i-Clicker Quiz #26

Bright light of wavelength 585 nm is incident perpendicularly on a soap film \((n = 1.33)\) of thickness 1.21 microns, suspended in air. Is the light reflected by the two surfaces of the film closer to interfering fully destructively or fully constructively?

(A) fully constructively

(B) fully destructively

(please don’t enter E...)
Interference

• Three ways to introduce a phase difference in two beams of coherent light:
  – Different index of refraction, (not relevant here)
  – Reflection from a material with a larger $n$, ($\delta_1 = \pi$, $\delta_2 = 0$)
  – Different path length: $t = 1.21 \, \mu m$, $n = 1.33$, $\lambda_{air} = 585 \, nm$
    \[
    \lambda_{soap} = \frac{(585 \, nm)\,/1.33}{\lambda_{air}} = 440 \, nm
    \]
    Number of wavelengths in the soap film: $2t/\lambda_{soap} = 5.5$
    Total phase difference between 1 & 2: $(\delta_2 + 2\pi \times 5.5) - \delta_1 = 10\pi$
    The two reflected waves are in phase with each other. (constructive interference)

LECTURE 27: Diffraction, Resolution
Why is this important?

- Photolithography
- Astronomical observations
- Ultimate limit for high-resolution photography. (Espionage 101)
- How much data can one usefully store on a CD or DVD?
- Radar dish size.

Moore’s Law

Diffraction

- Geometrical optics:
  - Edge, obstacle or aperture much larger than $\lambda$
  - Light travels in straight lines
  - Shadows have distinct, sharp edges
- Wave optics:
  - Feature sizes are similar to the wavelength, $\lambda$
  - Waves spread out around objects and can interfere constructively or destructively
  - Interference patterns of light and dark fringes
Fresnel Diffraction

Two waves that touch the top and bottom parts of the disk travel the same distance to the center and interfere constructively.

Fresnel believed in wave optics
Newton was a proponent of geometric optics

Diffraction from an Edge

The shadow has a diffraction pattern within a few wavelengths of the edge.

(DEMO, use 150 mm lens)
Interference, Diffraction and Resolution

Example application:

- Consider a plane monochromatic light incident on a screen containing a single narrow slit.
- Geometric optics predicts that the image has the same cross section as the slit.
- Wave optics predicts a central bright band that is wider than the width of the slit, with interference fringes on either side.

Diffraction from a Single Slit (screen far away)

(DEMO, use multiple mask)
Diffraction from a Single Slit

Central bright fringe
(the waves from all points in the slit travel the same distance to reach the center – and thus are in Phase)

Represent the slit as a number of point sources of equal amplitude. Divide the slit into two and pair a point from the upper half with its partner in the lower half.

When \( \frac{1}{2} a \sin \theta = \frac{1}{2} \lambda \),\,(first minimum) destructive interference between the paired rays occurs.
Diffraction from a Single Slit

Locations of minima for single - slit diffraction :
\[ a \sin \theta_m = m \lambda, \quad m = 1,2,3,... \]

\[ a \sin \theta_1 = \lambda \quad \text{(location 1st min)} \]
\[ a \sin \theta_2 = 2\lambda \quad \text{(location 2nd min)} \]

\[ \tan \theta_m = \frac{y_m}{L} \]

Single Slit Diffraction

- How to reduce the spread of the image, defined by the position of the first minimum:
  \[ y = \frac{L\lambda}{a} \]
  - Decrease \( L \): put the mask close to the screen
  - Decrease \( \lambda \): use blue or ultraviolet light (not easy)
  - Increase \( a \) (counter intuitive)
Diffraction from a Single Slit

Dependence of the width of the central maximum on $a$:

\[ a \sin \theta = m\lambda \implies a \sin \theta = \lambda \]
\[ \sin \theta \approx \theta = \frac{\lambda}{a} \]

Increasing aperture size

i-Clicker Question #1

Two waves, of wavelength 650 (red) and 430 nm (blue), are used separately in a single-slit diffraction experiment. The figure shows the results as graphs of intensity, $I$, versus angle $\theta$ for two diffraction patterns. If both wavelengths are then used simultaneously, what color will be seen in the combined diffraction pattern at angle D?

(A) 650 nm
(B) 650 and 430 nm
(C) 430 nm
(D) neither
Diffraction from a Circular Aperture

The diffraction pattern of a circular aperture of diameter $d$ is similar to a single slit of width $a$.

The central bright spot is called an Airy disk. About 85% of the power is in this area.

The dark fringes are found at:

\[
\sin \theta_1 = 1.22 \frac{\lambda}{d} \\
\sin \theta_2 = 2.23 \frac{\lambda}{d} \\
\sin \theta_3 = 3.24 \frac{\lambda}{d}
\]

The bright fringes are at:

\[
\sin \theta_1 = 1.63 \frac{\lambda}{d} \\
\sin \theta_2 = 2.68 \frac{\lambda}{d} \\
\sin \theta_3 = 3.70 \frac{\lambda}{d}
\]

Image of two nearby binary stars. Diffraction patterns overlap

The Airy disk limits the resolvability of nearby objects.
Rayleigh Criterion

\[ \alpha_c \approx 1.22 \frac{\lambda}{d} \]

Circular aperture of diameter \( d \)

Two incoherent point sources

Screen far from opening

Maximum falls on minimum

The minimum angular separation \( \alpha_c \) of two marginally resolvable points is such that the maximum of the diffraction pattern from one falls on the first minimum of the diffraction pattern of the other, when \( \alpha_c \approx 1.22 \frac{\lambda}{d} \)

The first minimum is at

Central Axis
Resolvability

• **Rayleigh Criteria:**
  
  - The minimum angular separation $\alpha_c$ of two marginally resolvable points is such that the maximum of the diffraction pattern from one falls on the first minimum of the diffraction pattern of the other.
  
  - First minima is at $\sin \theta = 1.22 \frac{\lambda}{d}$ so $\alpha_c = \theta \approx 1.22 \frac{\lambda}{d}$

![Intensity Distributions](image)

Not resolved  Resolved  Barely resolved

Resolvability

• **Rayleigh Criteria:**

$$\alpha_c = 1.22 \frac{\lambda}{d}$$

• The minimum resolvable angle can be decreased by:
  
  - Increasing $d$
  
  - Decreasing $\lambda$ (use ultraviolet light or observe it in a medium with larger $n$)
  
  - Electrons behave like waves with very short wavelengths $\lambda(e) \approx \lambda(light)/10^5$
Diffraction Gratings: Lots of Slits

- Consider a plane wave incident upon an opaque screen containing $N$ evenly-spaced slits.
- Constructive interference when: $\delta = d \sin \theta = m\lambda$

Lines become sharp when $N$ is large: half-width of central line is $\Delta \theta = \frac{\lambda}{Nd}$

For a given wavelength and $d$, if $N$ increases the half-width decreases

$$\Delta \theta_{hw} = \frac{\lambda}{Nd}$$

Fixed $d$ & wavelength, Changing the aperture to illuminate more lines….this is different then the demo
Diffraction Gratings

• Sharp, bright fringes occur when
  \[ d \sin \theta = m \lambda, \quad m = 1,2,3, \ldots \]

• Resolving power: can we detect two spectral lines separated by \( \Delta \lambda \)?
  \[ d \Delta \theta = m \Delta \lambda \]

• Half width of central line:
  \[ \Delta \theta = \frac{\lambda}{Nd} \]

• Resolving power: \( R = \frac{\lambda}{|\Delta \lambda|} = \frac{m \lambda}{d \Delta \theta} = mN \)