4th week of Lectures Jan. 30.– Feb. 03. 2017.

- **Circular motion**
- **Going round the bend**
- **Riding in a Ferris wheel**
- **Gravitation**

Our solar system, satellites

The tides, Dark matter

The Greatest Accomplishment of the Human Intellect, the Development of the Heliocentric World View

(Copernican Revolution)

Personalities participated in this development

- Aristotle BC (384-322)
- Ptolemaeus AD (100-160) Alexandria, Egypt(Cleopatra)
- Copernicus (1473-1543) Poland, Heliocentric theory
- Kepler (1571-1630) Prague, King Rudolf’s Astronomer
- Galileo (1564-1642) Italy, Telescope
- Newton (1643-1727) England, Gravitational force field
- Bohr (1885-1962) Denmark, Circular Atom

Ptolemaeus proposed a “Geocentric” world.

Copernicus 1383 years later proposed the “Heliocentric” world. He suggested that planets orbit around the Sun. It was a theory.

Kepler from experimental data claimed that planets rotate around the Sun on elliptical (circular) orbits.

Galileo constructed a telescope and by visual observation publicly supported views of Copernicus and Kepler.

However neither Kepler nor Galileo could provide a theory to describe their observations.

Galileo’s observation at Pisa (equal time of flight of all heavy objects) refuted Aristotle's theory.

Newton's ideas is that:

1. The necessary condition for circular motion is the existence of a central force.
2. In the Sun-planet case:
   a.) Sun creates an attractive field: $F_S = \dfrac{GM_S}{r^2}$

   b.) This field creates an attractive radial force between Sun-planet system $F_{Sp} = G\dfrac{M_S m_p}{r^2}$

   which provides the central force for circular or elliptical motions

Bohr suggested that atoms consist of a positively charged protons in the center which attract the negatively charged orbiting electrons. Just like the Sun holds on the planets, so do the protons hold on the electrons.

$$F_{pe} = k\frac{q_p q_e}{r^2}$$

Thus the understanding of our material world has its origin in the Heliocentric model of Newton.

Dynamics of Uniform Circular Motion

Uniform circular motion is the motion of an object which is traveling at a constant speed on a circular path.

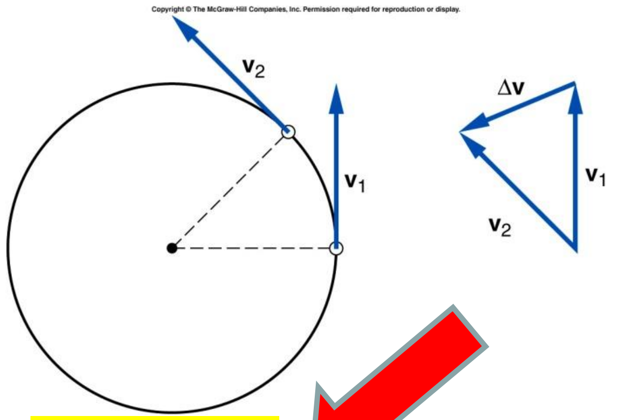
The period T of the motion is the time required to travel once around the circle.

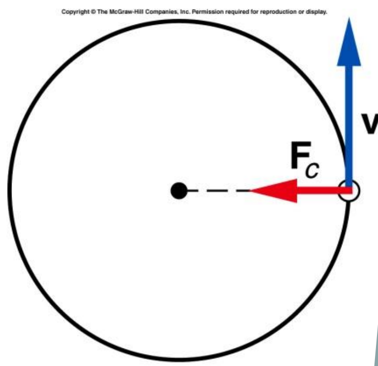
Since $2\pi r$ is the circumference of a circle, the constant speed v is:

$$\mathrm{v} = \frac{2\pi r}{T}$$

While the magnitude of $\vec{\mathrm{v}}$ is constant, the direction changes direction as the planet moves around the circle.

Circular motion





$a = v^2/r$

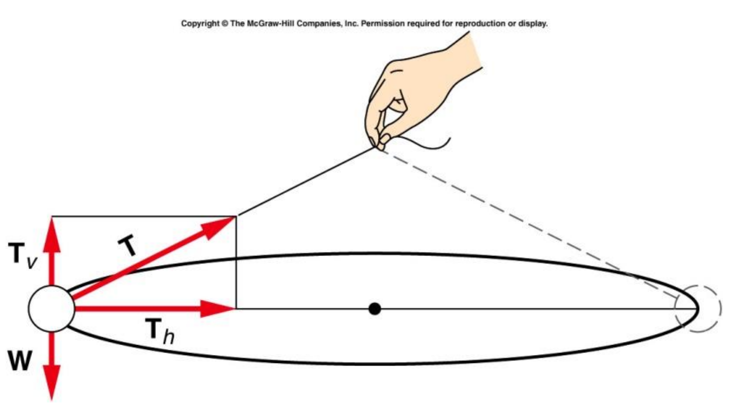
$F = ma = mv^2/r$

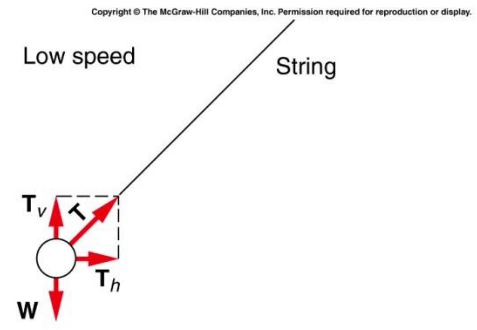
If the velocity of an object changes direction then the object experiences an acceleration and a force is required.

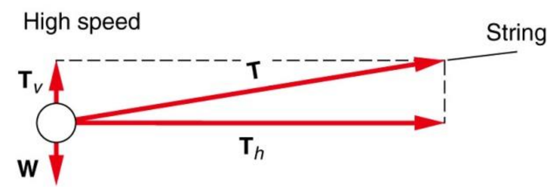
This is a centripetal acceleration and force. They are both directed toward the center of the circle.

This is the effect you feel rounding a corner in a car

Balance of forces







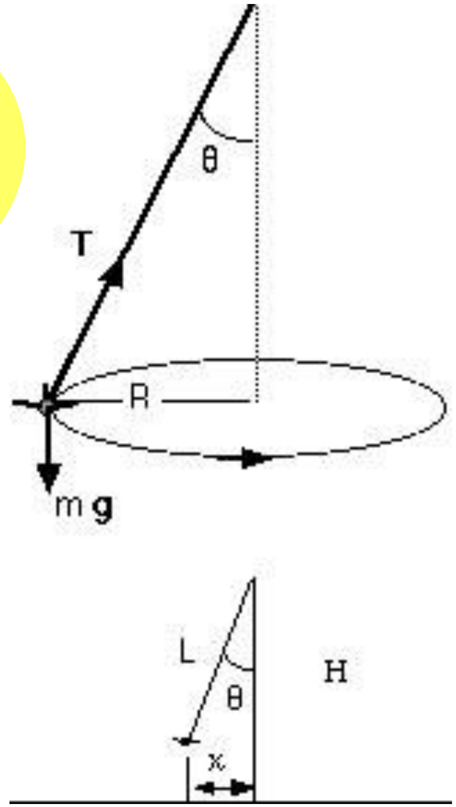
We need to understand the forces that are acting horizontally and vertically.

