PHYSICS 271
ELECTRICITY AND MAGNETISM
FINAL EXAMINATION

14 December 1998

INSTRUCTIONS: Answer all questions on the answer sheet provided, it will be the only paper that is collected. This is a closed book exam.

1. Positive charge Q is distributed uniformly throughout an insulating sphere of radius R, centered at the origin. A positive point charge Q is placed at $x = 2R$ on the x-axis. The magnitude of the electric field at $x = R/2$ on the x-axis is:

a) $\dfrac{Q}{4\pi\varepsilon_0 R^2}$

b) $\dfrac{Q}{8\pi\varepsilon_0 R^2}$

c) $\dfrac{Q}{72\pi\varepsilon_0 R^2}$

d) $\dfrac{17Q}{72\pi\varepsilon_0 R^2}$

2. Positive charge Q is distributed uniformly throughout a non-conducting sphere. The highest electric potential occurs

a) at the center

b) at the surface

c) halfway between the center and surface

d) far from the sphere

3. For the sphere in problem **2**, the highest potential is

a) $\dfrac{Q}{4\pi\varepsilon_0 R}$

b) $\dfrac{3Q}{8\pi\varepsilon_0 R}$

c) $\dfrac{-Q}{4\pi\varepsilon_0 R}$

d) $\dfrac{-3Q}{8\pi\varepsilon_0 R}$

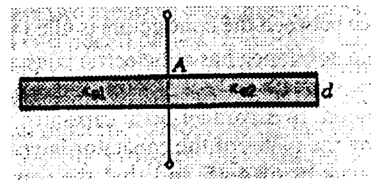
4. A parallel plate capacitor is filled with two dielectrics as shown below. The capacitance is given by

a) $C = \frac{\varepsilon_0 A}{d}\left(\frac{\kappa_{e1} + \kappa_{e2}}{2}\right)$

c) $C = \frac{2\varepsilon_0 A}{d}\left(\frac{1}{\kappa_{e1} + \kappa_{e2}}\right)$

b) $C = \frac{2\varepsilon_0 A}{d}\left(\frac{\kappa_{e1}\kappa_{e2}}{\kappa_{e1} + \kappa_{e2}}\right)$

d) $C = \frac{\varepsilon_0 A}{d}\frac{1}{2}\left(\frac{1}{\kappa_{e1}} + \frac{1}{\kappa_{e2}}\right)$



