Disclaimer:

This lecture reviews some but not all of the material that will be on the final exam that covers in Chapters 29-33.

Chapter 29 Alternating-Current Circuits (1)

- LC circuit: Energy stored 
  \[ U_c = \frac{1}{2} C V_{\text{max}}^2 \quad U_L = \frac{1}{2} L I_{\text{max}}^2 \]

- LC circuit: Oscillation frequency 
  \[ \omega_0 = \frac{1}{\sqrt{LC}} \]

- RLC circuit:
  \[ q = q_{\text{max}} e^{\frac{R t}{2L}} \cos(\omega t + \phi) \]
  \[ \omega = \sqrt{\omega_0^2 - \left( \frac{R}{2L} \right)^2} \quad \omega_h = \frac{1}{\sqrt{LC}} \]

- With time varying emf

Chapter 29 Alternating-Current Circuits (2)

- Time-varying emf
  \[ V_{\text{emf}} = V_{\text{max}} \sin \omega t \]

- Time-varying emf \( V_R \) with resistor
  \[ i_R = \frac{V_R}{R} = I_R \sin \omega t \]

- Time-varying emf \( V_C \) with capacitor
  \[ X_C = \frac{1}{\omega C} \quad i_C = \frac{V_C}{X_C} \sin(\omega t + 90^\circ) \]

- Time-varying emf \( V_L \) with inductor
  \[ X_L = \omega L \quad i_L = \frac{V_L}{X_L} \sin(\omega t - 90^\circ) \]

Chapter 29 Alternating-Current Circuits (3)

Summary: Phase and Phasors
Chapter 29 Alternating-Current Circuits (4)
Transformers
- Consider the second coil with \(N_s\) turns
- The time-varying emf in the primary coil induces a time-varying magnetic field in the iron core. This core passes through the secondary coil
- Thus a time-varying voltage is induced in the secondary coil described by Faraday’s Law
- Because both the primary and secondary coils experience the same changing magnetic field, we can write
  \[ V_s = N_s \frac{d\Phi_s}{dt}, \quad V_p = N_p \frac{d\Phi_p}{dt} \Rightarrow V_p = V_s \frac{N_p}{N_s} \]
  \[ N_p < N_s \Rightarrow V_p < V_s \quad \text{step-up} \]
  \[ N_p > N_s \Rightarrow V_p > V_s \quad \text{step-down} \]

Chapter 29 Alternating-Current Circuits (5)
- Single loop RLC Circuit with time-varying emf:
  \[ V = iZ \quad Z = \sqrt{R^2 + (X_C - X_L)^2} \quad \text{(impedance)} \]
  \[ X_L = \omega L \quad X_C = \frac{1}{\omega C} \]
  \[ \phi = \tan^{-1}\left(\frac{X_C - X_L}{V_s}ight) = \tan^{-1}\left(\frac{V_s - V_p}{R}\right) \]
  \[ \text{The average power} \quad \langle P \rangle = V_{rms} I_{rms} \cos \phi \quad V_{rms} = \frac{V_{peak}}{\sqrt{2}} \quad I_{rms} = \frac{I_{peak}}{\sqrt{2}} \]

Chapter 29 Alternating-Current Circuits (6)
Resonance
For given \(v_{\text{peak}}\), \(R\), \(L\), and \(C\), the current amplitude \(I_{\text{peak}}\) will be at the maximum when the impedance \(Z\) is at the minimum.
  \[ v_{\text{peak}} = \frac{I_{\text{peak}}}{Z} \sqrt{R^2 + (X_L - X_C)^2} \quad I_{\text{peak}} = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}} \]
  \[ X_L = X_C \Leftrightarrow \omega_m L = \frac{1}{\omega_m C}, \quad Z = R, \text{ and } I_{\text{peak}} = \frac{v_{\text{peak}}}{R} \]
  \[ i.e., \text{load purely resistive} \]
  This is called resonance.
  Resonance angular frequency:
  \[ \omega_{\text{res}} = \frac{1}{\sqrt{LC}} \]

Chapter 30 Maxwell’s Equations and Electromagnetic Waves (1)
Maxwell’s Equations
- Integral Form
  \[ \oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{\partial \mathbf{D}}{\partial t} \cdot \epsilon_0 \quad (1) \]
  \[ \oint_S \mathbf{B} \cdot d\mathbf{A} = 0 \quad (2) \]
  \[ \int_S \mathbf{E} \cdot d\mathbf{l} = -\frac{d\mathbf{F}_Z}{dt} \quad (3) \]
  \[ \int_S \mathbf{B} \cdot d\mathbf{l} = \mu_0 \mathbf{j} + \mu_e \frac{d\Phi_L}{dt} \quad (4) \]
Wave solutions:
  \[ \mathbf{E} = E_{\text{rms}} \sin(kx - \omega t) \hat{y} \]
  \[ \mathbf{B} = B_{\text{rms}} \sin(kx - \omega t) \hat{z} \]
  \[ c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \lambda f \]
where \(k = 2\pi/\lambda\) and \(\omega = 2\pi f\) with wavelength \(\lambda\) and frequency \(f\).
Characteristics