In the case shown the tension or force exerted by the string has components which balance the weight in the vertical direction and provide the centripetal force horizontally.

$$T_v = W = mg \qquad T_h = mv^2/r$$

1D-02 Conical Pendulum

Could you find the NET force?



$T \sin(\theta) = mv^2/R$
$T \cos(\theta) = mg$
Dividing the above two Equations we obtain:

$$v = \sqrt{gR\tan\theta}$$

Period of the pendulum
$T = 2\pi R/v$,
where $R = L / \sin(\theta)$

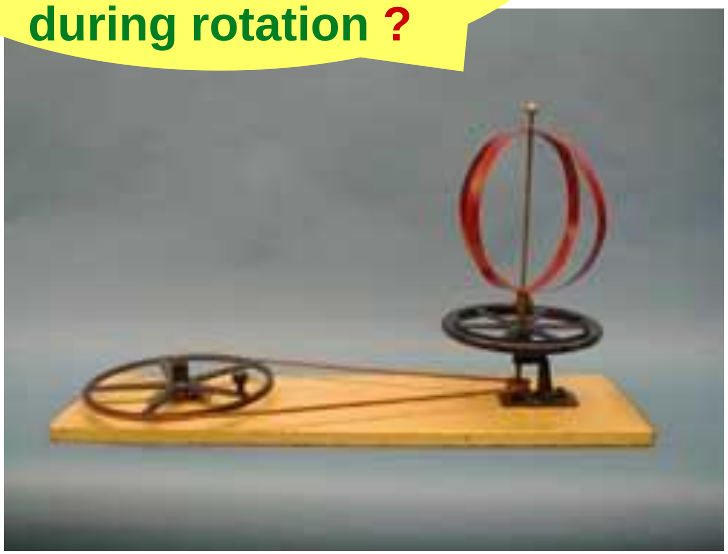
$$T = 2\pi\sqrt{L\cos\theta / g}$$

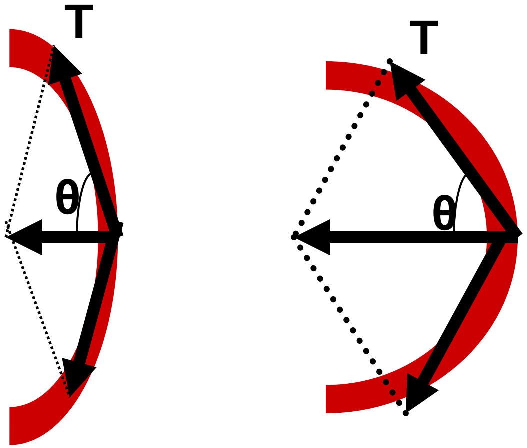
NET FORCE IS TOWARD THE CENTER OF THE CIRCULAR PATH

1D-03 Demonstrations of Central Force

THE SHAPES/SURFACES OF SEMI-RIGID OBJECTS BECOME MORE CURVED TO PROVIDE GREATER CENTRAL FORCES DURING ROTATION.

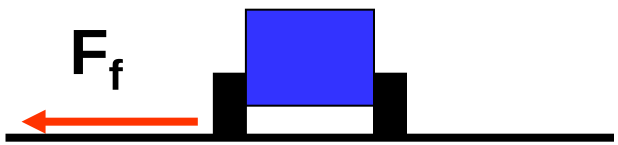
What will happen when it is subjected to forces during rotation ?





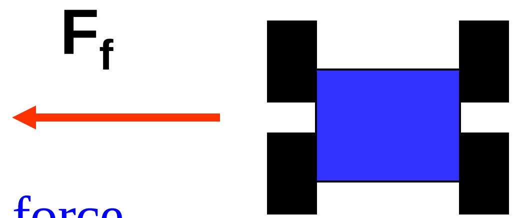
$$2T\cos(\theta) = mv^2/R$$

Centripetal Force and Safe Driving



Rear

$$F_F = \text{frictional force}$$



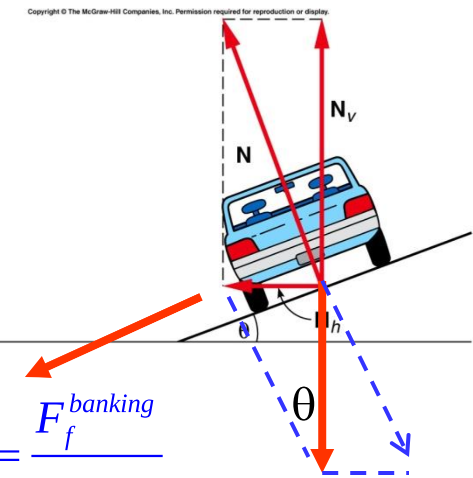
Above

When a car turns a corner it is friction between the tires and the road which provides the centripetal force. $F_f = \mu mg$ **where μ is the frictional coefficient and m is the mass of the car. The maximum speed safe driving is**

$$\frac{m\mathrm{v}^2}{r} = \mu mg \Rightarrow \mathrm{v} = \sqrt{\mu rg}$$

If the road is banked then the horizontal component of the car's weight provides a force which plays the role of the frictional force

$$\tan\theta = \frac{F_f^{banking}}{mg} \quad \Rightarrow \quad F_f^{banking} = mg\tan\theta$$



$$\tan\theta = \frac{F_f^{banking}}{mg}$$

W = mg

For a banked track there is a velocity for which no friction is required.

$$F_f^{banking} = mg\tan\theta = \frac{m\mathrm{v}^2}{r}$$

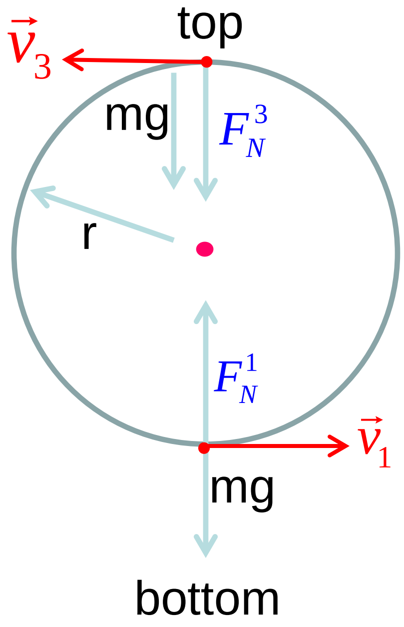
Thus the maximum velocity without slipping off the road is

$$\mathrm{v}_{\max} = \sqrt{rg\tan\theta}$$

irrespective of the surface quality.

