

2nd Week

Jan. 16. - Jan. 20. 2017.

Gravity: Dropping and throwing point objects vertically

Projectile motion of a package dropped from a plane

Two dimensional motion of baseballs and cannonballs

Announcement

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CHIP

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Free fall is a One dimensional motion with constant acceleration

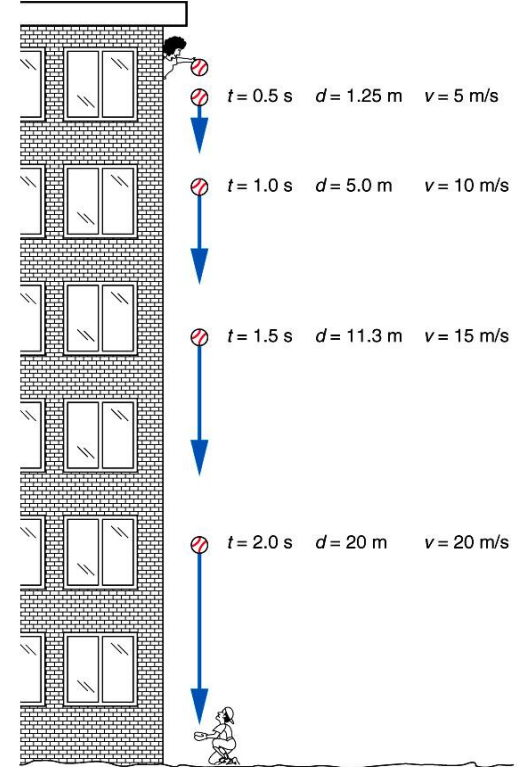
When we drop an object it's velocity continues to increase. That means there is an acceleration.



At our latitude and near the earth's surface, the value of this acceleration is a constant denoted by g where $g = 9.8\text{m/s}^2$.

This value is due to the attractive force of Earth's gravity and g does vary over the earth's surface because the earth is not a perfect sphere and because of the rotation of the earth.

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Origo
 $y=0$
 $v = gt$
 $y = \frac{gt^2}{2}$
 y

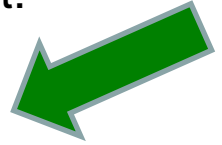
Last lecture we derived the equations of motion of an accelerating object:

$$v = v_0 + at \quad \text{Eq.1a}$$

$$d = v_0 t + \frac{1}{2} at^2 \quad \text{Eq.1b}$$

$$v^2 = v_0^2 + 2ad \quad \text{Eq.2a}$$

$$d = \frac{1}{2} (v + v_0)t \quad \text{Eq.2b}$$



Starting at $d=0$

Expressing the above equations in terms of y coordinates:

$$v_y = v_y^i + a_y t \quad \text{Eq.1a}$$

$$y = v_y^i t + \frac{1}{2} a_y t^2 \quad \text{Eq.1b}$$

$$v_y^2 = (v_y^i)^2 + 2a_y y \quad \text{Eq.2a}$$

$$y = \frac{1}{2} (v_y + v_y^i)t \quad \text{Eq.2b}$$

Throwing vertically

In the example a point object is tossed vertically.
The acceleration is negative:

