Surface Charge and Resistors

Just after connection:  
\( E \) may be the same everywhere

\[
\begin{align*}
    i &= nA\bar{v} = nAuE \\
    i_{\text{thin}} &= nA_{\text{thin}}uE \\
    i_{\text{thick}} &= nA_{\text{thick}}uE \\
    i_{\text{thin}} &= \frac{A_{\text{thin}}}{A_{\text{thick}}} i_{\text{thick}}
\end{align*}
\]

After steady state is reached:

\[
\begin{align*}
    i_{\text{thin}} &= i_{\text{thick}} \\
    i_{\text{thin}} &= nA_{\text{thin}}uE_{\text{thin}} \\
    i_{\text{thick}} &= nA_{\text{thick}}uE_{\text{thick}}
\end{align*}
\]

\[
\begin{align*}
    E_{\text{thin}} &= \frac{A_{\text{thick}}}{A_{\text{thin}}} E_{\text{thick}}
\end{align*}
\]
Surface Charge Distribution in a Circuit

1. Current must be constant: based on $i = nA\bar{v} = nAuE$
draw distribution of $E$

2. Based on $E$ draw approximate distribution of surface charges
A Wide Resistor

low mobility

\[ i = nA\bar{v} = nAuE \]

\[ i_{\text{thin}} = nAu_{\text{thin}}E_{\text{thin}} \]

\[ i_{\text{thick}} = nAu_{\text{thick}}E_{\text{thick}} \]

\[ E_{\text{thin}} = \frac{u_{\text{thick}}}{u_{\text{thin}}} E_{\text{thick}} \]
The current node rule
(Kirchhoff node or junction rule [law #1]):

In the steady state, the electron current entering a node in a circuit is equal to the electron current leaving that node.
Energy conservation (the Kirchhoff loop rule [2nd law]):

\[ \Delta V_{\text{wire}} = EL \quad \Delta V_{\text{battery}} = ? \]

\[ \Delta V_1 + \Delta V_2 + \Delta V_3 + \ldots = 0 \quad \text{along any closed path in a circuit} \]

\[ \Delta V = \Delta U / q \quad \text{← energy per unit charge} \]
The function of a battery is to produce and maintain a charge separation.

The *emf* is measured in Volts, but it is not a potential difference! The *emf* is the energy input per unit charge.

chemical, nuclear, gravitational...
Chemical Battery

http://www.howstuffworks.com/battery.htm

A battery maintains a potential difference across the circuit.
Chemical Battery

\[ \text{emf} = |\Delta V_{\text{batt}}| = 0.76 \text{ V} \]
The First Chemical Battery

1780: Luigi Galvani, bioelectrogenesis

Animal electricity

1800: Alessandro Volta: artificial electrical organ

Alessandro Giuseppe Antonio Anastasio Volta (1745-1827)

1881: \( J/C = \) Volt
Internal Resistance of a Battery

\[ F_{NC} = F_C = eE_C \]

\[ i = nAuE = nAu \frac{\Delta V}{L} = nAu \frac{(emf)}{L} \]

Increase mobility – current increases

Real battery cannot provide \( \infty \) current

Internal battery resistance – limits maximum current

Drift speed in battery:

\[ \bar{v}_b = u_b E_{net} = u_b \left( \frac{F_{NC}}{e} - E_C \right) \]

Maximum drift speed:

\[ E_C = 0 \rightarrow \bar{v}_{b,\text{max}} = u_b \frac{F_{NC}}{e} \rightarrow \Delta V = 0 \]

For large \( u_b \):

\[ E_C \approx \frac{F_{NC}}{e} \rightarrow \Delta V \approx (emf) \]
Field and Current in a Simple Circuit

Round-trip potential difference:

\[ \Delta V_{\text{batt}} + \Delta V_{\text{wire}} = 0 \]

\[ emf + (-EL) = 0 \]

\[ E = \frac{emf}{L} \]

\[ I = enAuE = enAu \frac{emf}{L} \]
Field and Current in a Simple Circuit

Round-trip potential difference:

**Path 1**

\[ \Delta V_{batt} + \Delta V_1 + \Delta V_3 = 0 \]

\[ emf + (-E_1 L_1) + (-E_3 L_3) = 0 \]

**Path 2**

\[ \Delta V_{batt} + \Delta V_2 + \Delta V_3 = 0 \]

\[ emf + (-E_2 L_2) + (-E_3 L_3) = 0 \]

\[ E_1 L_1 = E_2 L_2 \]
ΔV Across Connecting Wires

The number or length of the connecting wires has little effect on the amount of current in the circuit.