Vertical Circular Motion

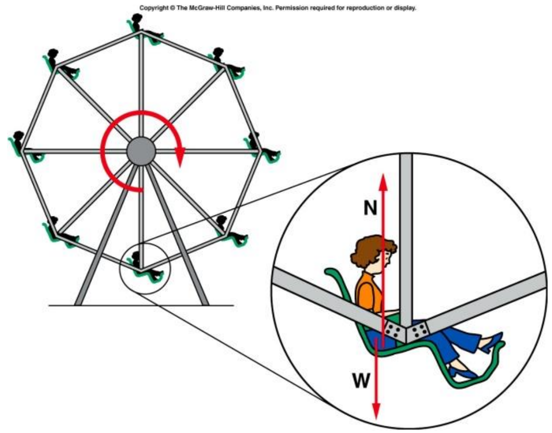
Stunt drivers drive their cycles around a vertical circular track inside of a barrel.



There are 2 points where the centripetal forces can be easily identified:
At the top and the bottom.
Let F_N indicate the normal force pushing on the circle. The centripetal forces are denoted by

$$F_c^1 = \frac{m v_1^2}{r} = F_N^1 - mg \quad \text{(Net force)}$$

$$F_c^3 = \frac{m v_3^2}{r} = F_N^3 + mg \quad \text{(Net force)}$$



Ferris wheel

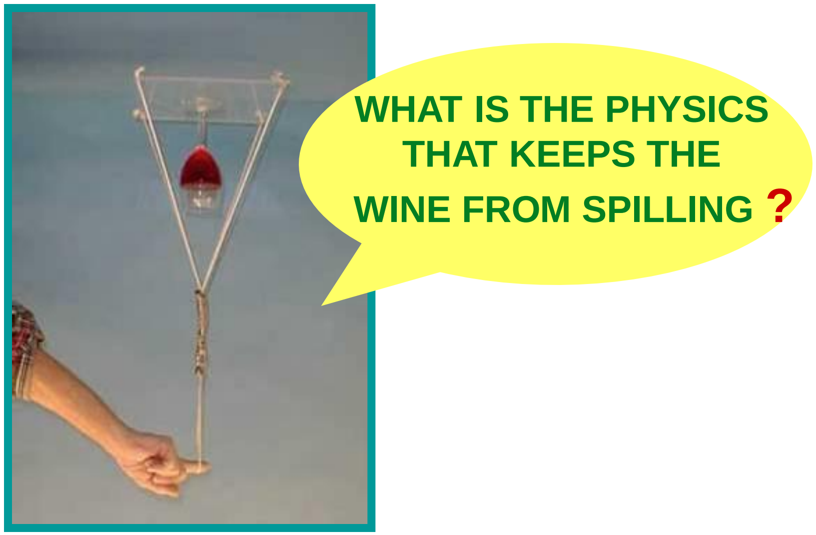
At the bottom
N - mg = mv²/r

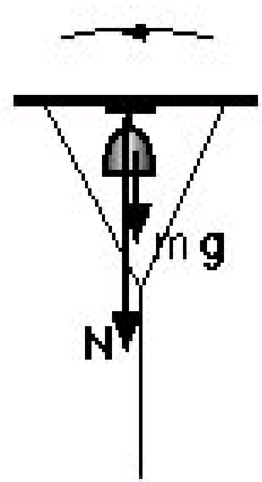
Where m denotes the mass of the bike plus the rider.

At the top
N+mg = mv²/r

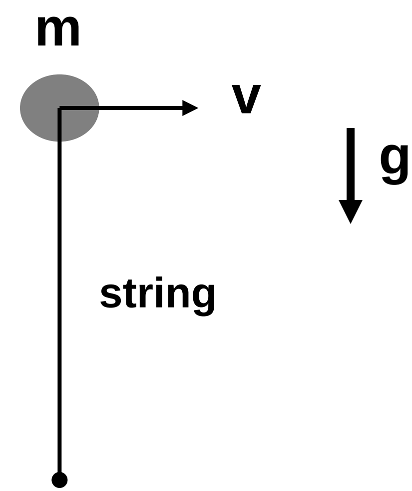
At the speed where $\frac{m v_3^2}{r} = mg$ the driver + motorbike become weightless $v_3 = \sqrt{rg}$

1D-05 Twirling Wine Glass





Same as



$N + mg = mv^2/R$ $\quad N > 0$

THE GLASS WANTS TO MOVE ALONG THE TANGENT TO THE CIRCLE AND THE REACTION FORCE OF THE PLATE AND GRAVITY PROVIDE THE CENTRIPETAL FORCE TO KEEP IT IN THE CIRCLE

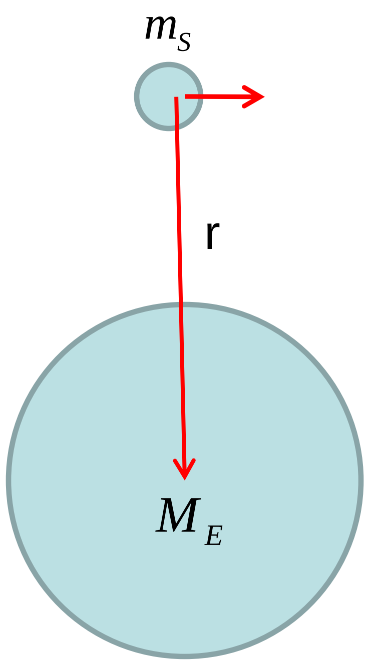
Satellites

Space station

As a satellite circles the earth in a stable orbit we have all the time that the gravitational force exactly equals the centripetal force since the center of the earth is the center of the circle and the direction of gravity.

So everything and everybody is weightless all the time independent of mass.

That is toothbrushes and people are both weightless.



$$G m_S M_E / r^2 = m_S v^2 / r \quad \text{or} \quad G M_E / r^2 = v^2 / r$$

Gravitation and the planets

Astronomy began as soon as man was able to observe the sky and records exist going back several thousand years. In particular the yearly variation of the stars in the sky and the motion of observable objects such as planets.

People observed the “fixed” North Star and, for example, the rising of Sirius signaling the flooding of the Nile.

Copernicus was the first person to advocate, in the year of 1543, a sun centered solar system. He was followed by Galileo who used the first telescopes.

Tycho Brahe was the most famous naked eye astronomer.

Kepler, his assistant, used the data to draw quantitative conclusions.