$$a_y = -g = -9.8 \text{ m/s}^2$$

$$v_y^i = v_0 \text{ is the initial velocity}$$

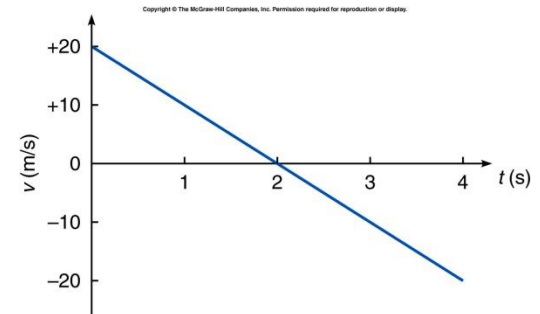
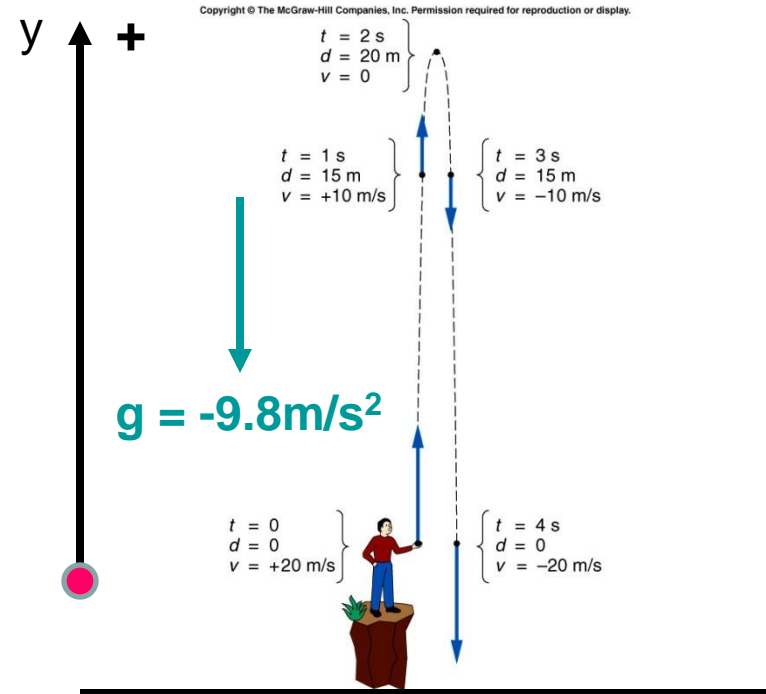
and thus the velocity is decreasing

$$v = v_0 + at = v_0 - gt = v_0 - 9.8t$$

As the object reaches the highest point its velocity become zero and it stops. The time when it happens t_{up} is

$$0 = v_0 - 9.8t_{up} \rightarrow t_{up} = v_0 / 9.8$$

Next it start falling and when a total time of flight (t_{tof}) elapsed the object returns to it initial $d=0$ position.



Substituting:

$$y = 0, \quad v_y^i = v_0 \quad t = t_{tof} \quad \text{and} \quad a = -9.8 \text{ m/s}^2 = -9.8 \text{ m/s}^2 \quad \text{into} \quad \text{Eq.1b}$$

$$\text{where Eq.1b is: } y = v_y^i t + \frac{1}{2} a_y t^2$$

$$\begin{aligned} \text{we obtain: } 0 &= v_0 t_{tof} - \frac{1}{2} g t_{tof}^2 \Rightarrow 0 = t_{tof} \left(v_0 - \frac{1}{2} g t_{tof} \right) \\ &\begin{array}{l} \nearrow t_{tof} = 0 \\ \searrow t_{tof} = \frac{2v_0}{g} \end{array} \\ t_{tof} = \frac{2v_0}{g} = \frac{2v_0}{9.8} &\Rightarrow t_{tof} = 2t_{up} \quad t_{up} = t_{down} \end{aligned}$$

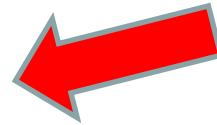
Using Eq.2a there are two velocities at $d = 0$. Since in this case $2ad = 0$, thus

$$v^2 = v_0^2$$

This equation has two solutions: $v = \pm v_0$

$$\text{at } t = 0 \quad v = +v_0$$

$$\text{at } t_{tof} \quad v = -v_0$$



1K-11 Coin and Feather

DROPPING A COIN AND A FEATHER ?



