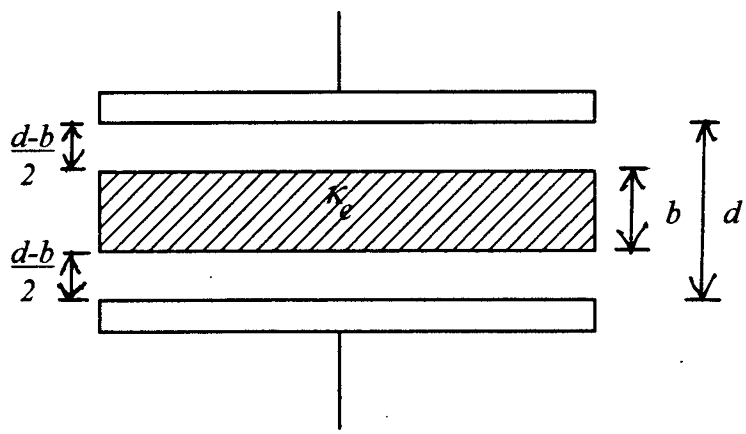
a) $\frac{2}{3}q$ and $\frac{1}{3}q$ c) $-q$ and $+2q$ e) $\frac{1}{2}q$ and $\frac{1}{2}q$.

b) $\frac{1}{3}q$ and $\frac{2}{3}q$ d) q and 0

3. b) A hollow metal sphere is charged to a potential V. The potential at its center is:

a) V c) $-V$ e) πV
b) 0 d) $2V$

4. A potential difference V_0 is applied to a parallel plate capacitor with plate area A and plate separation d. The battery is disconnected and then a dielectric slab of thickness b and dielectric constant κ_e is centrally inserted between the plates as shown below.



4. a) What is the magnitude of the electric field E_0 in the gaps between the plates and the dielectric slab?

a) $E_0 = \frac{q}{\varepsilon_0 A}$ c) $E_0 = \frac{q}{\varepsilon_0 \kappa_e A}$

b) $E_0 = \frac{V_0}{d}$ d) $E_0 = \frac{V_0}{\kappa_e d}$

4. b) What is the magnitude of the electric field E in the dielectric slab?

a) $E = E_0$

c) $E = \frac{V_0}{d}$

b) $E = \kappa_e E_0$

d) $E = \frac{1}{\kappa_e} E_0$

4. c) What is the potential difference V between the plates after the slab has been inserted?

a) $V = V_0$

c) $V = V_0\left[1 - \frac{b}{d}\left(\frac{\kappa_e - 1}{\kappa_e}\right)\right]$

b) $V = \frac{V_0}{\kappa_e}$

d) $V = E_0(d - b)$

4. d) What is the capacitance with the slab in place?

a) $C = \frac{\varepsilon_0 A}{\left[d - b + \frac{b}{\kappa_e}\right]}$

c) $C = \frac{q}{V}$

b) $C = \kappa_e \frac{\varepsilon_0 A}{d}$

d) $C = \frac{\varepsilon_0 \kappa_e A}{[\kappa_e(d - b) + b]}$

4. e) Find the ratio of the stored energy before to that after the slab is inserted.

a) $\frac{U_0}{U} = 1$, energy is conserved

c) $\frac{U_0}{U} = \frac{d}{\kappa_e(d - b) + b}$

b) $\frac{U_0}{U} = \kappa_e$

d) $\frac{U_0}{U} = \frac{\kappa_e d}{\kappa_e(d - b) + b}$

4. f) How much work is done on the slab as it is inserted?

a) $W = 0$, energy is conserved

c) $W = \frac{1}{2}CV_0^2 - \frac{1}{2}C_0V_0^2$

b) $W = \frac{1}{2}CV_0^2$

d) $W = \frac{1}{2}C_0V_0^2 - \frac{1}{2}CV^2$

Physics 271 Electricity and Magnetism
Examination 1: Equation Sheet

$$\vec{F}_q = q\vec{E} \tag{1}$$

$$\begin{aligned}\vec{E}(\vec{r}) &= \frac{1}{4\pi\epsilon_0}\left\{\sum_{i=1}^{N}\frac{\vec{r}-\vec{r}_i}{|\vec{r}-\vec{r}_i|^3}q_i + \int_V \frac{\vec{r}-\vec{r'}}{|\vec{r}-\vec{r'}|^3}\rho(\vec{r'})dV'\right.\\ &+ \left.\int_S \frac{\vec{r}-\vec{r'}}{|\vec{r}-\vec{r'}|^3}\sigma(\vec{r'})dA' + \int_C \frac{\vec{r}-\vec{r'}}{|\vec{r}-\vec{r'}|^3}\lambda(\vec{r'})ds'\right\}\end{aligned} \tag{2}$$

$$\vec{p} = qd\hat{k} \quad ; \quad \vec{\tau} = \vec{p}\times\vec{E} \quad ; \quad U = -\vec{p}\cdot\vec{E} \tag{3}$$

$$\epsilon_0 \oint_S \vec{E}\cdot d\vec{A} = Q_{\text{enclosed}} \tag{4}$$

$$W_{ab} = \int_a^b \vec{F}\cdot d\vec{s} \quad ; \quad \Delta U = U_b - U_a = -q\int_a^b \vec{E}\cdot d\vec{s} \tag{5}$$

$$U = \frac{q_1 q_2}{4\pi\epsilon_0|\vec{r_1}-\vec{r_2}|} \tag{6}$$

$$U = \frac{1}{2}\sum_{i=1}^{N} q_i V(\vec{r_i}) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1\neq i}^{N}\frac{q_i q_j}{4\pi\epsilon_0|\vec{r_i}-\vec{r_j}|} \tag{7}$$

$$\Delta V = V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{s} \quad ; \quad \Delta U = q\Delta V \tag{8}$$

$$\vec{E} = -\vec{\nabla}V \quad ; \quad V(\vec{r}) = \frac{q}{4\pi\epsilon_0}\frac{1}{r} \quad ; \quad V = \frac{\vec{p}\cdot\vec{r}}{4\pi\epsilon_0 r^3} \tag{9}$$

$$q = CV \quad ; \quad U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} \quad ; \quad u = \frac{1}{2}\epsilon_0|\vec{E}|^2 \tag{10}$$

$$\kappa_e \equiv \frac{C}{C_0} \quad ; \quad \kappa_e = \epsilon/\epsilon_0 = 1 + \chi_E \tag{11}$$

$$\sigma_P \Delta A = \vec{P}\cdot\Delta\vec{A} \tag{12}$$

$$\vec{P} = \chi_E\epsilon_0\vec{E} = (\kappa_e - 1)\epsilon_0\vec{E} \quad ; \quad \vec{D} \equiv \epsilon_0\vec{E} + \vec{P} = \kappa_e\epsilon_0\vec{E} \tag{13}$$

$$\oint_S \vec{D}\cdot d\vec{A} = q = \epsilon_0\oint_S \kappa_e\vec{E}\cdot d\vec{A} \tag{14}$$

$$u = \frac{1}{2}\vec{D}\cdot\vec{E} \quad ; \quad U = \frac{1}{2}\int_V \vec{D}\cdot\vec{E}dV \tag{15}$$