1. \( \vec{E} \perp \vec{B} \).
2. If \( E = E_0 \sin(kx - \omega t) \), \( B = B_0 \sin(kx - \omega t) \).
3. The Poynting vector \( \vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} \) is in the direction of propagation.
4. \( \vec{E} \) and \( \vec{B} \) are both \( \perp \vec{S} \).
5. \( c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 2.99792458 \times 10^8 \) m/s (exact)
6. \( E = cB \)

Chapter 30

**Pressure** \( P = \text{Force/Area} \)

**Total Absorption**

\[ P_r = \frac{I}{c} \]

**Total Reflection**

\[ P_r = \frac{2I}{c} \]

*Polarization*

\[ I_{\text{out}} = I_{\text{in}} \cos^2 \theta \]

\[ I_{\text{out}} = \frac{1}{2} I_{\text{in}} \] (unpolarized light)

Chapter 30 Maxwell’s Equations and Electromagnetic Waves (2)

- Intensity of an electromagnetic wave:
  \[ I = \frac{1}{c \mu_0} E_0^2 \quad (E_{\text{rms}} = E_{\text{max}} / \sqrt{2}) \quad u_E = \frac{1}{2} E_0^2 \quad u_B = \frac{1}{2 \mu_0} B^2 \]

- Energy density:
  \[ E = cB \quad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \Rightarrow u_E = \frac{E^2}{2 \mu_0 c^2} = \frac{1}{2} E_0^2 = u_E \quad \therefore u_B = u_E \]

Chapter 31 Properties of Light (5)

**Intensity after Polarizer**

The intensity of the light \( I_b \) before the polarizer is given by

\[ I_b = \frac{1}{c \mu_0} E_{\text{rms}}^2 = \frac{1}{2 c \mu_0} E_0^2 \]

- After the light passes through the polarizer, the intensity \( I \) is given by
  \[ I = \frac{1}{2 c \mu_0} E_0^2 \]

- The transmitted intensity in terms of the initial intensity is
  \[ I = \frac{1}{2 c \mu_0} E_0^2 = \frac{1}{2 c \mu_0} (E_0 \cos \theta)^2 = I_b \cos^2 \theta \]

- This result is called the Law of Malus
- This equation only applies to the case of polarized light incident on a polarizer
Chapter 31 Properties of Light (1)

- Law of Reflection: \( \theta_i = \theta_f \)
- Focal length of a spherical mirror:
  \[ f = \frac{R}{2} \]
- Mirror Equation:
  \[ \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \]
- Magnification:
  \[ m = \frac{d_i}{d_o} = \frac{h_i}{h_o} \]
- Law of Refraction (Snell’s Law):
  \[ n_1 \sin \theta_i = n_2 \sin \theta_f \]
- The critical angle for a total reflection:
  \[ \sin \theta_c = \frac{n_2}{n_1} \quad (n_2 \leq n_1) \]

Chapter 31 Properties of Light (2)

- Laws of Reflection and Refraction: Summary
  - Law of Reflection
    - A reflected ray lies in the plane of incidence
    - The angle of reflection is equal to the angle of incidence
      \[ \theta_f = \theta_i \]
  - Law of refraction
    - Index of refraction \( n \):
      - A refracted ray lies in the plane of incidence
      - The angle of refraction is related to the angle of incidence by
        \[ n_2 \sin \theta_2 = n_1 \sin \theta_1 \]
        **Snell’s Law**
        \[ \frac{\lambda}{f} = \frac{c}{n_2} = \frac{c}{n_1} \]
        \[ n_2 > n_1 \]

Chapter 31 Properties of Light (3)

- Total Internal Reflection
  - The critical angle, \( \theta_c \), at which total internal reflection takes place is given by
    \[ \frac{n_2}{n_1} = \frac{\sin \theta_i}{\sin \theta_f} = 90^\circ \]
    \[ \theta_c = \theta_i \text{ and } n_{\text{air}} = n_2, \quad \theta_c = \theta_i \]
    \[ \sin \theta_c = \frac{n_2}{n_1} \]
  - Which we can rewrite as
    \[ \sin \theta_c = \frac{n_2}{n_1} \]
    \[ n_2 \sin \theta_2 = n_1 \sin \theta_1 \]
  - For \( n_2 = 1 \) (air)
    \[ n_2 \sin \theta_2 = \frac{n_2}{n_1} \]
    \[ \sin \theta_c = 1 \Rightarrow (n_2 < n_1) \]
  - At angles less than \( \theta_c \), some light is reflected and some transmitted
  - At angles greater than \( \theta_c \), all the light is reflected and none is transmitted