**DO ALL OBJECTS
HAVE THE SAME
ACCELERATION
WHEN DROPPED ?**

**IN AIR WEIGHT AND SURFACE AREA MAKE OBJECTS
FALL AT DIFFERENT SPEEDS BECAUSE OF AIR FRICTION.**

1G-03 Measurement of g

Measuring g by dropping an object



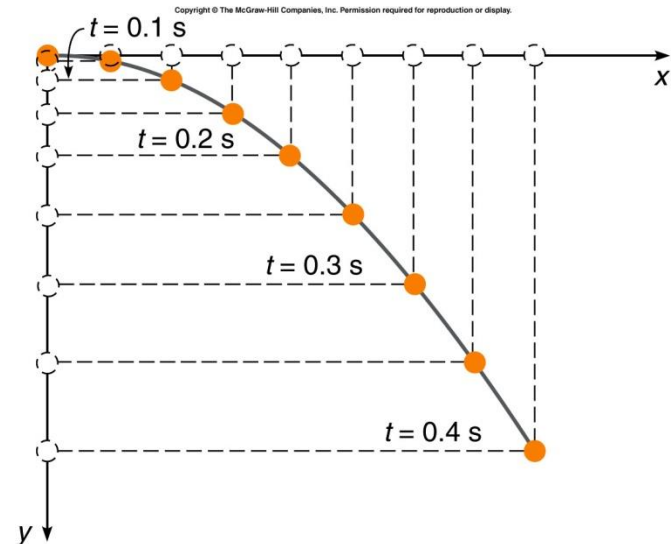
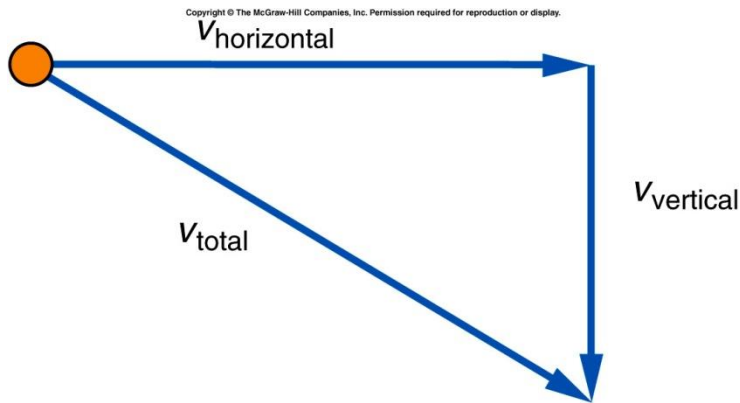
$$d = \frac{1}{2}gt^2 \quad t = \sqrt{2d/g}$$
$$g = 2d/t^2$$

What difficulties
might be
encountered in
measuring h & t of
fall ?

Hand timing would not be accurate because of the short fall time

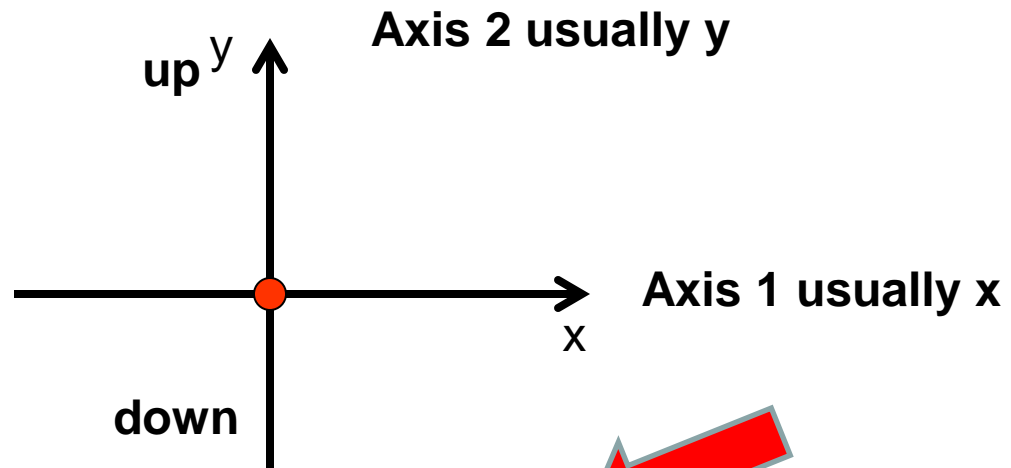
Projectile Motion

If we throw an object so that its initial velocity is horizontal then ignoring friction it will continue to move with this velocity in the horizontal direction but it will also start to fall so that its trajectory will be curved.



Motion in two dimensions

If we take two axes at right angles we can analyze the motion along each axis separately and determine properties of the whole motion. We will only deal with cases where there is a constant velocity along the x axis and a constant acceleration along the y axis. This means that for