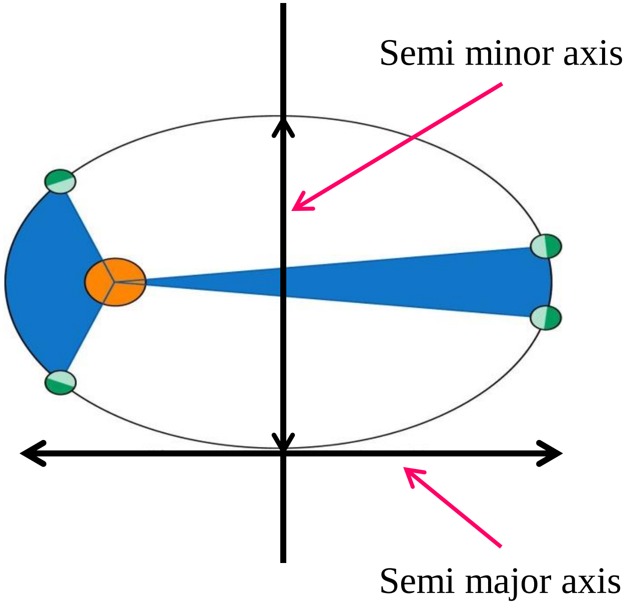
Kepler's Laws

1) **Orbits of planets are ellipses**

2) **An imaginary line drawn from the Sun to the planet is called the radius vector.**

It sweeps out equal areas in equal times.

Kepler published the above two laws in 1609, 66 years after Copernicus' book was published.



3) **T^2 is proportional to r^3.**

T is the period which for the earth is one year and r is the semi major axis

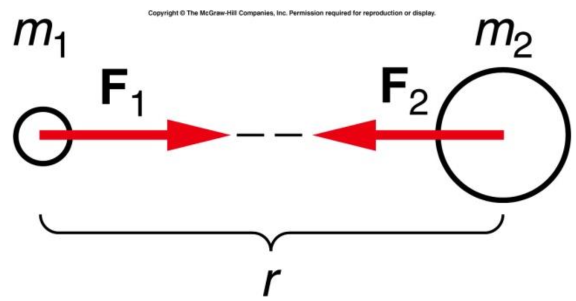
For circular motion the velocity v is constant

The circumference of a circle is 2πR and thus the period T is:

$$T = 2\pi r/v$$

Newton and Gravitation

Newton developed the Law of Gravitation



force between two objects is

$$F = Gm_2m_1 / r^2$$

$$F = G\frac{m_1m_2}{r^2}$$

The constant G of proportionality was measured by Cavendish more than 100 years after Newton

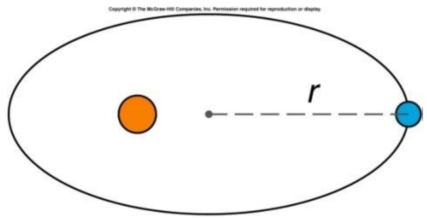
G = 6.67 x 10^{-11} N.m^2/kg^2.

Since at the earths surface $mg = GmM_e / r_e^2$
Knowing g and G Cavendish could measure the mass of the earth.

http://www.physics.purdue.edu/class/applets/NewtonsCannon/newtmtn.html

Planetary orbits

For any planet, orbiting the Sun on a circle we calculate the product of $T^2 \times \frac{1}{r^3}$ and find:



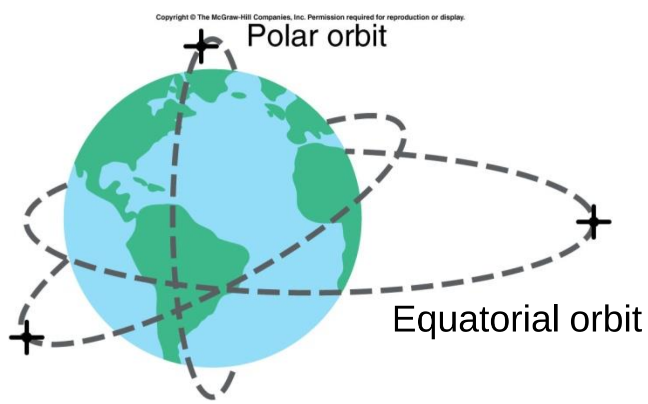
$$GmM_S / r^2 = mv^2 / r \;\Rightarrow \frac{1}{r^3} = \frac{v^2}{GM_S r^2}$$

$$\text{Period:} \quad T = 2\pi r / v \;\Rightarrow T^2 = 4\pi^2 r^2 / v^2$$

$$\boxed{T^2/r^3 = 4\pi^2/GM_s}$$

$$\frac{T^2}{r^3} = \text{constant}$$

where M_S is the mass of the Sun and m is the mass of the earth or M is the mass of the earth and m the mass of a satellite.



For a geosynchronous orbit the period T is 24 hours and the height is 22,000miles above the earth's surface.

Announcement

Please bring a Calculator for the next Lecture Quiz, Feb.08.

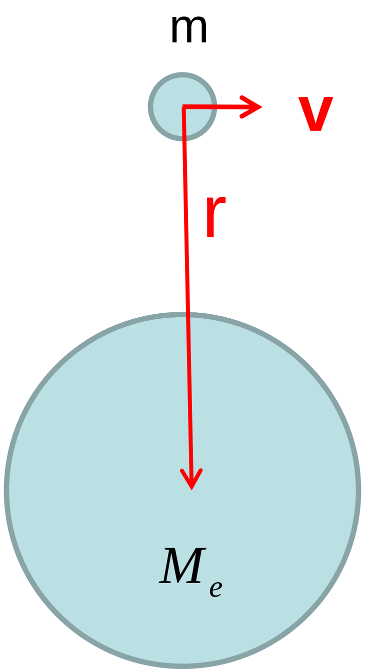
Exam Calculator: When taking a Physics 214 Exam, there is only one calculator model that is acceptable: The

CASIO fx-260 SLRSC FRACTION

NO OTHER BRAND or TYPE WILL BE ALLOWED!

Calculations of Satellite Velocities v as Function of Orbital Radius r

$GmM_e/r^2 = mv^2/r$ Eq.1



where M_e is the mass of the earth and m the mass of a satellite for any r.

