An arrow is located in front of a convex spherical mirror of radius $R = 50$ cm. The object distance is $s = 40$ cm.

What is the focal length of the mirror?

A) $f = 50$cm  
B) $f = 25$cm  
C) $f = -50$cm  
D) $f = -25$cm
Geometric Optics

• When the wavelength of light is much shorter than the sizes of objects it interacts with, we can ignore the wave-like nature and treat it as rays that propagate in straight lines.

\[ \theta_1' = \theta_1 \]  

Refraction:
\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

Types of Images

• **Virtual image**: image from which light *appears* to emanate (but doesn’t really!)
• **Real image**: image from which light actually does emanate (and therefore could be projected on a screen).
Spherical Refracting Surfaces

- **Real image:**

When the object is far from the surface, the refracted ray is directed towards the central axis, forming a real image.

- **Virtual image:**

When the object is close to the surface, the refracted ray bends, but is still directed away from the central axis, forming a virtual image.
Spherical Refracting Surfaces

• Real image:

When the object is far from the surface, the refracted ray is directed towards the central axis, forming a real image.

Spherical Refracting Surfaces

• Virtual image:

When the object is close to the surface, the refracted ray is directed away from the central axis, forming a virtual image.
**Small Angle Approximation**

- When $\theta \ll 1$ then $\sin \theta \approx \theta$
  - $\theta$ must be expressed in radians!
  - When $\theta < 14^\circ$ this is accurate to better than 1%
- Snell’s Law:
  $$n_1 \sin \theta_1 = n_2 \sin \theta_2 \Rightarrow n_1 \theta_1 = n_2 \theta_2$$

**Sign Conventions**

- Convex surface:
  - $s$ is positive for objects on the incident-light side
  - $s'$ is positive for images on the refracted-light side
  - $r$ is positive if $C$ is on the refracted-light side
**Sign Conventions**

- Concave surface:
  - $s$ is positive for objects on the incident-light side
  - $s'$ is negative for images on the incident-light side
  - $r$ is negative if $C$ is on the incident-light side

\[
\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r}
\]

(same formula)

**Example: Mosquito in Amber**

A mosquito is embedded in amber with an index of refraction of 1.6. One surface of the amber is spherically convex with a radius of curvature 3.0 mm. The mosquito head is on the central axis of that surface, and when viewed along the axis appears to be buried 5.0 mm into the amber. How deep is it really?
Example: Mosquito in Amber

1) $s_i = -5 \text{ mm}$ (b/c obj. and image are on same side of refracting surface, the image is virtual.)

2) $n_i = 1.6$ (use convention that obj. is in medium with index $n_1$)
   $$n_2 = 1 \text{ (air)}$$

3) $r = -3 \text{ mm}$ (negative b/c obj. faces concave surface)
   $$\frac{n_1 + n_2}{s_o} \frac{s_i}{r} \Rightarrow \frac{1.6 + 1}{s_o} \frac{1}{-5} = \frac{1 - 1.6}{-3} \Rightarrow s_o = 4 \text{ mm}$$

Summary: Spherical Refracting Surfaces

1. Real Images form on the side of a refracting surface that is opposite the object.

2. Virtual Images form on the same side as the object.

3. For light rays making only small angles with the central axis:
   $$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r}$$

4. When the object faces a convex refracting surface, the radius of curvature is positive ($r > 0$).

5. When object faces a concave surface, radius of curvature is negative ($r < 0$).
i-Clicker Question #1

You are looking out of the spherical window of a submarine at a sea creature that appears to be 1 meter away.

What signs should we use for \( r \) and \( s' \)?

(a) + and +  
(b) + and –  
(c) – and +  
(d) – and –

Response

- The surface is convex, but \( n_2 < n_1 \)
- \( C \) is on the refracted-light side: \( r \) is positive
- \( I \) is on the incident-light side: \( s' \) is negative
- \( O \) is on the incident-light side: \( s \) is positive
You are in a submarine looking out of a spherical window with a 1 meter radius of curvature at a sea creature.