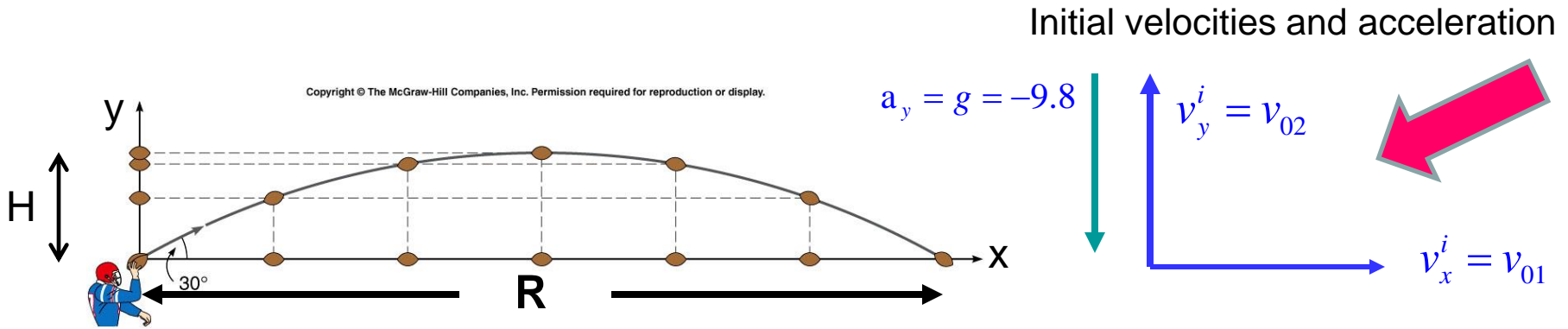
Along the x axis $v_x = v_x^i = \text{constant}$ and $x = v_x^i t$, $a_x = 0$

Along the y axis $v_y = v_y^i + a_y t$ and $y = v_y^i t + \frac{1}{2} a_y t^2$

$$v_y^2 = (v_y^i)^2 + 2a_y y \quad y = \frac{1}{2} (v_y^i + v_y) t$$

Projectile Motion

We will focus on projectiles with $|a| = g = 9.8\text{m/s}^2$ along the negative y axis. The highest point and the range are denoted by H and R



At the highest point H: the vertical velocity is zero,

$v_2 = 0$ and $t = t_{up}$ thus

$$v_2 = 0 = v_{02} + at_{up} \quad \text{so } t_{up} = v_{02}/9.8 = v_{02}/g$$

$$H = v_{02}t_{up} + \frac{1}{2}at_{up}^2 = \frac{v_{02}^2}{g} - \frac{1}{2}g\left(\frac{v_{02}}{g}\right)^2 = \frac{v_{02}^2}{2g}$$

The magnitude of the range R is:

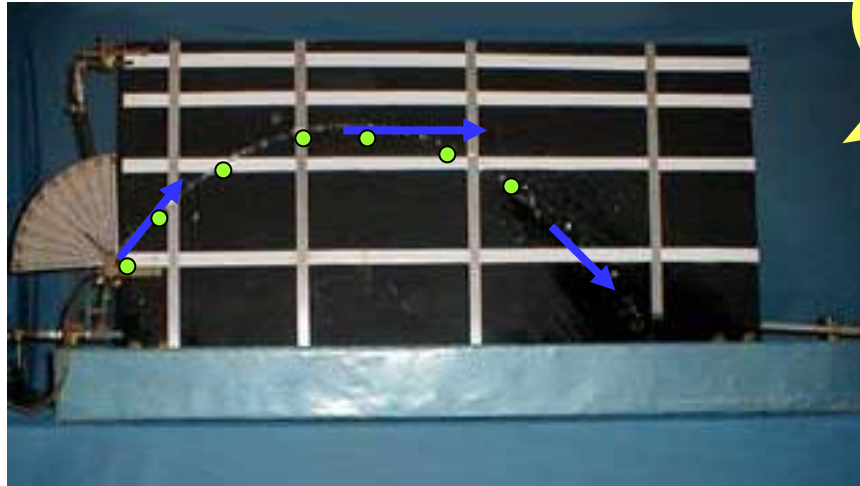
$$t = t_{tof} = 2v_{02}/9.8 \quad \text{and thus } R = v_{01} \times t_{tof} = v_{01} \times 2v_{02}/9.8$$