At the earths surface $mg = GmM_e / R_e^2 \Rightarrow mgR_e^2 = GmM_e$ Eq.2

Substituting Eq.2 into Eq.1, we obtain

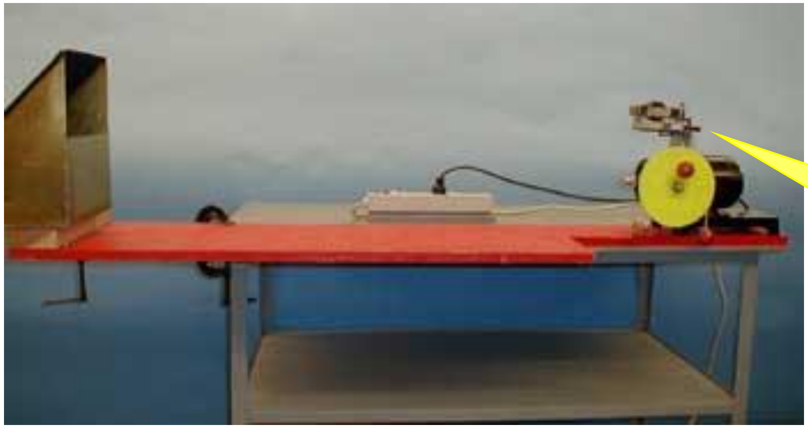
$v^2 = gR_e^2 / r$ for a stable orbit

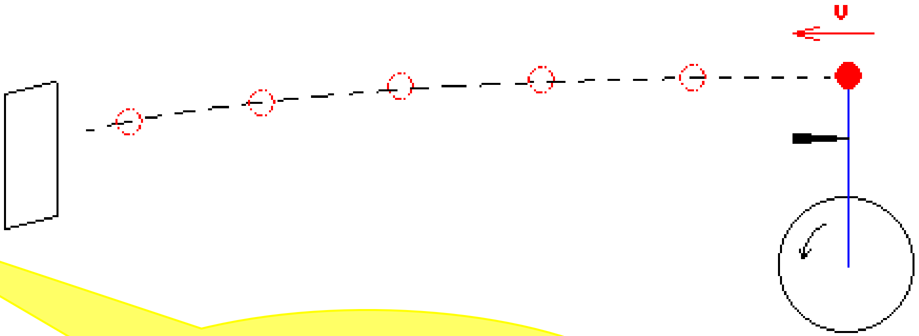
So at the earth's surface: $R_e = 6370km$ thus
V = ~ 17500mph

For a geosynchronous satellite at a height of 22,000 miles T is fixed, thus by Kepler's third law r and v are fixed, as shown on the previous page. There is one radius and one velocity for a geosynchronous satellite.

V = ~6500mph, V decreases with distance.
The moon is losing energy because of the tides so it is moving away from us.

1D-04 Radial Acceleration & Tangential Velocity



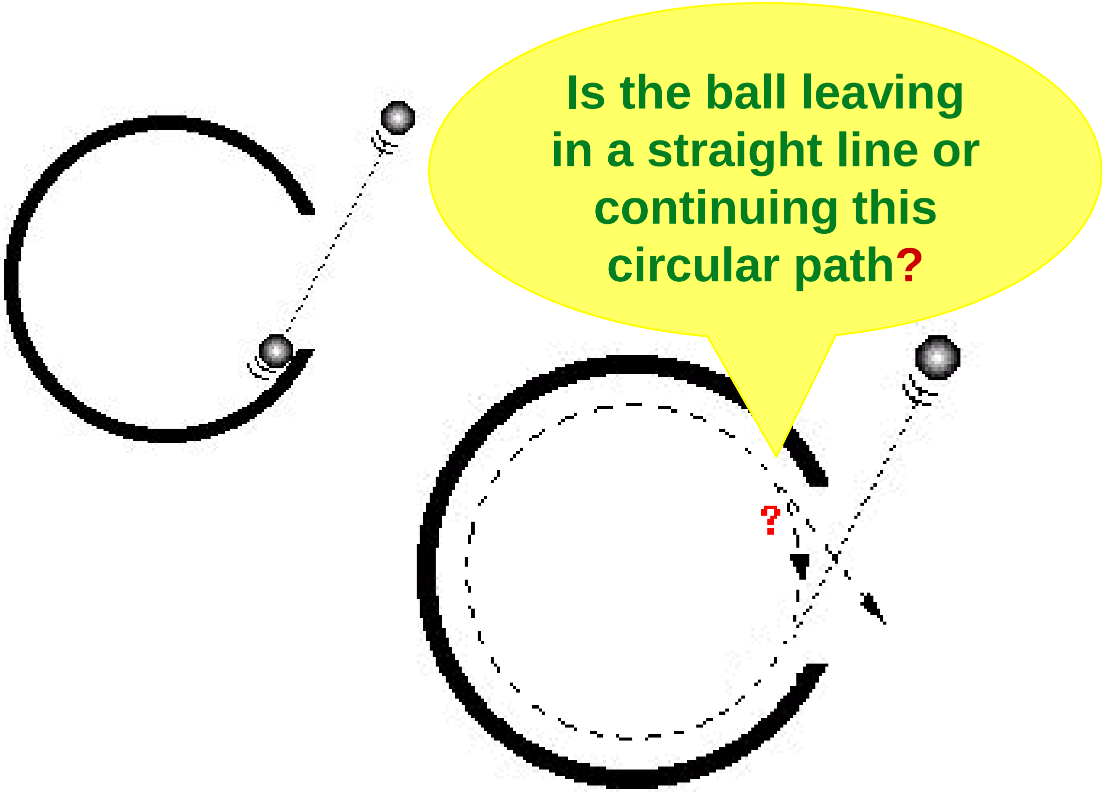


Once the string is cut, where is the ball going?

AT ANY INSTANT, THE VELOCITY VECTOR OF THE BALL IS DIRECTED ALONG THE TANGENT.

AT THE INSTANT WHEN THE BLADE CUTS THE STRING, THE BALL'S VELOCITY IS HORIZONTAL SO IT ACTS LIKE A HORIZONTALLY LAUNCHED PROJECTILE AND LANDS IN THE CATCH BOX.

1D-08 Ball in Ring

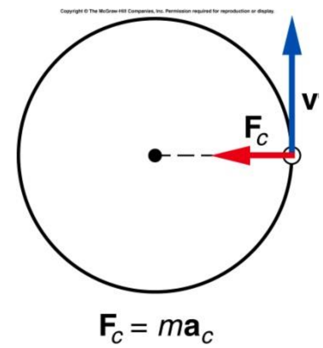


THE FORCE WHICH KEEPS THE BALL MOVING CIRCULAR IS PROVIDED BY THE RING. ONCE THE FORCE IS REMOVED, THE BALL CONTINUES IN A STRAIGHT LINE, ACCORDING TO NEWTON'S FIRST LAW.

Summary of Chapter 5

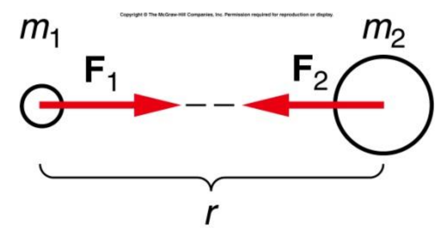
❖**Circular motion and centripetal acceleration and force.**

$$F_c = mv^2/r$$



Ferris wheel, car around a corner or over a hill.

❖**Gravitation and Planetary orbits**

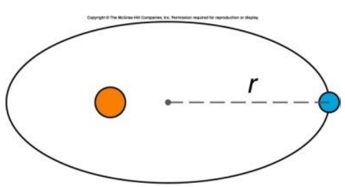


$$F = G\frac{m_1 m_2}{r^2}$$

For a simple circular orbit

$$GmM/r^2 = mv^2/r$$

M is the mass of the sun and m the mass of the earth.



