PHYSICS 271
ELECTRICITY AND MAGNETISM
FIRST EXAMINATION

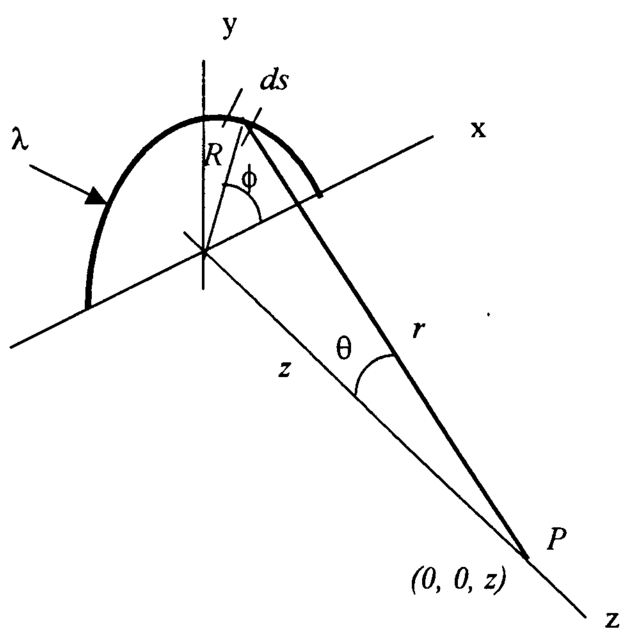
7 October 1998

INSTRUCTIONS: Answer all questions. Explain what you are doing in the calculations, do not simply write formulae. Stating the answer is not sufficient, you will lose points if you do not show steps in finding the solution (excluding the multiple choice questions). Neatness and clarity of presentation count. This is a closed book exam.

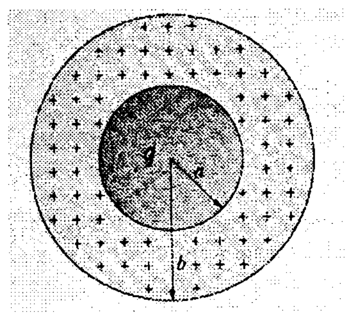
1) Suppose a thin semi-circular ring of radius R carrying a uniform linear charge density λ around its circumference lies in the x-y plane as shown below.

a) What is the electric field $\vec{E}$, that is E_x, E_y and E_z, at a point P along the z-axis a distance z from the origin? Hint: First find E_z and then E_x and E_y. Use symmetry arguments to simplify the calculation where applicable.

b) Suppose P is very far away from the half ring compared to its radius, $z >> R$. What is the approximate expression for $\vec{E}$?



2) A non-conducting spherical shell, with inner radius a and outer radius b, has a uniform volume charge density ρ, and hence charge $Q = \frac{4}{3}\pi\ (b^3 - a^3)\rho$. In addition a point charge q is at the center of the spherical shell. Using Gauss' Law, determine the magnitude of the radially directed electric field at all radii r .



3) a) Two conducting spheres are far apart. The smaller sphere carries a total charge of q. The larger sphere has a radius that is twice that of the smaller and is neutral. After the two spheres are connected by a conducting wire, the charges on the smaller and larger spheres, respectively, are:

a) $\frac{2}{3}q$ and $\frac{1}{3}q$
b) $\frac{1}{3}q$ and $\frac{2}{3}q$
c) $-q$ and $+2q$
d) q and 0
e) $\frac{1}{2}q$ and $\frac{1}{2}q$.

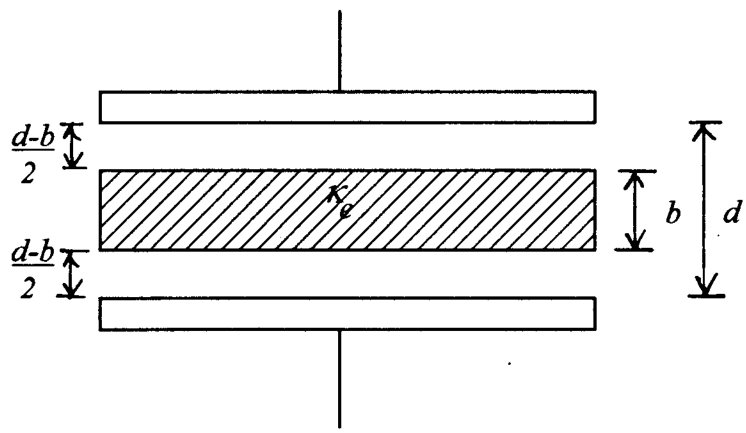
3) b) A hollow metal sphere is charged to a potential V. The potential at its center is:

a) V
b) 0
c) $-V$
d) $2V$
e) πV .