The vertical velocity at the end of the range is minus v_{02}

1D-22 Water Jets & Projectile Motion

PROJECTILE MOTION OF A WATER JET

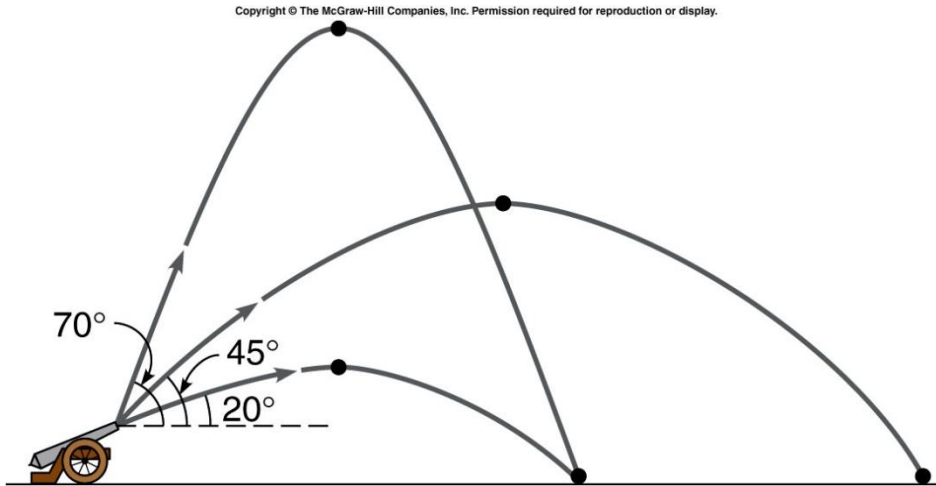
↓
g



What angle gives the maximum range?

**NEGLECTING FRICTION THE RANGE IS A MAXIMUM AT 45° .
TWO DIFFERENT ANGLES CAN GIVE THE SAME RANGE (ANGLES SYMMETRIC ABOUT 45°).
A LARGER ANGLE MEANS A LONGER TIME OF FLIGHT, BUT LESS HORIZONTAL VELOCITY. A SMALLER ANGLE MEANS A LARGER HORIZONTAL VELOCITY, AND LESS FLIGHT TIME.
THE TRAJECTORY IS SYMMETRIC.**

Trajectories



**With no friction
there are always
two angles which
give the same
range for the same
starting velocity
 $45^\circ + X$ and $45^\circ - X$**

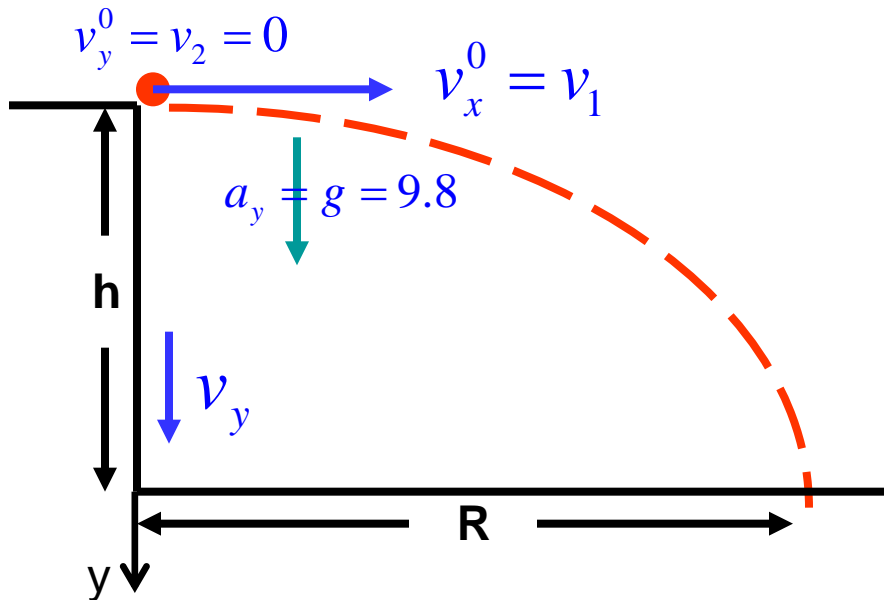
<http://faculty.tcc.fl.edu/scma/carrj/Java/baseball4.html>

<http://www.mhhe.com/physsci/physical/giambattista/proj/projectile.html>

<http://www.physics.purdue.edu/class/applets/phe/projectile.htm>

Throwing a ball horizontally from a building

Use +down so g is positive and y is positive



Thus we obtain the range $R = v_1 t_{tof}$

$$y = v_y^0 + \frac{1}{2} a_y t^2 \quad Eq.1a$$

$$v_y = v_y^0 + a_y t \quad Eq.1b$$

$$v_y^0 = 0 \quad , \quad v_x^0 = v_1$$

Since $v_y^0 = 0$ from *Eq.1a*
the vertical time of flight is:

