Magnetic Field

- Large Magnetic fields are used in MRI (Nobel prize for medicine in 2003)
- Extremely Large magnetic field are found in some stars
- Earth has a Magnetic Field
Bar Magnets

- Bar magnet
  Like poles repel; Unlike poles attract.
- Magnetic Field lines: (defined in same way as electric field lines, direction and density)

From North to South
DEMO 6A-1: Magnetic Field Lines

Magnetic Field Lines of a bar magnet

Electric Field Lines of an Electric Dipole
Question 4.

Which drawing shows the correct field lines for a bar magnet?

(1)
(2)
(3)
Magnetic Monopoles

• One explanation: there exists magnetic charge, just like electric charge. An entity which carried this magnetic charge would be called a magnetic monopole (having + or - magnetic charge).

• How can you isolate this magnetic charge?

Try cutting a bar magnet in half:

S N → S N S N

• In fact no attempt yet has been successful in finding magnetic monopoles in nature but scientists are looking for them.
Earth’s Magnetic Field

- By convention, the N end of a bar magnet is what points at the Earth’s North Geographic Pole.

- Since opposite poles attract (analogous to opposite electric charges), the “North Geomagnetic Pole” is in fact a magnetic SOUTH pole, by convention.

- Confusing, but it’s just a convention. Just remember that we define N for bar magnets as pointing to geographic North.
Earth’s Magnetic Field

Magnetic North Pole 1999
Earth’s Magnetic Field

Magnetically Quiet Day

Magnetically Disturbed Day
Earth’s Magnetic Field

Since 1904:  
750 km,  
an average of 9.4 km per year.

From 1973 to late 1983:  
120 km,  
an average of 11.6 km per year.
Earth’s Magnetic Field

Rapid Pole Shift

magnetic north pole shift (420 years)

Pole Shift
the last 150 years
(50 year periods)

faster and faster and faster...

2010

2000

1960

1910

1860
Lorentz Forces

The force $F$ on a charge $q$ moving with velocity $\mathbf{v}$ through a region of space with electric field $E$ and magnetic field $B$ is given by:

$$
\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}
$$

$$
\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}
$$

$$
|\mathbf{F}| = |q|\mathbf{v}B \sin \phi
$$
Right Hand Rule

Direction of $\mathbf{F}_B$ is perpendicular to plane containing $\mathbf{v}$ & $\mathbf{B}$.

If $q$ is positive, $\mathbf{F}_B$ has the same sign as $\mathbf{v} \times \mathbf{B}$.

If $q$ is negative, $\mathbf{F}_B$ has the opposite sign of $\mathbf{v} \times \mathbf{B}$.

$\mathbf{F}_B$ is never parallel to $\mathbf{v}$.

$\mathbf{F}_B$ can only change the direction of the particle not the speed.
More on Magnetic Force

- The magnetic force on a charged object that moves in a magnetic field does **not** do any work, because it’s perpendicular to v.
- The magnetic force cannot change the magnitude of the velocity of a charged object, but can change the direction of motion. B = steering wheel, E = accelerator or brake pedal, so to speak.
- The SI unit for magnetic field is *tesla* (T):

\[
1 \text{T} = 1 \frac{\text{N}}{\text{C} \cdot \text{m/s}} = 1 \frac{\text{N}}{\text{C/s} \cdot \text{m}} = 1 \frac{\text{N}}{\text{A} \cdot \text{m}}
\]

A common unit *gauss* (G): \( 1 \text{ G} = 10^{-4} \text{ T} \)

~Earth’s surface field!
The Magnetic Force

\[ \vec{F} = q \vec{v} \times \vec{B} \]

The direction of the force is:

\[ F_B \text{ (into the page)} \]

\[ F_B = 0 \ (\sin(0) = 0) \]
Quiz lecture 13

Three points are located in uniform magnetic field as shown. The B-field points into the page. A positive charge moved from point A toward B. The direction of the magnetic force on the particle is:

A) Right
B) Left
C) Into the page
D) Out of the page
E) zero
Quiz lecture 13

In the Figures below, a positive particle of velocity \( \mathbf{v} \) moves through a uniform magnetic field \( \mathbf{B} \) and experiences a magnetic force \( \mathbf{F}_B \). Which of the orientations of the vectors is physically possible?