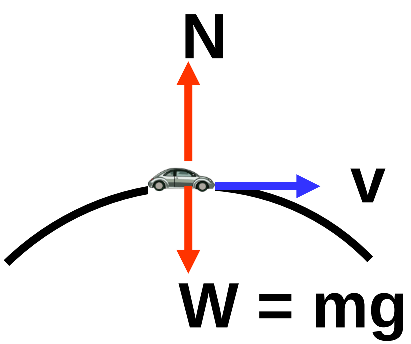
$$v^2 = GM/r \quad T = 2\pi r/v$$

$$T^2 = 4\pi^2 r^2/v^2 = 4\pi^2 r^3/GM_s$$

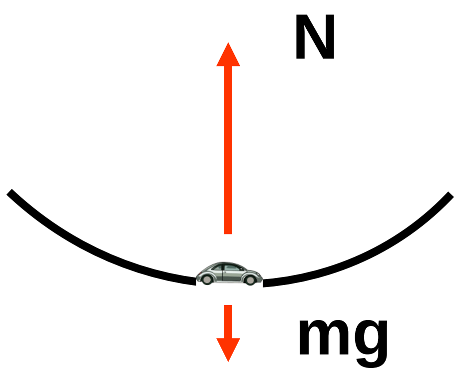
$$T^2/r^3 = 4\pi^2/GM_s$$

Examples of circular motion

Vertical motion

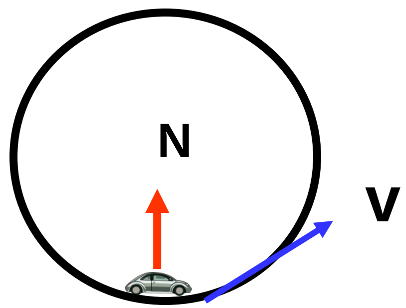


$mg - N = mv^2/r$

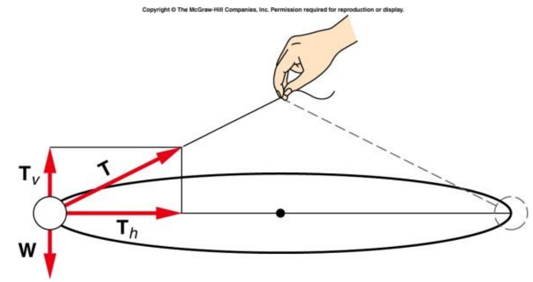


$N - mg = mv^2/r$

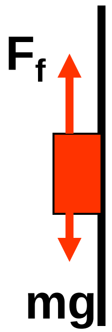
Looking down



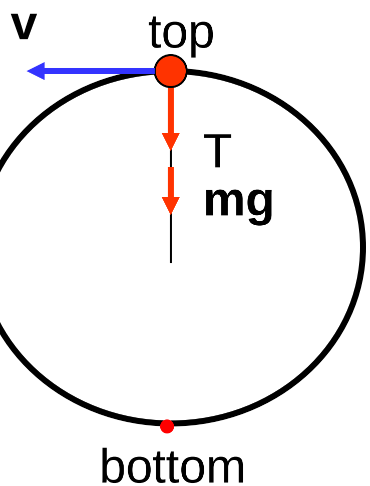
$N = mv^2/r$



Side



$mg = F_f$



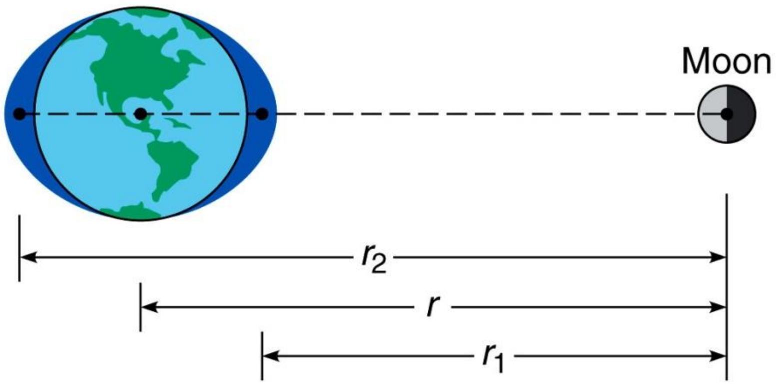
$mg + T = mv^2/r$ top

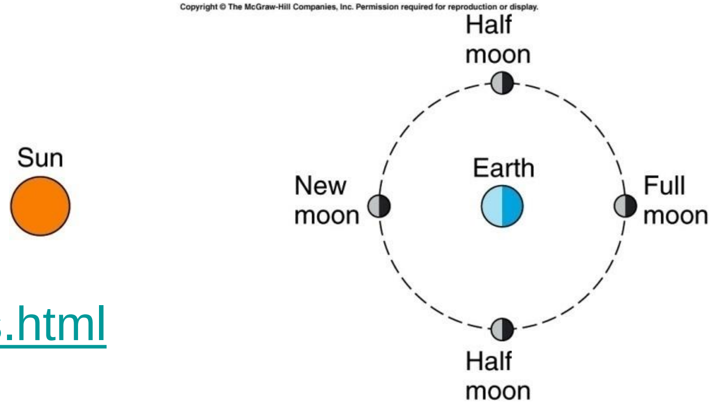
$T - mg = mv^2/r$ bottom

Moon and tides

anim0012.mov

Tides are dominantly due to the gravitational force exerted by the moon. Since the earth and moon are rotating this effect also plays a role. The moon is locked to the earth so that we always see the same face. Because of the friction generated by tides the moon is losing energy and moving away from the earth.





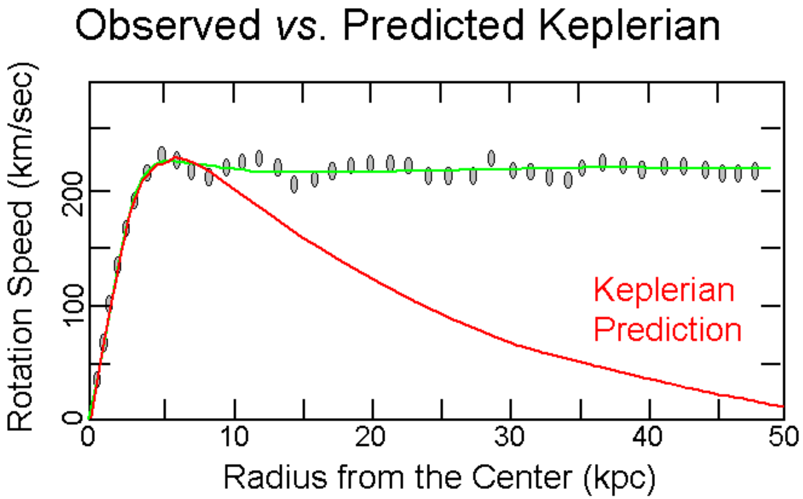
http://www.sfgate.com/getoutside/1996/jun/tides.html