5. The current in the 5.0 Ω resistor in the circuit shown is

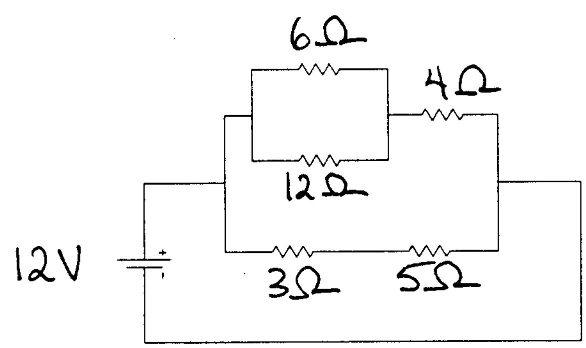
a) 0.42 A

b) 0.67 A

c) 1.5 A

d) 2.4 A

e) 3.0 A



6. The magnitude of the magnetic field at point P, at the center of the semi-circle shown below is

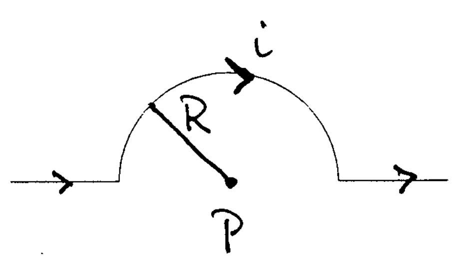
a) $\frac{\mu_0 i}{R^2}$

b) $\frac{\mu_0 i}{2\pi R}$

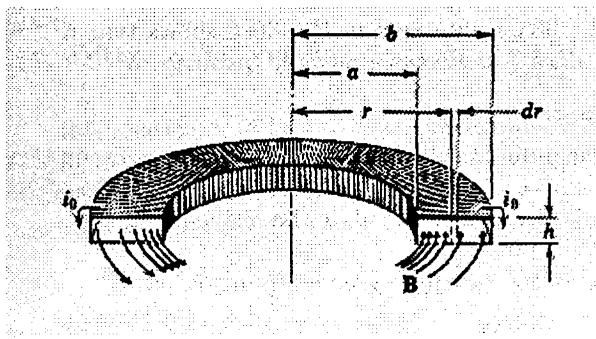
c) $\frac{\mu_0 i}{2R}$

d) $\frac{\mu_0 i}{4\pi R}$

e) $\frac{\mu_0 i}{4R}$



For questions **7-12** a rectangular toroid, whose cross section is shown below, is considered. It has inner radius a and outer radius b with a thickness h with a total of N turns of wire carrying current i_0.



7. A toroid with a rectangular cross section carries current i_0. The magnetic field has its largest magnitude

a) at the center of the hole

b) just inside the toroid at its inner surface

c) just inside the toroid at its outer surface

d) at any point inside (the field is uniform)

8. The value of the magnitude of the magnetic field at radius r inside the toroid is

a) $\dfrac{\mu_0 i_0 N}{h}$

b) $\dfrac{\mu_0 i_0 N}{2\pi r}$

c) $\dfrac{\mu_0 i_0 N}{4\pi h} \dfrac{b}{a}$

d) $\dfrac{\mu_0 i_0 N}{\pi r^2}$

9. The inductance of the toroid is given by

a) $L = \mu_0 n^2 h\pi(b^2 - a^2)$

b) $L = \dfrac{\mu_0 N^2 h}{2\pi}\ln\left(\dfrac{b}{a}\right)$

c) $L = \dfrac{\mu_0 N^2 ab}{2\pi\ h}$

d) none of the above

10. If the toroid is filled with magnetic material of permeability μ, its inductance L as compared to its inductance L_0 without the material is

a) $L = \mu\, L_0$

b) $L = \dfrac{\mu}{\mu_0} L_0$

c) $L = \dfrac{1}{\mu}\ L_0$

d) $L = \chi_M\ L_0$

11. The energy density as a function of radial distance r for the toroid is

a) $u_B = \dfrac{\mu_0 i_0^2 N^2}{8\pi^2 r^2}$

b) $u_B = \dfrac{\mu_0 i_0 N}{4\pi}\ln\left(\dfrac{r}{h}\right)$

c) $u_B = \left(\dfrac{\mu_0 i_0 N h}{8\pi}\right)^2$

d) $u_B = (\mu_0 i_0 n)^2$

12. The total energy stored in the magnetic field of the toroid is

a) $U_B = i_0^2 Rh\pi\,(b^2 - a^2)$

b) $U_B = \dfrac{1}{2} L i_0^2$

c) $U_B = (\mu_0 i_0 n)^2\, h\pi\,(b^2 - a^2)$

d) none of the above

13. A rod lies across frictionless rails in a uniform magnetic field B, as shown. The rod moves to the right with speed v. In order for the emf around the circuit to be zero, the magnitude of the magnetic field should

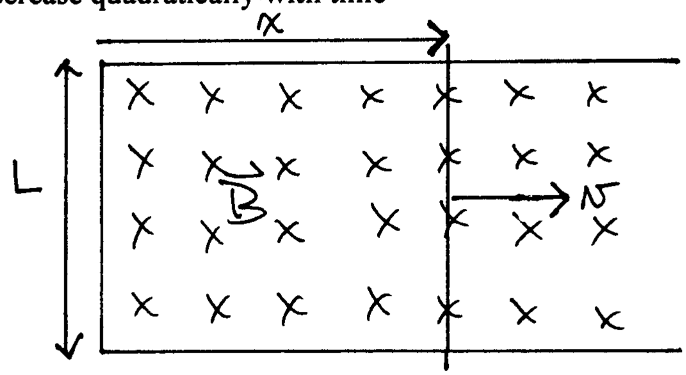
a) not change

b) increase linearly with time

c) decrease linearly with time

d) increase quadratically with time

e) decrease quadratically with time



14. A rod with resistance R lies across frictionless conducting rails in a uniform magnetic field B, as drawn for question **13**. Assume the rails have negligible resistance. The force that must be applied by a person to pull the rod to the right at constant speed v is