Chapter 31 Properties of Light (4)

- Chromatic Dispersion
  - The index of refraction of a medium is usually a function of the wavelength of the light. It is larger at shorter wavelengths.
  - This causes spreading of light which is called chromatic dispersion.
  - White light consists of components of nearly all the colors in the visible spectrum with approximately uniform intensities.
  - The component of a beam of white light with shorter wavelength tends to be bent more.
  - **Spectrometer** (such as a prism)
Chapter 31 Properties of Light (6)

Wave-Particle Duality of Light

- Refraction, diffraction (Huygens's Principle)
- Interference (Young’s double slit interference)
- Electromagnetic waves (Maxwell’s Equations)
  - Light and other EM radiation often come together (e.g., in Black-body radiation)
  - Doesn’t require a medium

Quantum Mechanics: Duality for all particles

Particle nature of light

- Collisions, scattering as in photoelectric effect (Albert Einstein)
  - Energy is quantized: \( E = hf = hc / \lambda \)
  - massless photons \( h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} \) Planck’s constant

Chapter 32 Optical Images (1)

- Thin-Lens Equation:
  \[
  \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}
  \]

- Lens-Maker’s Formula:
  \[
  \frac{1}{f} = (n - 1) \left( \frac{1}{R} - \frac{1}{R'} \right)
  \]

Chapter 32 Optical Images (2)

Mirror Equation and Magnification

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad (f = r/2)
\]

- \( |m| = \frac{P'}{P} = \frac{s'}{s} \)

\( \Rightarrow \quad m = -\frac{s'}{s} \)

- Using these sign conventions we can express the mirror equation in terms of the object distance, \( d_o \), and the image distance, \( d_i \), and the focal length \( f \) of the mirror

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad (f = r/2)
\]

\( s \to d_o \quad s' \to d_i \)

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}
\]

\( f = \frac{r}{2} \)

- The magnification \( m \) of the mirror is defined to be

\[
m = -\frac{d_i}{d_o} = \frac{h_i}{h_o}
\]

Chapter 32 Optical Images (3)

Lens Equation

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}
\]

Images Formed with Concave Lenses

The image formed is virtual, upright, and reduced

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}
\]
Chapter 32 Optical Images (4) Systems of Lenses

- Images formed by systems of lenses can be obtained using a set of equations

\[ \frac{1}{d_{11}} + \frac{1}{d_{12}} = \frac{1}{f_1} \quad \frac{1}{d_{21}} + \frac{1}{d_{22}} = \frac{1}{f_2} \]

\[ m_1 = \frac{h_{11}}{h_{12}} \Rightarrow m_2 = \frac{h_{21}}{h_{22}} \Rightarrow m_1 m_2 = \left( \frac{h_{11}}{h_{12}} \right) \left( \frac{h_{21}}{h_{22}} \right) = \frac{h_{11}}{h_{12}} \Rightarrow h_{11} = h_{12} \]

Chapter 33 Interference and Diffraction (1)

- Huygens’ Principle: every point on a propagating wave front serves as a source of spherical secondary wavelets

- The requirement for constructive interference of two coherent waves with \( \lambda \)

\[ \Delta x = m\lambda \quad (m = 0, m = \pm 1, m = \pm 2, \ldots) \]

- The requirement for destructive interference of two coherent waves with \( \lambda \)

\[ \Delta x = \left( m + \frac{1}{2} \right) \lambda \quad (m = 0, m = \pm 1, m = \pm 2, \ldots) \]

- Two slits with separation \( d \): bright interference fringes

\[ \Delta x = d \sin \theta = m \lambda \quad (m = 0, m = \pm 1, m = \pm 2, \ldots) \]

- On a screen a long distance \( L \) away:

\[ y = \frac{m\lambda L}{d} \quad (m = 0, m = \pm 1, m = \pm 2, \ldots) \]

Chapter 33 Interference and Diffraction (2)

- Two slits with separation \( d \): dark interference fringes

\[ \Delta x = d \sin \theta = \left( m + \frac{1}{2} \right) \lambda \quad (m = 0, m = \pm 1, m = \pm 2, \ldots) \]

- On a screen a long distance \( L \) away:

\[ y = \left( m + \frac{1}{2} \right) \frac{\lambda L}{d} \quad (m = 0, m = \pm 1, m = \pm 2, \ldots) \]

- For a thin film with thickness \( t \), the condition for constructive interference is

\[ \frac{m + \frac{1}{2}}{n} \frac{\lambda}{n} = 2t \quad (m = 0, m = \pm 1, m = \pm 2, \ldots) \]

