Lectures 6 & 7: Chapter 23

Electric Potential

Definitions

Path independence

Examples

Equipotential surfaces

\[ \frac{Q}{4\pi \varepsilon_0} R \]

\[ \frac{Q}{4\pi \varepsilon_0} r \]
From Mechanics (PHYS 172)

- **Energy**
  - Kinetic Energy: associated with the state of motion

  Potential Energy: associated with the configuration of the system

- **Conservative Forces:**
  - Work done by a conservative force is independent of path
From Mechanics (PHYS 172)

• **Work**

\[ W_{TOT} = \Delta K \]

\[ \begin{align*}
    \begin{array}{c|c|c}
        F & dr & W > 0 \\
        \text{or} & dr & W < 0 \\
        \text{or} & dr & W = 0 \\
    \end{array}
    \end{align*} \\ 
\]

Object speeds up (\( \Delta K > 0 \))

Object slows down (\( \Delta K < 0 \))

Constant speed (\( \Delta K = 0 \))
Electric Potential Energy

When an electrostatic force acts between two or more charges within a system, we can assign an Electric Potential Energy:

$$F \cdot \Delta x$$

If a Coulomb force does negative work, Potential energy increases.
You hold a positively charged ball and walk due west in a region that contains an electric field directed due east.

$W_H$ is the work done by the hand on the ball  
$W_E$ is the work done by the electric field on the ball

Which of the following statements is true:

A) $W_H > 0$ and $W_E > 0$

B) $W_H > 0$ and $W_E < 0$

C) $W_H < 0$ and $W_E < 0$

D) $W_H < 0$ and $W_E > 0$
Example: Electric Potential Energy

What is the change in electrical potential energy of a released electron in the atmosphere when the electrostatic force from the near Earth’s electric field (directed downward) causes the electron to move vertically upwards through a distance \( d \)?

1. \( \Delta U \) of the electron is related to the work done on it by the electric field:

   \[
   \Delta U = F \cdot d
   \]

2. Work done by a constant force on a particle undergoing displacement:

   \[
   W = F \cdot d
   \]

3. Electrostatic Force and Electric Field are related:
Example: Electric Potential Energy

What is the change in electrical potential energy of a released electron in the atmosphere when the electrostatic force from the near Earth’s electric field (directed downward) causes the electron to move vertically upwards through a distance $d$?

1. $\Delta U$ of the electron is related to the work done on it by the electric field:

   Key Idea: $\Delta U = -W$

2. Work done by a constant force on a particle undergoing displacement:

   Key Idea: $W = \vec{F} \cdot \vec{d}$

3. Electrostatic Force and Electric Field are related:

   Key Idea: $\vec{F} = q\vec{E}$

   $W = q\vec{E} \cdot \vec{d} = -qEd \cos \theta = -qEd \cos 180 = qEd$

   $\Delta U = -W = -qEd$

   Electric potential decreases as electron rises.
Electric Potential is a property of an electric field and is measured in J/C or V

Electric Potential Energy is an energy of system consisting of the charged object and the external electric field, and is measured in Joules.
Potential & Electric Fields

The electric field points in the direction in which the potential decreases most rapidly.
Example: Potential Difference

*Independent of path

(a) What is $\Delta V$ moving directly from point i to point f?

(b) What is $\Delta V$ moving from point i to point c to point f?
Quiz:

A single charge \( Q = -1 \mu C \) is fixed at the origin. Define point A at \( x = +5 \text{m} \) and point B at \( x = +2 \text{m} \).

What is the sign of the potential difference between A and B?

(Where \( V_{BA} = V_B - V_A \))

(a) \( V_{BA} < 0 \)  (b) \( V_{BA} = 0 \)  (c) \( V_{BA} > 0 \)
Potential due to a point charge:

Find V in space around a charged particle relative to the zero potential at infinity:
V(r) versus r for a positive charge at r = 0

For a point charge

\[ V(r = 0) = \infty \]
Electrical Potential Energy

Push $q_0$ “uphill” and its electrical potential energy increases according to

$$U = \frac{kq_0 q}{r}$$

The work required to move $q_0$ initially at rest at $\infty$ is

$$W = \frac{kq_0 q}{r}. \quad \text{Work per unit charge is} \quad V = \frac{kq}{r}.$$
Gauss’ law says the sphere looks like a point charge outside $R$. 

$$V(R) = \frac{kQ}{R}$$
Demo 5A-35

\[ \Delta U = q \Delta V \]

Get energy out

\[ V = \frac{kQ}{R} \]

charge flow

Also try an elongated neon bulb.