a) 0

b) BLv

c) $\frac{BLv}{R}$

d) $\frac{B^2L^2v}{R}$

e) $\frac{B^2Lxv}{R}$

15. In a purely resistive circuit the current

a) leads the voltage by $\frac{1}{4}$ cycle

b) leads the voltage by $\frac{1}{2}$ cycle

c) lags the voltage by $\frac{1}{4}$ cycle

d) lags the voltage by $\frac{1}{2}$ cycle

e) is in phase with the voltage

16. In a LCR series circuit, which is connected to a source $E_M \cos(\omega t)$ the current lags the voltage by 45° if

a) $R = \dfrac{1}{\omega C} - \omega L$

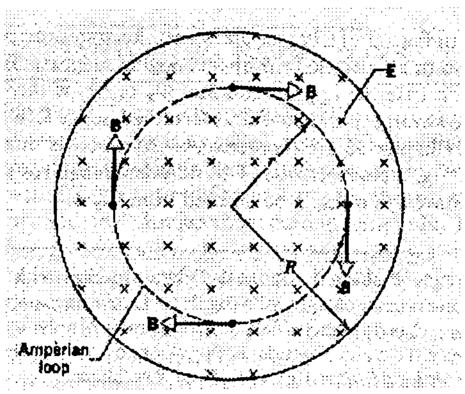
b) $R = \dfrac{1}{\omega L} - \omega C$

c) $R = \omega L - \dfrac{1}{\omega C}$

d) $R = \omega C - \dfrac{1}{\omega L}$

e) $\omega L = \dfrac{1}{\omega C}$

17-18. A parallel plate capacitor with circular plates is being charged. Determine the induced magnetic field at various radii r.



17. In the region between the plates, $r \le R$, the magnetic field is

a) $B = \dfrac{1}{2} \varepsilon_0 \mu_0 r \dfrac{dE}{dt}$

b) $B = \dfrac{\mu_0 i}{2\pi r}$

c) $B = \dfrac{\mu_0 i}{8\pi^2 r}$

d) $B = -\dfrac{1}{2\pi r} \dfrac{dE}{dt}$

18. In the region outside the plates, $r \ge R$, the magnetic field is

a) $B = \dfrac{\varepsilon_0 \mu_0 R^2}{2r} \dfrac{dE}{dt}$

b) $B = \dfrac{\mu_0 i}{2\pi R}$

c) $B = \dfrac{\mu_0 i}{8\pi^2 R}$

d) $B = -\dfrac{1}{2\pi R} \dfrac{dE}{dt}$

19. An electromagnetic wave is travelling in the positive x-direction with its electric field along the z-axis and its magnetic field along the y-axis, the fields are related by

a) $\dfrac{\partial E}{\partial x} = \varepsilon_0 \mu_0 \dfrac{\partial B}{\partial x}$

b) $\dfrac{\partial E}{\partial x} = \varepsilon_0 \mu_0 \dfrac{\partial B}{\partial t}$

c) $\dfrac{\partial B}{\partial x} = \varepsilon_0 \mu_0 \dfrac{\partial E}{\partial x}$

d) $\dfrac{\partial B}{\partial x} = \varepsilon_0 \mu_0 \dfrac{\partial E}{\partial t}$

e) $\dfrac{\partial B}{\partial x} = -\varepsilon_0 \mu_0 \dfrac{\partial E}{\partial t}$

20. For an electromagnetic wave the direction of the Poynting vector $\vec{S} = \dfrac{1}{\mu_0}\vec{E} \times \vec{B}$ gives

a) the direction of the electric field

b) the direction of the magnetic field

c) the direction of wave propagation

d) the direction of the electromagnetic force on a positive test charge

e) the direction of the emf induced by the wave.