4) A parallel plate capacitor has plates of area A and separation d and is charged to a potential difference V. The charging battery is then disconnected and the plates are pulled apart until their separation is 2d. Determine expressions, in terms of A, d and V, for

a) the new potential difference
b) the initial and final stored energy
c) the work required to separate the plates.

5) A potential difference V_0 is applied to a parallel plate capacitor with plate area A and plate separation d. The battery is disconnected and then a dielectric slab of thickness b and dielectric constant κ_e is centrally inserted between the plates as shown below.



a) What is the electric field E_0 in the gaps between the plates and the dielectric slab?

b) What is the electric field E in the dielectric slab?

c) What is the potential difference between the plates after the slab has been inserted?

d) What is the capacitance with the slab in place?

e) Find the ratio of the stored energy before to that after the slab is inserted.

f) How much work is done on the slab as it is inserted?

Physics 271 Electricity and Magnetism
Examination 1: Equation Sheet

$$\vec{F}_q = q\vec{E} \tag{1}$$

$$\begin{aligned}\vec{E}(\vec{r}) &= \frac{1}{4\pi\epsilon_0}\left\{\sum_{i=1}^{N}\frac{\vec{r}-\vec{r_i}}{|\vec{r}-\vec{r_i}|^3}q_i + \int_V \frac{\vec{r}-\vec{r'}}{|\vec{r}-\vec{r'}|^3}\rho(\vec{r'})dV' \right.\\ &+ \left. \int_S \frac{\vec{r}-\vec{r'}}{|\vec{r}-\vec{r'}|^3}\sigma(\vec{r'})dA' + \int_C \frac{\vec{r}-\vec{r'}}{|\vec{r}-\vec{r'}|^3}\lambda(\vec{r'})ds'\right\}\end{aligned} \tag{2}$$

$$\vec{p} = qd\hat{k} \quad ; \quad \vec{\tau} = \vec{p}\times\vec{E} \quad ; \quad U = -\vec{p}\cdot\vec{E} \tag{3}$$

$$\epsilon_0 \oint_S \vec{E}\cdot d\vec{A} = Q_{\text{enclosed}} \tag{4}$$

$$W_{ab} = \int_a^b \vec{F}\cdot d\vec{s} \quad ; \quad \Delta U = U_b - U_a = -q\int_a^b \vec{E}\cdot d\vec{s} \tag{5}$$

$$U = \frac{q_1 q_2}{4\pi\epsilon_0|\vec{r_1}-\vec{r_2}|} \tag{6}$$

$$U = \frac{1}{2}\sum_{i=1}^{N} q_i V(\vec{r_i}) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1\neq i}^{N}\frac{q_i q_j}{4\pi\epsilon_0|\vec{r_i}-\vec{r_j}|} \tag{7}$$

$$\Delta V = V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{s} \quad ; \quad \Delta U = q\Delta V \tag{8}$$

$$\vec{E} = -\vec{\nabla}V \quad ; \quad V(\vec{r}) = \frac{q}{4\pi\epsilon_0}\frac{1}{r} \quad ; \quad V = \frac{\vec{p}\cdot\vec{r}}{4\pi\epsilon_0 r^3} \tag{9}$$

$$q = CV \quad ; \quad U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} \quad ; \quad u = \frac{1}{2}\epsilon_0|\vec{E}|^2 \tag{10}$$

$$\kappa_e \equiv \frac{C}{C_0} \quad ; \quad \kappa_e = \epsilon/\epsilon_0 = 1 + \chi_E \tag{11}$$

$$\sigma_P \Delta A = \vec{P}\cdot\Delta\vec{A} \tag{12}$$

$$\vec{P} = \chi_E\epsilon_0\vec{E} = (\kappa_e - 1)\epsilon_0\vec{E} \quad ; \quad \vec{D} \equiv \epsilon_0\vec{E} + \vec{P} = \kappa_e\epsilon_0\vec{E} \tag{13}$$

$$\oint_S \vec{D}\cdot d\vec{A} = q = \epsilon_0\oint_S \kappa_e\vec{E}\cdot d\vec{A} \tag{14}$$

$$u = \frac{1}{2}\vec{D}\cdot\vec{E} \quad ; \quad U = \frac{1}{2}\int_V \vec{D}\cdot\vec{E}dV \tag{15}$$