Four long, straight parallel wires each carry a current $I$. Each wire is at the corner of a square, and the direction of each current is shown in the figure.

Determine the direction of the magnetic field at the center.

(A) $+\hat{i}$
(B) $-\hat{i}$
(C) $+\hat{j}$
(D) $-\hat{j}$
Review of Biot-Savart Law

\[ \vec{B} = \frac{\mu_0}{4\pi} \int \frac{I \, d\vec{\ell} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \int \frac{I \, d\vec{\ell} \times \hat{r}}{r^3} \]

\[ |\vec{B}| = \frac{\mu_0 \, 2I}{4\pi \, R} \]

\[ |\vec{B}| = \frac{\mu_0}{2} \frac{IR^2}{(R^2 + z^2)^{3/2}} \]

\[ |\vec{B}| = \mu_0 n \, I \]

Outline for Today

• Force between Current Carrying Wires

• Gauss’ Law for Magnetism

• Ampere’s Law

• Magnetism in Matter
Forces on Parallel Wires

Forces on Current-Carrying Wires

• Two wires carrying currents $I_1$ and $I_2$ will exert forces on each other:
  - Magnetic field from $I_1$ is $\vec{B} = \frac{\mu_0}{4\pi} \int \frac{l_1 \cdot \vec{d} \times \vec{r}}{r^2}$
  - Force on $I_2$ is $d\vec{F}_{12} = I_2 \ d\vec{l}_2 \times \vec{B}$

• Conversely
  - Magnetic field from $I_2$ is $\vec{B} = \frac{\mu_0}{4\pi} \int \frac{l_2 \cdot \vec{d} \times \vec{r}}{r^2}$
  - Force on $I_1$ is $d\vec{F}_{21} = I_1 \ d\vec{l}_1 \times \vec{B}$
**Force Between two Parallel Current Carrying Wires**

*Parallel currents attract
*Anti-parallel currents repel

**i-Clicker Question #1**
Upon turning on the current $I$ in this ‘tangle’ of wire, what reaction will we observe?

(A) The wire loop will expand.
(B) The wire loop will collapse.
(C) The wire loop will spin about a horizontal axis.
(D) There will be no effect at all.
Remember Gauss’s Law for Electric Fields?

- Electric field:
  \[ \vec{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{\hat{r}}{r^2} dQ \]

- Gauss’s Law:
  \[ \oint_S \hat{n} \cdot \vec{E} \, dA = \frac{Q_{\text{inside}}}{\varepsilon_0} \]

- If \( \vec{E} \) is constant over the surface then we can bring it outside the integral
  - The integral is just the surface area
  - This works only when there is sufficient symmetry

Gauss’s Law Applied Magnetism

- In magnetism we can have dipoles or currents but no magnetic charges, or monopoles
- Gauss’s law:
  \[ \oint_S \hat{n} \cdot \vec{B} \, dA = \frac{Q_{\text{inside}}}{\varepsilon_0} = 0 \]  
  **Always zero!**

- One of Maxwell’s Equations:
  \[ \nabla \cdot \vec{B} = 0 \]

Since all lines of \( \vec{B} \) are closed loops, any \( \vec{B} \) line leaving a closed surface MUST re-enter it somewhere. TRUE IN GENERAL, not just for this “dipole” example
Ampere’s Law

• But can we do something similar to calculate the magnetic field in cases with lots of symmetry?
  • Yes:
    \[ \oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_C \]
    \( I_C \) is the current passing through the contour \( C \).

Example

• What is the magnetic field around a long, straight wire?
  • From symmetry, we expect that the magnetic field is always azimuthal: \( \vec{B} = B\hat{\phi} \)
    • We choose a circular path centered on the wire.
    • The path length element is also azimuthal: \( d\vec{l} = d\ell \hat{\phi} \)
    \[
    \oint_C \vec{B} \cdot d\vec{l} = B \oint_C d\ell = \mu_0 I_C \\
    B \cdot 2\pi r = \mu_0 I_C \quad \Rightarrow \quad B = \frac{\mu_0 I_C}{2\pi r} 
    \]
Magnetic Field Inside a Long Straight Wire

\[ I_C = I \frac{r^2}{R^2} \]

\[ B(r) = \frac{\mu_0 I_C}{2\pi r} \]

\[ B = \frac{\mu_0 I r}{2\pi R^2} \quad \text{(Inside)} \]

\[ B = \frac{\mu_0 I}{2\pi r} \quad \text{(Outside)} \]

Magnetic Field Inside a Solenoid

- Symmetry principles:
  - The magnetic field always points along the axis of the solenoid: \( \vec{B} = B\hat{k} \)
  - It is independent of \( z \), except at the ends.
- Outside the solenoid, we expect \( \vec{B} \rightarrow 0 \) as \( r \rightarrow \infty \)
- Inside the solenoid, does \( \vec{B} \) depend on \( r \)?
Magnetic Field Inside a Solenoid

- Enclosed current: \( I_C = n I h \)

\[
\oint_C \vec{B} \cdot d\vec{l} = B h = \mu_0 I_C
\]

\[
B = \mu_0 n I
\]

\( n \) is the number of turns per unit length.