Physics 271 Electricity and Magnetism
Final Examination: Equation Sheet

$$\vec{F}_q = q\vec{E} \tag{1}$$

$$\begin{aligned}\vec{E}(\vec{r}) &= \frac{1}{4\pi\epsilon_0}\left\{\sum_{i=1}^{N}\frac{\vec{r}-\vec{r_i}}{|\vec{r}-\vec{r_i}|^3}q_i + \int_V \frac{\vec{r}-\vec{r'}}{|\vec{r}-\vec{r'}|^3}\rho(\vec{r'})dV'\right.\\ &+ \left.\int_S \frac{\vec{r}-\vec{r'}}{|\vec{r}-\vec{r'}|^3}\sigma(\vec{r'})dA' + \int_C \frac{\vec{r}-\vec{r'}}{|\vec{r}-\vec{r'}|^3}\lambda(\vec{r'})ds'\right\}\end{aligned} \tag{2}$$

$$\vec{p} = qd\hat{k} \quad ; \quad \vec{\tau} = \vec{p}\times\vec{E} \quad ; \quad U = -\vec{p}\cdot\vec{E} \tag{3}$$

$$\epsilon_0 \oint_S \vec{E}\cdot d\vec{A} = Q_{\text{enclosed}} \tag{4}$$

$$W_{ab} = \int_a^b \vec{F}\cdot d\vec{s} \quad ; \quad \Delta U = U_b - U_a = -q\int_a^b \vec{E}\cdot d\vec{s} \tag{5}$$

$$U = \frac{q_1 q_2}{4\pi\epsilon_0|\vec{r_1}-\vec{r_2}|} \tag{6}$$

$$U = \frac{1}{2}\sum_{i=1}^{N} q_i V(\vec{r_i}) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1\neq i}^{N}\frac{q_i q_j}{4\pi\epsilon_0|\vec{r_i}-\vec{r_j}|} \tag{7}$$

$$\Delta V = V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{s} \quad ; \quad \Delta U = q\Delta V \tag{8}$$

$$\vec{E} = -\vec{\nabla}V \quad ; \quad V(\vec{r}) = \frac{q}{4\pi\epsilon_0}\frac{1}{r} \quad ; \quad V = \frac{\vec{p}\cdot\vec{r}}{4\pi\epsilon_0 r^3} \tag{9}$$

$$q = CV \quad ; \quad U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} \quad ; \quad u_E = \frac{1}{2}\epsilon_0|\vec{E}|^2 \tag{10}$$

$$\kappa_e \equiv \frac{C}{C_0} \quad ; \quad \kappa_e = \epsilon/\epsilon_0 = 1 + \chi_E \tag{11}$$

$$\sigma_P \Delta A = \vec{P}\cdot\Delta\vec{A} \tag{12}$$

$$\vec{P} = \chi_E\epsilon_0\vec{E} = (\kappa_e - 1)\epsilon_0\vec{E} \quad ; \quad \vec{D} \equiv \epsilon_0\vec{E} + \vec{P} = \kappa_e\epsilon_0\vec{E} \tag{13}$$

$$\oint_S \vec{D}\cdot d\vec{A} = q = \epsilon_0\oint_S \kappa_e\vec{E}\cdot d\vec{A} \tag{14}$$

$$u_E = \frac{1}{2}\vec{D}\cdot\vec{E} \quad ; \quad U_E = \frac{1}{2}\int_V \vec{D}\cdot\vec{E}dV \tag{15}$$

$$i = \frac{dq}{dt} \quad ; \quad \vec{j} = Nq\vec{v}_d \quad ; \quad i = \int_S \vec{j} \cdot d\vec{A} \tag{16}$$

$$\int_V \frac{\partial \rho}{\partial t} dV + \oint_S \vec{j} \cdot d\vec{A} = 0 \tag{17}$$

$$\vec{j} = \sigma \vec{E} \quad ; \quad V = iR \quad ; \quad R = \frac{\rho L}{A} \tag{18}$$

$$P = iV \quad ; \quad P = i^2 R = \frac{V^2}{R} \tag{19}$$

Kirchhoff's First Rule: At any junction, the sum of the currents leaving the junction equals the sum of the currents entering the junction.

Kirchhoff's Second Rule: The algebraic sum of the changes in potential encountered in a complete traversal of any closed circuit is zero.

