Point Charge in Uniform E field

\[ \vec{F} = m\vec{a} = mq\vec{E} \]

\[ \vec{a} = \frac{q}{m} \vec{E} \]

\[ v_x = v_0 \quad x = v_0 t \]

\[ y = \frac{1}{2} a_y t^2 = \frac{1}{2} \frac{q}{m} E \frac{x^2}{v_0^2} \]

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**Last Week…..Discrete Charges**

- Magnitude of the Electric Field due to a Point Charge:
  \[ E = \frac{1}{4\pi\varepsilon_0} \frac{|q|}{r^2} \]

- Electric Field due to a Dipole at point P:
  \[ E = \frac{qd}{2\pi\varepsilon_0 r^3} = \frac{p}{2\pi\varepsilon_0 \pi^3} \]

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**Today…**

- More on Electric Field:
  - Continuous Charge Distributions

**Text Reference:** Chapter 22.1
May useful examples 22-1-8 etc.
Calculate $E$-field due to charge distributions

1. Extension of point charge approach:
   \[ \vec{E} = \frac{kq}{r^2} \hat{r} \rightarrow \vec{dE} = \frac{k dq}{r^2} \hat{r} \text{ and then integrate.} \]

2. Use Gauss' law:
   \[ \oint \vec{E} \cdot d\vec{A} = \frac{1}{\varepsilon_0} Q_{\text{inside}} \]

Electric Fields from Continuous Charge Distributions

- **Principles (Coulomb's Law + Law of Superposition) remain the same.**

Only change:

\[ \sum \rightarrow \oint \]

Charge Distributions Problems

Step 1: Understand the geometry
Step 2: Choose $dq$
Step 3: Evaluate $dE$ contribution from the infinitesimal charge element
Step 4: Exploit symmetry as appropriate
Step 5: Set up the integral
Step 6: Solve the integral
Step 7: The Result!
Step 8: Check Limiting Cases

Charge Densities

- How do we represent the charge “$Q$” on an extended object?
  - Total charge
  - Small pieces of charge $dq$

Line of charge:
\[ \lambda = \text{charge per unit length [C/m]} \]

Surface of charge:
\[ \sigma = \text{charge per unit area [C/m}^2] \]

Volume of Charge:
\[ \rho = \text{charge per unit volume [C/m}^3] \]
**Problem:** calculate the electric field along z-axis due to a (circular) ring (of radius \( R \)) of uniform positive charge (with density \( \lambda \)).

- **Charge of element**
- **The electric field due to this element**

**Field due to charge on a ring**

**Steps 5-7**
- Integrate over the entire ring with \( \lambda \) uniform; \( z \) and \( R \) fixed

\[
E = E_z = \int dE_z = \frac{z \lambda}{4 \pi \varepsilon_0 \left( z^2 + R^2 \right)^{3/2}} \int_0^{2\pi} ds
\]

**Limiting cases**

**Step 8**

\[
E = \frac{q z}{4 \pi \varepsilon_0 \left( z^2 + R^2 \right)^{3/2}}
\]
**Charged Disk**

**Problem:** Calculate the electric field along the z-axis due to a (circular) disk (of radius $R$) of uniform positive charge (with density $\sigma$).

Pick any ring element (of infinitesimal width $dr$) of the disk. The charge of the element is:

The electric field due to the ring element

**Field due to the disk**

- Integrate over the entire disk

\[
E = \int dE = \int_0^R \frac{\sigma z}{4\varepsilon_0} \frac{2rdr}{(z^2 + r^2)^{3/2}}
\]

- Use substitution of variables

\[
\int X^m dX = \frac{X^{m+1}}{m+1}
\]

**Field due to the disk**

- Check Limiting Cases

\[
E = \int dE = \int_0^R \frac{\sigma z}{4\varepsilon_0} \frac{2rdr}{(z^2 + r^2)^{3/2}} = \frac{\sigma}{2\varepsilon_0} \left[ 1 - \frac{z}{\sqrt{z^2 + R^2}} \right]
\]
Important limiting cases for charged disk

\[ R \to \infty : \quad E_z \rightarrow \frac{\sigma}{2\varepsilon_0} = 2\pi k \sigma \quad \text{for} \quad z > 0 \]

\[ E_z \rightarrow -\frac{\sigma}{2\varepsilon_0} = -2\pi k \sigma \quad \text{for} \quad z < 0 \]

Same results for \( z \approx 0 \).

There is a discontinuity at \( z = 0 \).

Infinite sheet of positive charge

Uniform electric fields generated on both sides of sheet.

discontinuity in electric field at \( x = 0 \)

90° arc of charge

In this coordinate system you have to deal with \( E_x \) and \( E_y \).

In this coordinate system you have to deal with the horizontal component of \( E \) only.

uniform charge distribution

total charge = \( Q \)

90° arc of charge (continued)

\[ \lambda = \frac{Q}{2\pi r/4} = \frac{2Q}{\pi r} \]

\[ dq = \lambda r \, d\theta \]

\[ dE_x = \frac{k \, dq}{r^2} \cos \theta = \frac{k \lambda r \cos \theta \, d\theta}{r^2} \]

\[ E_x = \frac{k \lambda}{r} \left( \int_{\theta_1}^{\theta_2} \cos \theta \, d\theta \right) \]

\[ = \frac{k \lambda}{r} \left( \sin \theta_2 - \sin \theta_1 \right) \]