http://astro.unl.edu/classaction/animations/lunarcycles/tidesim.html

Dark Matter

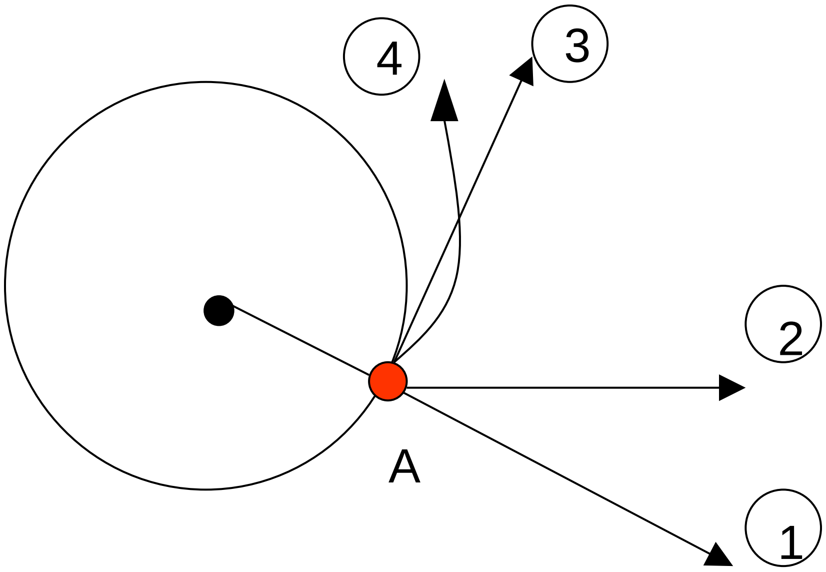
For the orbit of a body of mass m about a much more massive body of mass M $GmM/r^2 = mv^2/r$ and

$GM/r = v^2$. In fact M is the mass inside the orbit, that is the sun could be nearly as big as the orbit of the earth and it would not change anything. If we look at stars in motion in galaxies we find there is not enough normal matter to provide the necessary gravitational force. We believe this is caused by a new form of matter, which we call dark matter, and it comprises 25% of the energy in our Universe.(normal matter = 4.4%)



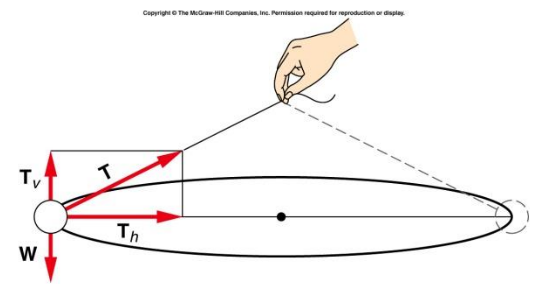
Questions Chapter 5

Q6 **A ball on the end of a string is whirled with constant speed in a counterclockwise horizontal circle. At point A in the circle, the string breaks. Which of the curves sketched below most accurately represents the path that the ball will take after the string breaks (as seen from above)? Explain.**



Path number 3

Q8 For a ball twirled in a horizontal circle at the end of a string, does the vertical component of the force exerted by the string produce the centripetal acceleration of the ball? Explain.

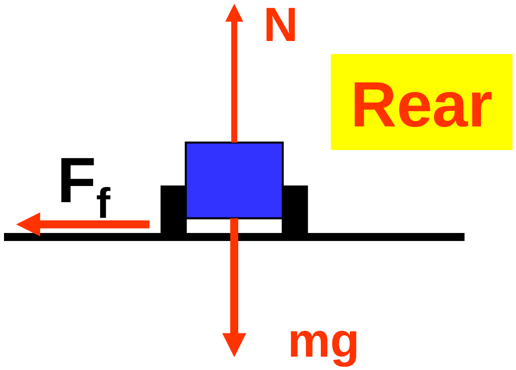


Vertical component balances the weight
Horizontal component provides the acceleration

Q9 A car travels around a flat (unbanked) curve with constant speed.

A. Show all of the forces acting on the car.

B. What is the direction of the net force act.



The force acts toward the center of the turn circle

Q10 **Is there a maximum speed at which the car in question 9 will be able to negotiate the curve? If so, what factors determine this maximum speed? Explain.**

Yes. The friction between the tires and the road

Q11 **If a curve is banked, is it possible for a car to negotiate the curve even when the frictional force is zero due to very slick ice? Explain.**

Yes there is just one speed. If the car moves too slowly it will slide down. If it moves to fast it will slide up.

Q12 **If a ball is whirled in a vertical circle with constant speed, at what point in the circle, if any, is the tension in the string the greatest? Explain. (Hint: Compare this situation to the Ferris wheel described in section 5.2).**

The tension is the greatest at the bottom because the string has to support the weight and provide the force for the centripetal acceleration.

Q19 **Does a planet moving in an elliptical orbit about the sun move fastest when it is farthest from the sun or when it is nearest to the sun? Explain by referring to one of Kepler's laws.**

When it is nearest

Q20 **Does the sun exert a larger force on the earth than that exerted on the sun by the earth? Explain.**

The magnitude of the forces is the same they are a reaction/action pair

Q23 **Two masses are separated by a distance *r*. If this distance is doubled, is the force of interaction between the two masses doubled, halved, or changed by some other amount? Explain.**

The force reduces by a factor of 4

Ch 5 E 14

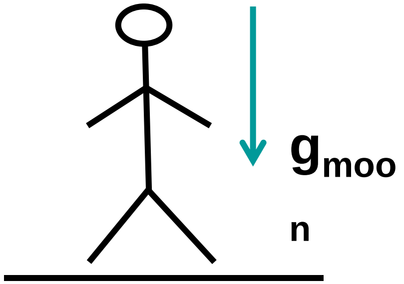
The acceleration of gravity at the surface of the moon is about 1/6 that at the surface of the Earth (9.8 m/s^2). What is the weight of an astronaut standing on the moon whose weight on earth is 180 lb?

$W_{earth} = m\ g_{earth} = 180\ \text{lb}$

$W_{moon} = m\ g_{moon}$

$g_{moon} = 1/6\ g_{earth}$

$W_{moon} = m\ 1/6\ g_{earth} = 1/6\ m\ g_{earth} = 1/6\ (180\ \text{lb})$



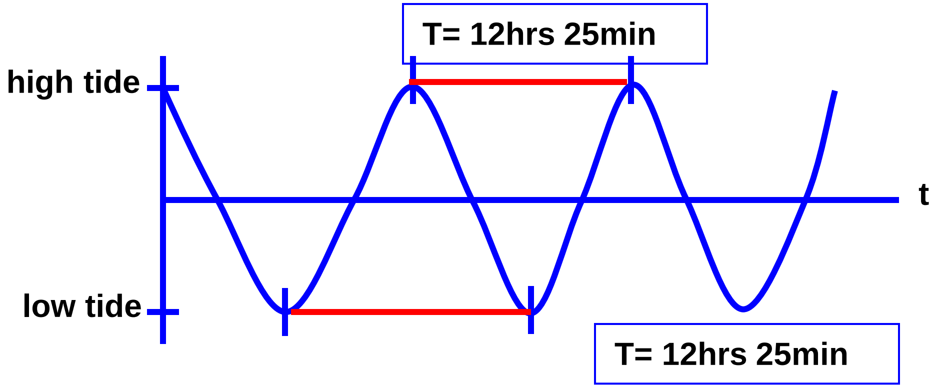
Ch 5 E 16

Time between high tides = 12 hrs 25 minutes.
High tide occurs at 3:30 PM one afternoon.
a) When is high tide the next afternoon
b) When are low tides the next day?