Parallel Resistors: $\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i}$ Series Resistors: $R_{eq} = \sum_{i=1}^{N} R_i$

Series Capacitors: $\frac{1}{C_{eq}} = \sum_{i=1}^{N} \frac{1}{C_i}$ Parallel Capacitors: $C_{eq} = \sum_{i=1}^{N} C_i$

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \quad ; \quad \oint_S \vec{B} \cdot d\vec{A} = 0 \tag{20}$$

$$d\vec{F} = i d\vec{s} \times \vec{B} \quad ; \quad \vec{F} = i\vec{L} \times \vec{B} \tag{21}$$

$$\vec{\mu} = NiA\hat{n} \quad ; \quad \vec{\tau} = \vec{\mu} \times \vec{B} \quad ; \quad U = -\vec{\mu} \cdot \vec{B} \tag{22}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \vec{r}}{r^3} \quad ; \quad \vec{B} = \frac{\mu_0}{4\pi} \int_C \frac{i d\vec{s} \times \vec{r}}{r^3} \tag{23}$$

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enclosed}} \quad ; \quad \mathcal{E} = -\frac{d\Phi_B}{dt} \tag{24}$$

$$\Phi_B = \int_S \vec{B} \cdot d\vec{A} \quad ; \quad \mathcal{E} = \oint_C \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{A} \tag{25}$$

$$\vec{\mu} = -\frac{e}{2m}\left(\vec{L}+\vec{S}\right) = -\frac{e}{2m}\vec{J} \quad ; \quad \vec{M} = \left[\frac{c}{T} - \frac{NZe^2r_0^2}{6m}\right]\vec{B}_0 \tag{26}$$

$$\vec{j}_{MS} = \vec{M}\times\hat{n} \quad ; \quad \vec{H} = \frac{1}{\mu_0}\left(\vec{B}-\mu_0\vec{M}\right) \tag{27}$$

$$\vec{M} = \chi_M\vec{H} \quad ; \quad \vec{B} = \mu\vec{H} \quad ; \quad \oint_C \vec{H}\cdot d\vec{s} = i_{\text{conduction}} \tag{28}$$

$$\mathcal{E}_L = -L\frac{di}{dt} \quad ; \quad L = \frac{\Phi_B}{i} \quad ; \quad U_B = \frac{1}{2}Li^2 = \frac{1}{2}i\phi_B \tag{29}$$

$$u_B = \frac{1}{2}\vec{B}\cdot\vec{H} \quad ; \quad U_B = \frac{1}{2}\int_V \vec{B}\cdot\vec{H}dV \tag{30}$$

$$\text{ELI the ICE man} \quad ; \quad X_L = \omega L \quad ; \quad X_C = \frac{1}{\omega C} \tag{31}$$

$$i_m = \frac{\mathcal{E}_m}{Z} \quad ; \quad Z = \sqrt{R^2+(X_L-X_C)^2} \quad ; \quad \tan\phi = \frac{X_L-X_C}{R} \tag{32}$$

$$\bar{P} = i_{\text{rms}}^2 R \quad ; \quad \bar{P} = \mathcal{E}_{\text{rms}} i_{\text{rms}}\cos\phi \quad ; \quad \cos\phi = \frac{R}{Z} \tag{33}$$

$$\oint_C \vec{H}\cdot d\vec{s} = i + \frac{d}{dt}\int_S \vec{D}\cdot d\vec{A} \tag{34}$$

Maxwell's Equations In Vacuum

$$\begin{aligned}
\oint_S \vec{E}\cdot d\vec{A} &= \frac{1}{\epsilon_0}q \\
\oint_S \vec{B}\cdot d\vec{A} &= 0 \\
\oint_C \vec{E}\cdot d\vec{s} &= -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{A} \\
\oint_C \vec{B}\cdot d\vec{s} &= \mu_0 i + \epsilon_0\mu_0\frac{d}{dt}\int_S \vec{E}\cdot d\vec{A}
\end{aligned} \tag{35}$$

$$\vec{S} = \vec{E}\times\vec{H} \quad ; \quad I = \bar{S} \quad ; \quad I = \frac{1}{\mu_0}E_{\text{rms}}B_{\text{rms}} \tag{36}$$