$$t_{tof}^2 = \frac{2h}{a_y}$$

NOTE h is measured down

Summary Chapter 3

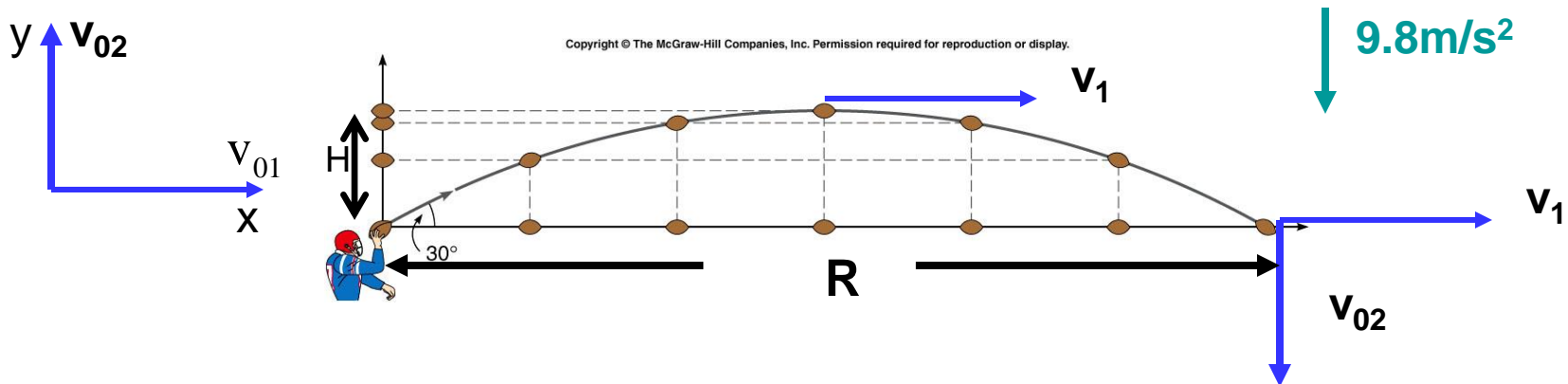
- ❖ Any motion in a plane can be analyzed using two axes at right angles.
- ❖ The motion along each axis is independent of the other
- ❖ The two dimensional motion can be analyzed as two one dimensional motions linked by time. In our special case.

axis 1=X $v_{01} = \text{constant and } d_1 = v_{01}t$

axis 2=Y $v_2 = v_{02} + at, h = v_{02}t + \frac{1}{2}at^2, v_2^2 = v_{02}^2 + 2ah$

At the maximum height H: $v_2 = 0$ thus $0 = v_{02}^2 + 2aH$ thus $H = \frac{v_{02}^2}{2g}$

At the end of range R: $t_{\text{tof}} = 2v_{02}/9.8$ thus $R = v_{01} \times t_{\text{tof}} = v_{01} \times 2v_{02}/9.8$

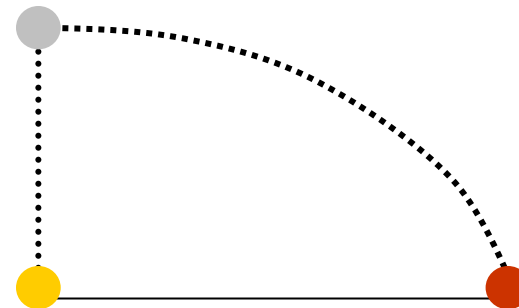
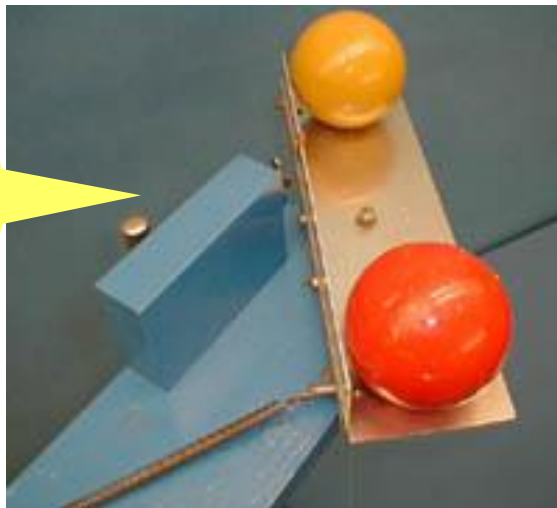


1D-20

Independence of Vertical & Horizontal Motions (Drop-Kick)

One ball drops from rest. The other ball is simultaneously projected horizontally

Which ball will hit the ground first ?