\[ \Delta V = V(r_1) - V(r_2) \]

across fluorescent light bulb
Question from last class
Potential due to a Group of Point Charges

\[ V(r) = -\int_{r=\infty}^{r=r} \vec{E} \cdot d\vec{l} \]

\[ V(r) = -\int_{r=\infty}^{r=r} \sum_{n} \vec{E}_n \cdot d\vec{l}_n \]

\[ V(r) = \sum_{n} \left[ -\int_{r=\infty}^{r=r} \vec{E}_n \cdot d\vec{l}_n \right] = \sum_{n} \left[ -\int_{r=\infty}^{r=r} \frac{q_n}{4\pi \varepsilon_0 r_n^2} dr_n \right] \]

\[ = \sum_{n} \left[ -\frac{-q_n}{4\pi \varepsilon_0 r_n} \right] = \sum_{n} \frac{q_n}{4\pi \varepsilon_0 r_n} \]

\[ V(r) = \sum_{n=1}^{N} V_n(r) = \frac{1}{4\pi \varepsilon_0} \sum_{n=1}^{N} \frac{q_n}{r_n} \]

Negative sign from integration of E field cancels the negative sign from the original equation.
Potential due to a Group of Point Charges

Find the Potential at the center of the square.

\[ V(r) = \sum_{n=1}^{N} V_n(r) = \frac{1}{4\pi\epsilon_0} \sum_{n=1}^{N} \frac{q_n}{r_n} \]

q_1 = +12 nC  q_2 = -24 nC  q_3 = +31 nC  q_3 = +17 nC
Chapter 23

- **Electrostatic Potential Energy** of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system, bringing each charge in from an infinite distance.

![Diagram of two points with a charge](image)
Electrical Potential Energy of a System of Point Charges

$U$ of a system of fixed point charges equals $W$ done by an external agent to assemble the system by bringing each charge in from infinity.

If $q_1$ & $q_2$ have the same sign, we must do positive work to push against mutual repulsion.

If $q_1$ & $q_2$ have opposite signs, we must do negative work against mutual attraction.
Example: Electrostatic Potential Energy

If $q_1$ & $q_2$ have the same sign, $U$ is positive because positive work by an external agent must be done to push against their mutual repulsion.

If $q_1$ & $q_2$ have opposite signs, $U$ is negative because negative work by an external agent must be done to work against their mutual attraction.
In Case A, two negative charges which are equal in magnitude are separated by a distance $d$. In Case B, the same charges are separated by a distance $2d$. Which configuration has the higher potential energy?

A) Case A
B) Case B
Clicker Question Discussion

As usual, choose $U = 0$ to be at infinity:

$$U(r) = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{1}{r}$$

**Case A**

$$U_A = \frac{q^2}{4\pi\varepsilon_0} \frac{1}{r}$$

**Case B**

$$U_B = \frac{q^2}{4\pi\varepsilon_0} \frac{1}{2d}$$

$$U_A > U_B$$
Example: Three Point Charges

Point 1 \( q_1 \)

Point 2 \( q_2 \)

Point 3 \( q_3 \)

\[ r_{1,3} \]

\[ r_{2,3} \]

\[ r_{1,2} \]
Electrostatic Potential Energy

• We can conclude that the total work required to assemble the three charges is the electrostatic potential energy $U$ of the system of three point charges:

$$U = W_{total} = W_{12} + W_{13} + W_{23} = \frac{kq_2q_1}{r_{1,2}} + \frac{kq_3q_1}{r_{1,3}} + \frac{kq_3q_2}{r_{2,3}}$$

The electrostatic potential energy of a system of point charges is the work needed to bring the charges from an infinite separation to their final position.
Another Method: Electrostatic Potential Energy

Leave the charges in place and add all the electrostatic energies of each charge and divide it by 2.

where:

\[ q_i \] -- is the charge at point \( i \)

\[ V_i \] -- is the potential at location \( i \) due to all of the other charges.
Another Method: Electrostatic Potential Energy

\[
U = \frac{1}{2} \left[ q_1 \left( \frac{kq_2}{r_{1,2}} + \frac{kq_3}{r_{1,3}} \right) + q_2 \left( \frac{kq_1}{r_{1,2}} + \frac{kq_3}{r_{2,3}} \right) + q_3 \left( \frac{kq_1}{r_{1,3}} + \frac{kq_2}{r_{2,3}} \right) \right]
\]

\[
U = \frac{1}{2} \left[ 2 \frac{kq_1q_2}{r_{1,2}} + 2 \frac{kq_3q_1}{r_{1,3}} + 2 \frac{kq_2q_3}{r_{2,3}} \right] = \frac{kq_1q_2}{r_{1,2}} + \frac{kq_3q_1}{r_{1,3}} + \frac{kq_2q_3}{r_{2,3}}
\]
Summary: Electrostatic Potential Energy for a Collection of Point Charges

1. Bringing charge by charge from infinity

2. Or using the equation: 
   \[ U = \frac{1}{2} \sum_{i=1}^{n} q_i V_i \]
   and trying not to forget the factor of \( \frac{1}{2} \) in front.

Note: The second way, could involve more calculations for point charges. However, it becomes very handy whenever you try to calculate the electrostatic potential energy of a continuous distribution of charge, the sum becomes an integral.