\[ i = nAuE = nAu \frac{\Delta V}{L} \quad \rightarrow \quad \Delta V = \frac{L}{nAu} i \]

\[ \Delta V_{\text{wires}} + \Delta V_{\text{filament}} + \Delta V_{\text{battery}} = 0 \]

\[ \Delta V_{\text{battery}} \approx \text{emf} \]

\[ u_{\text{wires}} > > u_{\text{filament}} \quad \rightarrow \quad \Delta V_{\text{wires}} << \Delta V_{\text{filament}} \]

\[ \Delta V_{\text{filament}} \approx (\text{emf}) \]

Work done by a battery goes mostly into energy dissipation in the bulb (heat).
General Use of the Loop Rule

\[
\Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0
\]

\[
(V_B - V_A) + (V_C - V_B) + (V_F - V_C) + (V_A - V_F) = 0
\]
Application: Energy Conservation

\[ \Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_{\text{batt}} = 0 \]

\[ (-E_1L_1) + (-E_2L_2) + (-E_3L_3) + \text{emf} = 0 \]

1. If battery \textit{emf}=1.5V, what is the voltage drop on thin wire?

2. What is the current in this circuit?
A Nichrome wire 50 cm long and 0.5 mm thick is connected to a 1.5 V battery.

1. What is the electric field inside the wire?

2. How would it change if we change the wire diameter to 1 mm?

3. What is the current in this circuit?
Quantitative measurement of current with a compass

\[ i = n \Delta V \frac{E}{L} \]

\[ i_{2L} = \frac{1}{2} i_L \]

Current is halved when increasing the length of the wire by a factor of 2.
Electron mobility in metals decreases as temperature increases! Conversely, electron mobility in metals increases as temperature decreases. Thus, the current in the 2-bulb circuit is slightly more than that in the one-bulb circuit.
Doubling the Cross-Sectional Area

Electron current in the wire increases by a factor of two if the cross-sectional area of the wire doubles.

\[ i = nA u E = nA u \frac{\Delta V}{L} \]

Loop: \( emf - EL = 0 \)
Two Light Bulbs in Parallel

\[ i = nAuE = nAu \frac{\Delta V}{L} \quad L \ldots \text{length of bulb filament} \]

1. Path ABDFA:
\[ -2(emf) + E_B L = 0 \quad \rightarrow \quad E_B = \frac{2(emf)}{L} \]

2. Path ACDFA:
\[ -2(emf) + E_C L = 0 \quad \rightarrow \quad E_C = \frac{2(emf)}{L} \quad \rightarrow \quad E_B = E_C \]

3. Path ABDCA:
\[ E_B L - E_C L = 0 \quad \rightarrow \quad E_B = E_C \]

\[ i_B = i_C \]
\[ i_{\text{batt}} = 2i_B \]

We can think of the two bulbs in parallel as equivalent to increasing the cross-sectional area of one of the bulb filaments.
How Do the Currents Know How to Divide?
Two Batteries in Series

Why light bulb is brighter with two batteries?

Two batteries in series can drive more current: Potential difference across two batteries in series is $2emf \rightarrow$ doubles electric field everywhere in the circuit $\rightarrow$ doubles drift speed $\rightarrow$ doubles current.

\[
emf - EL = 0
\]
\[
E = \frac{emf}{L}
\]
\[
i = nAuE = nAu \frac{emf}{L}
\]
\[
P_{1\text{batt}} = eLnAu \left( \frac{emf}{L} \right)^2
\]

Work per second:
\[
P = \frac{(q / T)EL}{ieEL}
\]
\[
P = nAueLE^2
\]

\[
2emf - EL = 0
\]
\[
E = \frac{2emf}{L}
\]
\[
i = nAu \frac{2emf}{L}
\]
\[
P_{2\text{batt}} = eLnAu \left( \frac{2emf}{L} \right)^2
\]
\[
P_{2\text{batt}} = 4 \times P_{1\text{batt}}
\]
Analysis of Circuits

The current node rule (Charge conservation)
Kirchhoff node or junction rule [1st law]:

In the steady state, the electron current entering a node in a circuit is equal to the electron current leaving that node

Electron current: \( i = nAuE \)
Conventional current: \( I = |q|nAuE \)

The loop rule (Energy conservation)
Kirchhoff loop rule [2nd law]:

\[ \Delta V_1 + \Delta V_2 + \Delta V_3 + \ldots = 0 \] along any closed path in a circuit

\[ \Delta V = \Delta U/q \quad \leftarrow \text{energy per unit charge} \]