LECTURE 15

• Force between Current Carrying Wires
• Gauss’ Law for Magnetism
• Ampere’s Law
• Magnetism in Matter

\[ \vec{F} = I L \times \vec{B} \]

*Parallel currents attract
*Anti-parallel currents repel

**Force between Two Parallel Current Carrying Wires**

- Parallel currents attract
- Anti-parallel currents repel

**DEMO**

\[ |I_1| + |I_2| \approx 500 \text{A} \]
**Force Between two Parallel Current Carrying Wires**

- **Attraction**
- **Repulsion**

**Gauss' Law for Magnetism**

\[
\Phi_{B\text{net}} = \oint \mathbf{B} \cdot d\mathbf{A} = 0
\]

Since all lines of \( B \) are closed loops, any \( B \) line leaving a closed surface MUST reenter it somewhere. TRUE IN GENERAL, not just for this “dipole” example.

**Compare Gauss’ Law for Electric Fields**

\[
\Phi_{\text{Enet}} = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}
\]

**Ampere's Law**

- Calculate field at distance \( r \) from wire using Ampere’s Law:

\[
\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{Enclosed}}
\]

\[
\begin{align*}
2\pi r B &= \mu_0 I \\
B &= \frac{\mu_0 I}{2\pi r}
\end{align*}
\]
Magnetic Field Inside a Long Straight Wire

\[ \oint \vec{B} \cdot d\vec{l} = B \int_0^{2\pi r} dl = 2\pi r B = \mu_o I_{enc} \]

Magnetic Field Inside & Outside a Long Straight Wire

Inside

\[ B = \frac{\mu_o I}{2\pi R^2} \]

Outside

\[ B \approx \frac{1}{r} \]

Solenoid

*Direction of \( \vec{B} \): Use the right hand rule. Curl fingers in direction of current flow in loop and thumb points in the direction of \( \vec{B} \).

Magnetic Field Inside an Ideal Solenoid

\[ \oint \vec{B} \cdot d\vec{l} = \int_a^b \vec{B} \cdot d\vec{l} + \int_b^c \vec{B} \cdot d\vec{l} + \int_c^d \vec{B} \cdot d\vec{l} + \int_d^a \vec{B} \cdot d\vec{l} \]
When Ampere’s Law doesn’t Help

- **$B$ can’t be factored out of the integral.**
- **insufficient symmetry**
- **finite length current segment**
- **current is not continuous**

Dipole Moments in Applied Fields

- **Electric dipole**
  - $\vec{p}$
  - External fields tend to align dipoles.
- **Magnetic Dipole**
  - $\vec{\mu}$
  - $\vec{E}_{app}$ decreases at center
  - $\vec{B}_{app}$ increases at center
Magnetization and “Bound Current”

- Strong externally applied field $B_{app}$ aligns the magnetic moments in matter. → **Magnetization**
- Amperian current

$$\bar{M} = \frac{\mu}{V} = \left( \frac{d\mu}{dV} \right)$$

- Amper: Aligned magnetic moments in magnetized matter arise due to microscopic current loops inside the material. → **Bound current**

Net current inside the material is zero. We are left with a surface current and therefore a magnetic moment.

Magnetization and Magnetic Susceptibility

- Magnetization $M$
  $$\bar{M} \propto \bar{B}_{app} \rightarrow \bar{M} = \frac{\chi_m}{\mu_0} \bar{B}_{app} \rightarrow \bar{B}_m = \mu_0 \bar{M} = \chi_m \bar{B}_{app}$$

- Magnetic Susceptibility $\chi_m$
  $$\therefore \bar{B}_m = \chi_m \bar{B}_{app}$$

$$\bar{B} = \bar{B}_{app} + \bar{B}_m = \bar{B}_{app} + \mu_0 \bar{M} = \left( 1 + \chi_m \right) \bar{B}_{app}$$

Relative permeability $K_m$

- $\chi_m > 0$, small $\chi_m$ **Paramagnetism** (aluminum, tungsten….)
- $\chi_m < 0$, small $|\chi_m|$ **Diamagnetism** (bismuth, copper, silver….)
- $\chi_m > 0$, large **Ferromagnetism** (iron, cobalt, nickel, and their alloys)

Magnetic Materials fall into Three Categories

<table>
<thead>
<tr>
<th>Category</th>
<th>$\chi_m$</th>
<th>$K_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Paramagnetism</strong></td>
<td>Order of $(+10^{-5})$, depends on temperature</td>
<td>$1 + \varepsilon$</td>
</tr>
<tr>
<td><strong>Diamagnetism</strong></td>
<td>Decrease in $B$ is small aligns opposite $B_{app}$</td>
<td>Order of $(-10^{-5})$</td>
</tr>
<tr>
<td><strong>Ferromagnetism</strong></td>
<td>High degree of alignment even in weak $B_{app}$</td>
<td>positive and large</td>
</tr>
</tbody>
</table>
**Magnetic Susceptibility** \( \chi_m \)

\[ B = B_{\text{app}} (1 + \chi_m) \]

<table>
<thead>
<tr>
<th>Material</th>
<th>( \chi_m )</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bi</td>
<td>(-1.66 \times 10^{-5})</td>
<td>diamagnetic</td>
</tr>
<tr>
<td>Ag</td>
<td>(-2.6 \times 10^{-5})</td>
<td>diamagnetic</td>
</tr>
<tr>
<td>Al</td>
<td>(2.3 \times 10^{-5})</td>
<td>paramagnetic</td>
</tr>
<tr>
<td>Fe (annealed)</td>
<td>5,500</td>
<td>ferromagnetic</td>
</tr>
<tr>
<td>Permalloy</td>
<td>25,000</td>
<td>ferromagnetic</td>
</tr>
<tr>
<td>Mu-metal</td>
<td>100,000</td>
<td>ferromagnetic</td>
</tr>
<tr>
<td>Superconductor</td>
<td>(-1)</td>
<td>diamagnetic (perfect)</td>
</tr>
</tbody>
</table>

\( \chi_m \) negative for diamagnetics, small and positive for paramagnetics, large and positive for ferromagnetics.

**Classify Types of Magnetic Materials: DEMO**

Use a strong electromagnet to produce \( B_{\text{ext}} \).
Put samples in \( B_{\text{ext}} \).

**Paramagnetic**

\[ B = (1 + 10^{-5}) B_{\text{app}} \]

Al sample orients itself as a weak magnet.

**Diamagnetic**

\[ B = (1 - 10^{-5}) B_{\text{app}} \]

Bi (bismuth) sample orients itself \( \perp \) to \( B_{\text{ext}} \).

Practical summary:

\( s \) N ~ slight increase

\( s \) slight reduction