**LC Circuits**

- Consider the LC and RC series circuits shown:
- Suppose that at \( t=0 \) the capacitor is charged to a value of \( Q \).

Is there a qualitative difference in the time development of the currents produced in these two cases. Why??

**LC Oscillations**

Kirchhoff’s loop rule

\[
V_L + V_C = L \frac{dI}{dt} + \frac{Q}{C} = 0
\]

**LC Oscillations: Energy Check**

- Oscillation frequency \( \omega_o = \frac{1}{\sqrt{LC}} \) has been found from the loop equation.
- The other unknowns \( (Q_0, \phi) \) are found from the initial conditions. Eg in our original example we took as given, initial values for the charge \( (Q_0) \) and current \( (0) \). For these values: \( Q_0 = Q_0, \phi = 0 \).
- Question: Does this solution conserve energy?
Energy Check

Energy in Capacitor

\[ U_E(t) = \frac{1}{2} Q_C^2 \cos^2(\omega_0 t + \phi) \]

Energy in Inductor

\[ U_B(t) = \frac{1}{2} L \omega_0^2 Q_L^2 \sin^2(\omega_0 t + \phi) \]

\[ \omega_0 = \frac{1}{\sqrt{LC}} \]

\[ U_B(t) = \frac{1}{2} Q_L^2 \sin^2(\omega_0 t + \phi) \]

Therefore,

\[ U_E(t) + U_B(t) = \frac{Q_C^2}{2C} \]

LC Oscillations with Finite R

- If \( L \) has finite \( R \), then energy will be dissipated in \( R \) and the oscillations will become damped.

\[ Q \]

\[ t \]

\[ R = 0 \]

\[ Q \]

\[ t \]

\[ R \neq 0 \]

Driven Oscillations

- An LC circuit is a natural oscillator.

\[ \omega_{\text{resonance}} = \frac{1}{\sqrt{LC}} \text{ in absence of resistive loss} \]

- In a real LC circuit, we must account for the resistance of the inductor. This resistance will damp out the oscillations.

\[ R \]

\[ L \]

\[ Q \]

\[ t \]
AC Circuits: Series LCR

- Statement of problem:
  Given $\varepsilon = \varepsilon_m \sin \omega t$, find $I(t)$.
  Everything else will follow.

Phasors: LCR

\[ \varepsilon = \varepsilon_m \sin \omega t \]
\[ I = I_m (\omega t - \phi) \]
\[ Q = -\frac{I_m}{\omega} \cos (\omega t - \phi) \]
\[ V_R = R I = R I_m \sin (\omega t - \phi) \]
\[ V_C = \frac{Q}{C} = -\frac{1}{\omega C} I_m \cos (\omega t - \phi) \]
\[ V_L = L \frac{dI}{dt} = \omega L I_m \cos (\omega t - \phi) \]

- From these equations, we can draw the phasor diagram at the right.

Resonance

- For fixed $R, C, L$ the current $I_m$ will be a maximum at the resonant frequency $\omega_0$, which makes the impedance $Z$ purely resistive.

\[ I_m = \frac{\varepsilon_m}{Z} = \frac{\varepsilon_m}{\sqrt{R^2 + (X_L - X_C)^2}} \]

reaches a maximum when:

\[ X_L = X_C \]

the frequency at which this condition is obtained is given from:

\[ \omega_0 L = \frac{1}{\omega_0 C} \]

\[ \omega_0 = \frac{1}{\sqrt{LC}} \]

- Note that the resonant frequency is identical to the natural frequency of the LC circuit by itself.

- At this frequency, the current and the driving voltage are in phase!

\[ \tan \phi = \frac{X_L - X_C}{R} = 0 \]
Power in LCR Circuit

- The power supplied by the emf in a series LCR circuit depends on the frequency $\omega$. The maximum power is supplied at the resonant frequency $\omega_0$.
  - The instantaneous power (for some frequency, $\omega$) delivered at time $t$ is given by:
    $$ P(t) = \epsilon(t) I(t) = (\epsilon_m \sin \omega t)(I_m \sin(\omega t - \phi)) = I^2(t)R $$
  - The most useful quantity to consider here is not the instantaneous power but rather the average power delivered in a cycle.

$$ \langle P(t) \rangle = \epsilon_m I_m \sin \omega t \sin(\omega t - \phi) $$

$$ \langle P(t) \rangle = \frac{1}{2} V_m I_m \cos \phi $$

- This result is often rewritten in terms of rms values:
  $$ \epsilon_{rms} = \frac{1}{\sqrt{2}} \epsilon_m \quad I_{rms} = \frac{1}{\sqrt{2}} I_m \quad \langle P(t) \rangle = \epsilon_{rms} I_{rms} \cos \phi $$

- Power delivered depends on the phase, $\phi$, the “power factor”

- Phase depends on the values of $L$, $C$, $R$, and $\omega$ and therefore...

$$ \tan \phi = \frac{X_L - X_C}{R} \quad \cos \phi = \frac{R}{Z} $$