- The radii of the bright circles in Newton’s rings are given by

\[ R = \frac{m + \frac{1}{2}}{n} \frac{\lambda}{n} \quad (m = 1, 2, 3, \ldots) \]

\[ \frac{R}{\lambda} \ll 1 \]

- The angle of dark diffraction fringes from a single slit of width \( a \):

\[ a \sin \theta = m \lambda \quad (m = 1, 2, 3, \ldots) \]

Chapter 33 Interference and Diffraction (3)

- The angle of the first minimum from a circular aperture with diameter \( D \) illuminated with light of the wave length \( \lambda \):

\[ \sin \theta = \frac{1.22 \lambda}{D} \]

- Rayleigh’s Criterion:

\[ \theta_s = \sin^{-1} \left( \frac{1.22 \lambda}{D} \right) = \frac{1.22 \lambda}{D} \]

- The angle of the maxima from a diffraction grating illuminated with light of the wave length \( \lambda \) (\( d \) is the distance between rulings of the grating):

\[ \theta = \sin^{-1} \left( \frac{m \lambda}{d} \right) \quad (m = 0, 1, 2, \ldots) \]

Because diffraction gratings produce widely spaced narrow maxima, they can be used to determine the wavelength of monochromatic light.
A capacitor in an LC circuit oscillator has a maximum potential difference of 15 V and a maximum energy 360 µJ. At a certain instant, the energy in the capacitor is 40 µJ. At that instant what is the potential difference across the capacitor?

Example

An AC generator producing 10 V (rms) at 200 rad/s is connected in series with a 50 ohm resistor, 400 mH inductor and a 200 µF capacitor. The rms voltage (in volts) across the resistor is:

Consider a point \((x,y,z)\) at time \(t\) when \(E_x\) is negative and has its maximum value. At \((x,y,z)\) at time \(t\), what is \(B_y\)?

A) \(B_y\) is positive and has its maximum value
B) \(B_y\) is negative and has its maximum value
C) \(B_y\) is zero
D) We do not have enough information

Which equation correctly describes this electromagnetic wave?

- \(E_x = E_0 \sin(kz - \omega t)\)
- \(E_y = E_0 \sin(kz + \omega t)\)
- \(B_y = B_0 \sin(kz - \omega t)\)
Example

An unpolarized beam of light has intensity $I_o$. It is incident on two ideal polarizing sheets. The angle between the axes of polarization of these sheets is $\theta$. Find $\theta$ if the emerging light has intensity $I_o/4$.

Problem

Question 3. (5 points)
In a stack of three polarizing sheets, the first and third are crossed while the middle one has its axis at 45° to the axes of the other two. The fraction of the intensity of an incident unpolarized beam of light that is transmitted by the stack is:

A) 0
B) 1/8
C) 1/4
D) 1/2
E) 1/3

Example

A laser beam of power 4.60 W and diameter 2.60 mm is directed upward at one circular face (of diameter less than 2.60 mm) of a perfectly reflecting cylinder, which is made to “hover” by the beam’s radiation pressure. The cylinder’s density is 1.20 g/cm³. What is the height (in meters) of the cylinder?
Example: Snell’s Law

**Question 8. (5 points)** The diagram shows the passage of a ray of light from air into a substance X. The index of refraction of X is: 

- A) 0.53
- B) 0.88
- C) 1.9
- D) 2.2
- E) 3.0

![Diagram of light rays](image)

(b) Calculate the critical angle for a diamond under water.

1) 30.61
2) 33.44

Example: Convex Mirror

In a subway station, a convex mirror allows the attendant to view activity on the platform. A woman 1.64 m tall is standing 4.5 m from the mirror. The image formed of the woman is 0.500 m tall.

(a) What is the radius of curvature of the mirror?

1) 3.9 m;
2) 4.5 m;

Example: Chapter 24 Geometric Optics

(b) Calculate the critical angle for a diamond under water.

1) 30.61
2) 33.44

Example

A concave spherical mirror has a focal point of 12 cm. If an object is placed 6 cm in front of the mirror, the image position is
Example

How thick is an anti-reflection coating for a glass lens which is chosen for light of wavelength 500 nm if the index of refraction of the coating is 1.29?

Example: Double Slit

In a double-slit interference experiment, the wavelength is 562 nm, the slit separation is 0.1 mm, and the screen is 30 cm away from the slits. What is the linear distance between adjacent maxima on the screen?

\[ d \sin \theta = m \lambda \]
\[ \sin \theta \sim \theta = \frac{\lambda}{d} = 0.00562 \]
\[ \tan \theta = \frac{y}{L} \]
\[ y = L \tan \theta = 1.68 \text{ mm} \]

Example: Thin Film

A thin film of oil (n = 1.48) is spread over a puddle of water (n = 1.33). In a region where the film looks red from directly above (λ = 631 nm), what is the minimum possible thickness of the film?