Make the path \( cd \) very far away, where \( \vec{B} \approx 0 \).

i-Clicker Question #2

Now apply Ampere’s Law on the red contour.

\[
\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_C
\]

Consider the current \( I_C \) passing through the red contour. What does this tell us about the magnetic field \( \vec{B} \) inside the solenoid?

(A) \( |\vec{B}| = 0 \) inside the solenoid.

(B) \( |\vec{B}| \) gets larger close to the axis.

(C) \( |\vec{B}| \) is uniform inside the solenoid.
Force between a Solenoid & a Current Carrying Wire (DEMO)

\[ \vec{F} = i \vec{L} \times \vec{B} \]

\( \vec{i} \)

\( \vec{B} \)

\( \vec{F} \) is into the page\( \imes \) and tangential to the surface of solenoid.
When the current is reversed, \( \vec{F} \) is out of the page. 😊

Magnetic Field Inside a Toroid
When Ampere’s Law doesn’t Help

- $B$ can’t be factored out of the integral.
- Insufficient symmetry
- Finite length current segment is (unphysical)
- Current is not continuous (time dependent)

Dipole Moments in Applied Fields

- Electric dipole: $\vec{E}$ decreases at center
- Magnetic dipole: $\vec{B}$ increases at center
- External fields tend to align dipoles.
Magnetic Properties of Materials

- Atoms in many materials act like magnetic dipoles.
- Magnetization is the net dipole moment per unit volume:
  \[ \vec{M} = \frac{d\vec{\mu}}{dV} \]
- In the presence of an external magnetic field, these dipoles can start to line up with the field:

Net current inside the material is zero. We are left with an effective surface current and therefore a magnetic moment.

Magnetization and “Bound Current”

Magnetic dipole for a current loop: \( \vec{\mu} = A I \hat{n} \)

Magnetic moment per unit length:
\[ d\mu = A \, di \]

Magnetization:
\[ M = \frac{d\mu}{dV} = \frac{d\mu}{A \, dl} = \frac{di}{dl} \]

This is the “effective surface current,” with units [A/m]. Magnetic field due to the surface current is the same as in a solenoid:
\[ B = \mu_0 n I = \mu_0 M \]

current per unit length
Magnetization and Magnetic Susceptibility

• How well do the microscopic magnetic dipoles align with an external applied magnetic field?
• Simplest model: linear dependence on $\vec{B}_{app}$
  – Magnetization: $\vec{M} \propto \vec{B}_{app}$
  – Magnetic field due to surface current:
    $$\vec{B}_{m} = \mu_0 \vec{M} \equiv \chi_m \vec{B}_{app}$$
  – Magnetic susceptibility: $\chi_m$
• Total magnetic field:
  $$\vec{B} = \vec{B}_{app} + \vec{B}_{m} = (1 + \chi_m) \vec{B}_{app} \equiv K_m \vec{B}_{app}$$
  – Relative permeability: $K_m$

Magnetic Susceptibility

• Different materials react differently to external magnetic fields:

| $\chi_m < 0$, small $|\chi_m|$ | Diamagnetic | bismuth, copper, silver |
| $\chi_m > 0$, small $\chi_m$ | Paramagnetic | aluminum, tungsten |
| $\chi_m > 0$, large $\chi_m$ | Ferromagnetic | iron, cobalt, nickel |

• Dipoles in diamagnetic materials align opposite $\vec{B}_{app}$
• Dipoles in paramagnetic materials align with $\vec{B}_{app}$
• Ferromagnetic materials align strongly even in weak $\vec{B}_{app}$
## Magnetic Susceptibility

<table>
<thead>
<tr>
<th>Material</th>
<th>$\chi_m$</th>
<th>Type</th>
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<tbody>
<tr>
<td>Bi</td>
<td>$-1.66 \times 10^{-5}$</td>
<td>diamagnetic</td>
</tr>
<tr>
<td>Ag</td>
<td>$-2.6 \times 10^{-5}$</td>
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</tr>
<tr>
<td>Al</td>
<td>$2.3 \times 10^{-5}$</td>
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<tr>
<td>Fe (annealed)</td>
<td>5,500</td>
<td>ferromagnetic</td>
</tr>
<tr>
<td>Permalloy</td>
<td>25,000</td>
<td>ferromagnetic</td>
</tr>
<tr>
<td>Mu-metal</td>
<td>100,000</td>
<td>ferromagnetic</td>
</tr>
<tr>
<td>Superconductor</td>
<td>$-1$</td>
<td>diamagnetic (perfect)</td>
</tr>
</tbody>
</table>