A) 1  
B) 2  
C) 3  
D) 1 & 2  
E) 1, 2 & 3
Motion of a Point Charge in a Magnetic Field

\[ \vec{B} \text{ cannot change } |\vec{v}| \text{ of a charged particle.} \]
\[ \Rightarrow \vec{B} \text{ cannot change the kinetic energy of a charged particle.} \]
\[ \vec{B} \text{ can only change the direction of a particle.} \]
The work done by a magnetic field

• A proton, moving at speed $v$, enters a region of space with a constant B field in the $-z$-direction and is deflected.

• Another proton, moving at speed $v_1 = 2v$, enters the same region of space and is deflected as shown.

  - Compare the work done by the magnetic field ($W$ for $v$, $W_1$ for $v_1$) to deflect the protons.

(a) $W_1 < W$  (b) $W_1 = W$  (c) $W_1 > W$
Motion of a Point Charge in a Magnetic Field

\[ qvB = m \frac{v^2}{r} ; \quad \omega_c = \frac{v}{r} = \frac{qB}{m} \]

The period of the circular motion is

\[ T = \frac{2\pi r}{v} ; \quad f = \frac{1}{T} = \frac{v}{2\pi r} = \frac{\omega_c}{2\pi} \]

or, the angular frequency

\[ \omega_c = 2\pi f \]
Question

Each chamber has a unique magnetic field. A positively charged particle enters chamber 1 with velocity 75 m/s up, and follows the dashed trajectory.

What is the direction of the magnetic field in region 2?
1) up
2) Left
3) Right
4) into page
5) out of page
Quiz lecture 13

Each chamber has a unique magnetic field. A positively charged particle enters chamber 1 with velocity 75 m/s up, and follows the dashed trajectory.

Compare the magnitude of the magnetic field in chamber 1 to the magnitude of the magnetic field in chamber 2?

1) $|B_1| < |B_2|$
2) $|B_1| = |B_2|$
3) $|B_1| > |B_2|$
Mass Spectrometer

- Ions of different masses can have the same charge $q$ and the same velocity $v$.
- If we shoot them to a uniform magnetic field $\vec{B}$ perpendicularly, question:
  - The circular trajectories followed by the ions once they enter the B field would show:
    a) The same radii (the radius has to remain the same)
    b) Different radii (the radius of the trajectories depend on the mass)
    c) One cannot tell (we need more information to decide)
The purpose of a mass spectrometer is to separate ions by mass and measure the mass of each type of ion.

If positive ions start from rest and move through a potential difference, $V$, the ions’ kinetic energy when they enter the magnetic field equals their loss in potential energy:

What kind of charge do the ions in the picture have?
Mass Spectrometer (Ions with same KE)

Working with both equations:

\[ qBr = mv \]

\[ KE = \frac{1}{2}mv^2 = q\Delta V \]

First solve for the velocity on the first one,

Then substitute it on the kinetic energy equation

A mass spectrometer can be improved if instead of having ions with the same kinetic energy entering the B field we have ions with the same velocity.
Combine an Electric Field and a Magnetic Field

• If we shoot charged particles into a region of space that has both an electric and a magnetic field, we would end up with a net electro-magnetic force that is equal to the vector sum of the electric and magnetic forces acting on the charge:

\[ \vec{F} = \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v} \times \vec{B} \]

• A very interesting effect can be achieved when we apply an electric and a magnetic force to a charged particle in such a way that these forces balance.

\[ \vec{F} = \vec{F}_E + \vec{F}_B = 0 \]
Crossed $\vec{E}$ and $\vec{B}$ Fields

• Question: In which direction is the magnetic force, once the positive charge reaches the region with the B field?

  a) Up  b) Down  c) into the page

• Question: If we would like to balance this magnetic force with an electric force, we would have to apply an electric field in which direction?

  a) Up  b) Down  c) into the page
CONCLUSION:
There is only one particular velocity of a + charged particle that will balance the magnetic and electric forces.