Listen to the **SOUND** when they hit the ground and when they bounce.

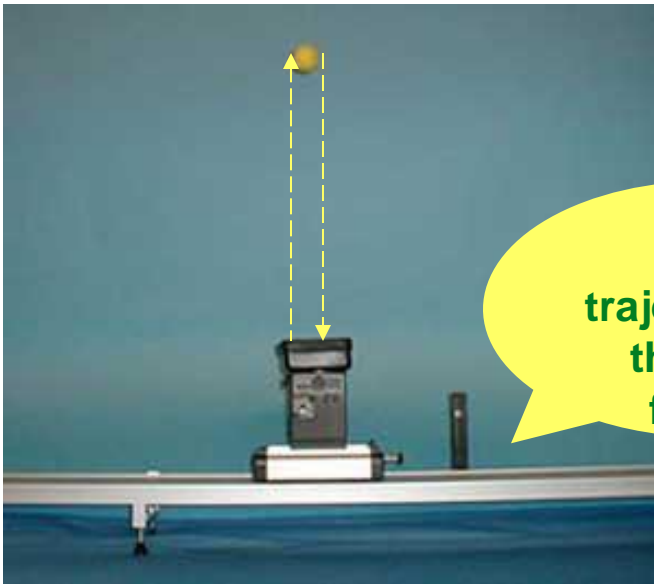
**THE VERTICAL & HORIZONTAL MOTIONS ARE INDEPENDENT.
THE HORIZONTAL VELOCITY DOES NOT AFFECT THE VERTICAL MOTION.**

THE VERTICAL FALL TIME IS THE SAME AS LONG AS THE BALLS DROP SIMULTANEOUSLY FROM THE SAME HEIGHT.

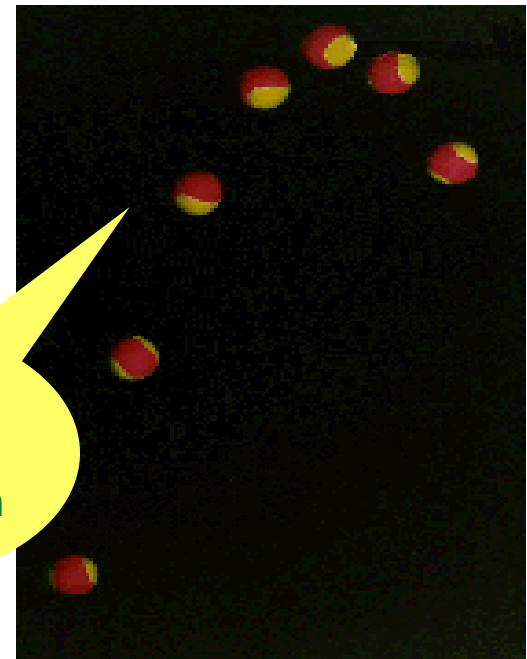
1D-21

Independence of Vertical and Horizontal Motions

A ball is projected vertically from a cart traveling horizontally



The trajectory in the cart frame



The trajectory in the room frame

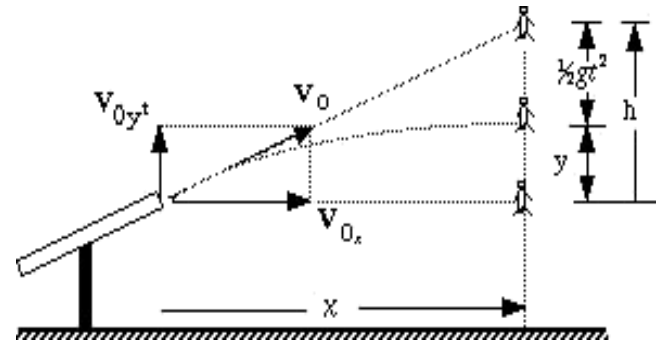
THE HORIZONTAL MOTION OF THE BALL IS UNAFFECTED BY ITS VERTICAL MOTION.

1D-23 Shoot the Monkey

The monkey falls out of the tree at the instant the gun is fired



WHERE SHOULD ONE AIM, ABOVE, BELOW OR AT?