a) 3:30 PM + 2 (12 hrs 25 min)

= 3:30 PM + 24 hrs + 50 min

= 4:20 PM



b) Low tide the next day = 4:20 PM - 6 hr 12 min 30 s = 10:07:30 AM

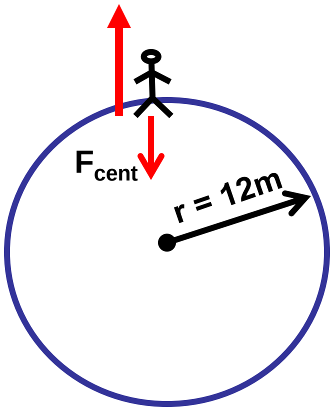
2nd Low tide = 10:07:30 AM + 12 hrs 25 min = 10:32:30 PM

Ch 5 CP 2

A Ferris wheel with radius 12 m makes one complete rotation every 8 seconds.

a) **Rider travels distance $2\pi r$ every rotation. What speed do riders move at?**
b) **What is the magnitude of their centripetal acceleration?**
c) **For a 40 kg rider, what is magnitude of centripetal force to keep him moving in a circle? Is his weight large enough to provide this centripetal force at the top of the cycle?**
d) **What is the magnitude of the normal force exerted by the seat on the rider at the top?**
e) **What would happen if the Ferris wheel is going so fast the weight of the rider is not sufficient to provide the centripetal force at the top?**

Ch 5 CP 2 (con't)



a) $S = d/t = 2\pi r/t = 2\pi(12m)/8s =$ 9.42 m/s

b) $a_{cent} = v^2/r = s^2/r = (9.42 m/s)^2/12m =$ 7.40 m/s^2

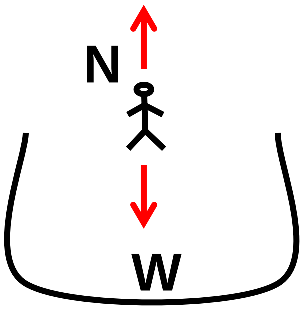
c) $F_{cent} = m\,v^2/r = m\,a_{cent} = (40\text{ kg})(7.40\text{ m/s}^2) =$ 296 N

$W = mg = (40\text{ kg})(9.8\text{ m/s}^2) =$ 392 N

Yes, his weight is larger than the centripetal force required.

d) $W - N_f = 296$ N = 96 newtons

e) rider is ejected



Ch 5 CP 4

A passenger in a rollover accident turns through a radius of 3.0m in the seat of the vehicle making a complete turn in 1 sec.

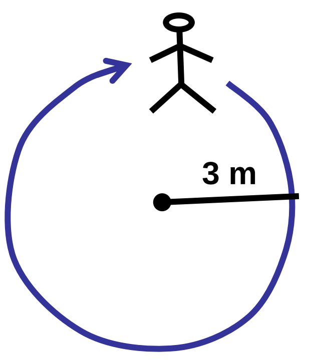
a) Circumference = $2\pi r$, what is speed of passenger?

b) What is centripetal acceleration? Compare it to gravity (9.8 m/s^2)

c) Passenger has mass = 60 kg, what is centripetal force required to produce the acceleration? Compare it to passengers weight.

a) $s = d/t = 2\pi(3.0m)/1 =$ **19m/s**

b) $a = v^2/r = s^2/r = (19\ m/s)^2/3m = 118\ m/s^2 =$ **12g**



c) $F = ma = (60\ kg)(118\ m/s^2) = 7080\ N$

$F = ma = m\ (12\ g) = 12\ mg =$ **12 weight**

Ch 5 CP 6

The period of the moons orbit about the earth is 27.3 days, but the average time between full moons is 29.3 days. The difference is due to the Earth's rotation about the Sun.

a) Through what fraction of its total orbital period does the Earth move in one period of the moons orbit?

b) Sketch the sun, earth & moon at full moon condition. Sketch again 27.3 days later. Is this a full moon?

c) How much farther does the moon have to move to be in full moon condition? Show that it is approx. 2 days.

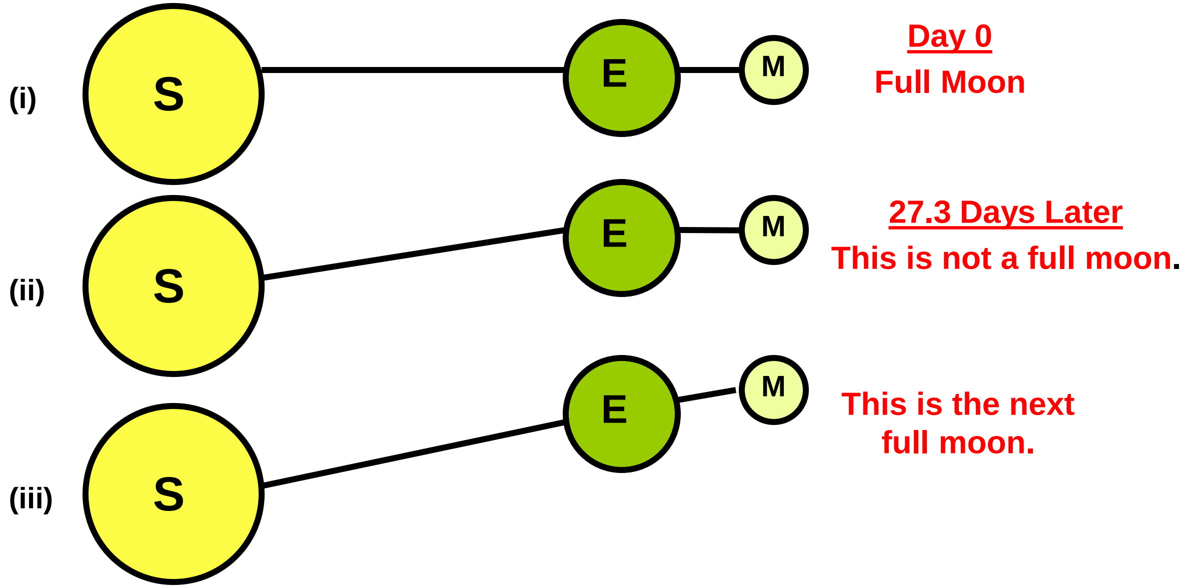
a) Earth orbital period = 365 days = 0_E

Moon orbital period = 27.3 days = 0_M

$0_M/0_E = 27.3/364 \approx 0.075$

Ch 5 CP 6 (con't)

b)



c) For moon to achieve full moon condition, it must sit along the line connecting sun & earth. In part (a) we found that the earth has moved thru 0.075 of its full orbit in 27.3 days (see diagram (ii)). To be inline w/ sun and earth, moon must move thru same fraction of orbit (see diagram (iii)).

0.075 (27.3 days) ≈ 2 days.