This device is called a Velocity Selector.
Magnetic Force on a Current-Carrying Wire
Top view of Current-Carrying Bar Sliding on two current carrying frictionless rails in a magnetic field.

![Diagram of a motor](image)

**Note** that this example assumes that the magnetic field caused by the currents in the rails is negligible compared to the external magnetic field $B$ shown.

The length $L$ is the distance between the rail and $B$ is the magnetic field. The current $I$ flows in the green bar.

**motion by reversing direction of $I$, by reversing $V$**
Torque on a Current Loop

• We first have to define an unambiguous direction of the loop, perpendicular to the plane of the loop $\hat{n}$.  
• We do this with our right hand (again)

• Curl your fingers of your right hand in the direction of the current, then your thumb should point in the direction of $\hat{n}$
\( \hat{n} \) wants to align with \( \vec{B} \) : 

\[ \tau = NIabB \sin \theta = NIAB \sin \theta \]

where \( A = ab \) and the formula does NOT depend on the shape of the loop, only on the area \( A \).

\( N \) counts the number of turns of wire in this loop, each turn contributes.
Torque on a Magnetic Dipole

Flat current loop of arbitrary shape

The magnetic dipole moment \( \vec{\mu} \) is given by \( \vec{\mu} = NIA\hat{n} \), where:

- \( N \) is the number of turns in the loop.
- \( I \) is the current in the loop.
- \( A \) is the area of the loop.
- \( \hat{n} \) is the normal vector to the loop.

The magnetic torque \( \vec{\tau} \) on the dipole is given by the cross product:

\[ \vec{\tau} = \vec{\mu} \times \vec{B} \]

Remember that an electric dipole in an \( \vec{E} \) field experiences a torque where \( \vec{\tau} = \vec{p} \times \vec{E} \), where \( \vec{p} \) is the electric dipole moment.
Quiz lecture 13

A square wire loop of side s lies in the x-y plane and carries a current I flowing in the counter-clockwise direction (as viewed from z > 0). A constant uniform magnetic field B points in the +x direction. The torque exerted by the magnetic force on this loop is pointing in the direction:

A) -x  
B) +x  
C) -y  
D) +y  
E) -z
Magnetic Dipole in a Uniform \( \mathbf{B} \) Field

\[
\mathbf{\tau} = \mathbf{\mu} \times \mathbf{B} \Rightarrow \tau = \mu B \sin \theta
\]

When \( \theta = 0^\circ \) or \( 180^\circ \), \( \tau = 0 \).

However, \( \theta = 180^\circ \) is unstable.

When a torque is exerted through an angle, work is done. When a dipole is rotated through an angle \( d\theta \)

\[
dW = -\tau d\theta = -\mu B \sin \theta d\theta
\]

\[
dU = -dW = +\mu B \sin \theta d\theta
\]

\[
U = \int dU = \mu B \int \sin \theta = -\mu B \cos \theta + U_0
\]

Choose \( U_0(\theta = 90^\circ) = 0 \Rightarrow U = -\mu B \cos \theta = -\mathbf{\mu} \cdot \mathbf{\vec{B}} \)
Potential Energy of Magnetic Dipole

A magnetic dipole has its highest energy when its dipole moment is anti-parallel to the magnetic field.

\[ U = -\mu B \cos 180^\circ \]
\[ U = +\mu B \]

A magnetic dipole has its lowest energy when its dipole moment is lined up with the magnetic field.

\[ U = -\bar{\mu} \cdot \bar{B} \]
\[ U = -\mu B \cos 0 = -\mu B \]