LECTURE 17

• Motional EMF
• Eddy Currents
• Self Inductance

Reminder: How to Change Magnetic Flux in a Coil

1. $|\vec{B}|$ changes: $\frac{d\Phi_m}{dt} = \frac{dB}{dt} NA \cos(\vec{B},\vec{A})$

2. $A$ changes: $\frac{d\Phi_m}{dt} = NB \frac{dA}{dt} \cos(\vec{B},\vec{A})$

3. $(\vec{B},\vec{A})$ changes: $\frac{d\Phi_m}{dt} = NBA \frac{d\left[\cos(\vec{B},\vec{A})\right]}{dt}$

4. $N$ changes (unlikely): $\frac{d\Phi_m}{dt} = \frac{dN}{dt} BA \cos(\vec{B},\vec{A})$

Example Problem: Solenoid within a Coil

- 120 turn coil of radius 2.4 cm and resistance 5.3 $\Omega$
- Solenoid with radius 1.6 cm and $n = 220$ turns/cm

Initial current in the solenoid is 1.5 A. Current is reduced to zero in 25 ms. What is the current in the coil while the current is being reduced?
Motional EMF

Motional EMF is any emf induced by the motion of a conductor in a magnetic field.

Energy Conservation

- Rate of work by applied force:

- The induced current gives rise to a net magnetic force $\mathbf{F}$ in the loop which opposes the motion:

- External Agent must exert equal but opposite force $\mathbf{F}_R$ to move the loop with velocity $v$; therefore, agent does work at rate $P$.

- Energy is dissipated in circuit at rate $P'$.

Calculation

A thin conducting rod is pulled with velocity $v$ along conducting rails that are connected by a resistor, $R$. A uniform magnetic field $\mathbf{B}$ is directed into the page. Surface $S$ is increasing therefore flux is increasing. What will be the magnitude & direction of the induced current?

Eddy Currents

*Relative motion between a $\mathbf{B}$ field and a conductor induces a current in the conductor.

*The induced current gives rise to a net magnetic force, $\mathbf{F}_M$, which opposes the motion.
Eddy Currents

- During the last lecture we considered currents confined to a loop or a coil.
- Electrical equipment contains other metal parts which are often located in changing B fields. Currents in such parts are called Eddy currents.

Induced current
- anti-clockwise
- clockwise

Flux
- increasing
- decreasing

B field into the page

Demonstration: Eddy Currents
Reduce Eddy Currents

Metal strips with insulating glue

Cut slots into the metal.

Inductors & Inductance

*An inductor can be used to produce a desired B field.

Symbol for inductor

*Inductance:

\[ L = \frac{N\Phi_M}{i} \]

Units of L:

1 Henry = 1H = 1Tm^2/A

Self Inductance

An induced emf, \( \varepsilon_L \), appears in any coil in which the current is changing.

Self-Induction: Changing current through a loop induces an opposing voltage in that same loop.

Self Inductance in a Coil
Magnetic Energy in an Inductor

Upon closing switch, S, apply Kirchoff’s loop rule:

\[ \varepsilon - IR - L \frac{dI}{dt} = 0 \]

Multiply through by I:

\[ \varepsilon I = I^2 R + LI \frac{dI}{dt} \]

- power delivered by battery
- power dissipated by resistor
- power delivered to inductor

\[ U_m = \frac{1}{2} LI^2 \]

energy stored in an inductor

Magnetic Energy in a Solenoid

\[ B = \mu_0 n I \Rightarrow I = \frac{B}{\mu_0 n} \]

\[ L = \mu_0 n^2 Al \]

\[ U_m = \frac{1}{2} LI^2 = \frac{1}{2} \mu_0 n^2 Al \left( \frac{B}{\mu_0 n} \right)^2 = \frac{B^2}{2\mu_0} Al \]

Both of these are general results.

\[ u_m = \frac{B^2}{2\mu_0} \]

magnetic energy density

\[ u_e = \frac{1}{2} \varepsilon_0 E^2 \]

electric energy density