The sea creature appears to be 1 meter away. How far away is it really?

\[ n_{air} = 1 \quad n_{water} = 1.333 \]

\[
\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r}
\]

\[
\frac{1.333}{s} + \frac{1}{-1 \text{ m}} = \frac{1-1.333}{1 \text{ m}} \Rightarrow s = +2 \text{ m}
\]
Magnification

- Using the same sign convention, the magnification is
  \[ m = - \frac{n_1 s'}{n_2 s} \]

- For the previous example,
  \[ m = - \frac{1.33 \times (-1 \text{ m})}{1 \times (+2 \text{ m})} = +0.66 \]

- The sea creature outside the submarine appears smaller (by x 2/3) than it really is.
- The image is not inverted.
Thin Lenses: Converging Lens

- Parallel rays refract twice
- Converge at $F_2$ a distance $f$ from center of lens
- $F_2$ is a real focal pt because rays pass through
- $f > 0$ for real focal points

\[ \frac{1}{f} = (n - 1) \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \]

- $n - 1 > 0$ because $n_{\text{glass}} > n_{\text{air}}$
- $r_1 > 0$ because object facing convex surface
- $r_2 < 0$ because object facing concave surface

Thin Lenses: Diverging Lens

- Rays diverge, never pass through a common point
- $F_2$ at a distance $f$
- $F_2$ is virtual focal point

\[ \frac{1}{f} = (n - 1) \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \]

- $f < 0$ for virtual focal points
- $f$ is negative
- Image is virtual

- For the front surface, $C_1$ is on the incident-light side so $r_1$ is negative.
- For the back surface, $C_2$ is on the refracted-light side so $r_2$ is positive.
Locating Images by Drawing Rays

1. Ray initially parallel to central axis will pass through $F_2$.
2. Ray passing through $F_1$ will emerge parallel to the central axis.
3. Ray passing through center of lens will emerge with no change in direction because the ray encounters the two sides of the lens where they are almost parallel.

Thin Lens formula

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Magnitude

$$m = \frac{h'}{h} = -\frac{s'}{s}$$

1. Ray initially parallel to central axis will pass through $F_2$.
2. Backward extension of ray 2 passes through $F_1$
3. Ray 3 passes through center of lens will emerge with no change in direction.
Locating Images by Drawing Rays

1. Backward extension of ray 1 passes through $F_2$
2. Extension of ray 2 passes through $F_1$
3. Ray 3 passes through center of lens will emerge with no change in direction.

Thin Lens formula

$$m = \frac{h'}{h} = -\frac{s'}{s}$$  
(magnification)

Two Lens Systems

- Calculate $s_1'$ using $\frac{1}{f_1} = \frac{1}{s_1} + \frac{1}{s_1'}$
- Use the image of the first lens as the object of the second.
- Use $s_2 = d - s_1'$ as the object distance for the second lens. Notes: Ignore the first lens; If image 1 is located beyond lens 2, $s_2$ for lens 2 is negative.
- Calculate $s_2'$ using $\frac{1}{f_2} = \frac{1}{s_2} + \frac{1}{s_2'}$
- Overall magnification: $m = m_1m_2 = \left(-\frac{s_1'}{s_1}\right)\left(-\frac{s_2'}{s_2}\right)$
Example: Two Lens System

A seed is placed in front of two thin symmetrical coaxial lenses (lens 1 & lens 2) with focal lengths $f_1 = +24 \text{ cm}$ & $f_2 = +9.0 \text{ cm}$, with a lens separation of $d = 10.0 \text{ cm}$. The seed is 6.0 cm from lens 1. Where is the image of the seed?

Lens 1: $\frac{1}{f_1} = \frac{1}{s_1} + \frac{1}{s_1'} \Rightarrow s_1' = -8 \text{ cm} \quad m_1 = \left( -\frac{s_1'}{s_1} \right) = \frac{4}{3}$

Image 1 is virtual.

Lens 2: Treat image 1 as O_2 for lens 2. O_2 is outside the focal point of lens 2. So, image 2 will be real & inverted on the other side of lens 2.

$s_2 = d - s_1' = 18 \text{ cm}$

$\frac{1}{f_2} = \frac{1}{s_2} + \frac{1}{s_2'} \Rightarrow s_2' = 18.0 \text{ cm} \quad m_2 = \left( -\frac{s_2'}{s_2} \right) = -1$

$m = m_1 m_2 = -\frac{4}{3}$

Image 2 is real.

Table for Lenses

<table>
<thead>
<tr>
<th>Lens Type</th>
<th>Object Location</th>
<th>Image Location</th>
<th>Image Type</th>
<th>Image Orientation</th>
<th>Sign of $f, s', m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Converging</td>
<td>Inside F</td>
<td>Same side as object</td>
<td>Virtual</td>
<td>Same as object</td>
<td>$+, -, +$</td>
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<td>Side of lens opposite the object</td>
<td>Real</td>
<td>Inverted</td>
<td>$+, +, -$</td>
</tr>
<tr>
<td>Diverging</td>
<td>Anywhere</td>
<td>Same side as object</td>
<td>Virtual</td>
<td>Same as object</td>
<td>$-, -, +$</td>
</tr>
</tbody>
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