Ignoring friction $y = v_{0y}t - 1/2gt^2$ $t = x/v_{0x}$, $v_{0y}/v_{0x} = h/d$
at $x = d$ $y = h - 1/2gt^2$ In the same time the monkey falls $1/2gt^2$
So the bullet always hits the monkey no matter what the value of v_0

THE VERTICAL MOTION IS INDEPENDENT OF THE HORIZONTAL MOTION
THE EFFECT OF FRICTION IS MINIMIZED BY USING A LARGE TARGET

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What causes motion

In our everyday life we observe that objects change their state of motion.

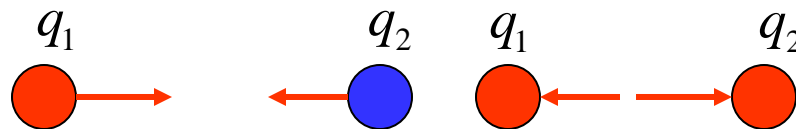
These changes are produced by forces and in our everyday life there are just two forces.

Gravitational force acts on mass



$$F = Gm_1m_2/r^2$$

Electrical force acts on charge



$$F = kq_1q_2/r^2$$

Extinction of the dinosaurs

There are accurate ways to look at events in geologic time back to the formation of the earth about 4.5 billion years ago.

- ❖ Radioactive elements are very accurate clocks and sedimentary layers reveal geologic events as a function of time.
- ❖ 65 million years ago the extinction of the dinosaurs and 70% of all species
- ❖ 250 million years ago over 90% of all species became extinct.

Experimental measurements show that the 65 million event was due to an asteroid which impacted in the gulf of Mexico near the Yucatan peninsula.

An asteroid about 6 miles in diameter and traveling at 45,000 miles a second, slammed into the Gulf of Mexico causing a crater 24 miles deep and 125 miles wide. The blast was equivalent to 100 million tons of TNT.

Sedimentary layers

Looking back in time



The discovery of iridium in a layer corresponding to 65 million years ago gave the key to the extinction of the dinosaurs

Asteroid impact simulation

PARAMETERS	
Projectile Diameter:	0 m
Projectile Density:	0 kg/m ³
Angle of Impact:	45 degrees
Velocity:	11 km/s
Target Type:	Sedimentary Rock
Distance from Impact:	0 km

* All fields are required

PROJECTILE PARAMETERS	IMPACT PARAMETERS	TARGET PARAMETERS
Diameter: <input type="text"/> m <small>Select from List</small>	Impact Angle (in degrees): 45 degrees 0 <input type="range"/> 90	Target Type: <input type="radio"/> Water of Depth <input type="text"/> m <input type="radio"/> Sedimentary Rock <input type="radio"/> Crystalline Rock
Density: <input type="text"/> (kg/m ³) <small>Select from a list</small>	Impact Velocity: <input type="text"/> km/s 11 km/s 11 <input type="range"/> 72	
DISTANCE FROM IMPACT: <input type="text"/> km		<input type="button" value="CALCULATE IMPACT"/>

<http://www.purdue.edu/impactearth>

Less than a minute after impact, the dissipation of the asteroid kinetic energy produces a stupendous explosion that melts, vaporizes, and ejects a substantial volume of calcite, granite, and water.

Questions Chapter 3

Q1 A small piece of paper is dropped and flutters to the floor. Is the piece of paper accelerating at any time during this motion? Explain?

Yes at the start

Q4 A lead ball and an aluminum ball, each 1 in. in diameter, are released simultaneously and allowed to fall to the ground. Due to its greater density, the lead ball has a substantially larger mass than the aluminum ball. Which of these balls, if either, has the greater acceleration due to gravity? Explain.

They both have the same gravitational acceleration. Any difference in how they fall is due to friction so if the balls have identical shapes, size and surface polish the two motions will be identical

Q8 A rock is dropped from the top of a diving platform into the swimming pool below. Will the distance traveled by the rock in a 0.1-second interval near the top of its flight be the same as the distance covered in a 0.1-second interval just before it hits the water? Explain.

No because the velocity is increasing

Q10 A ball is thrown downward with a large starting velocity.

A. Will this ball reach the ground sooner than one that is just dropped at the same time from the same height? Explain.

B. Will this ball accelerate more rapidly than one that is dropped with no initial velocity? Explain.

Yes because it will have a higher average velocity

No the acceleration is the same

Q14 A ball is thrown straight upward. At the very top of its flight, the velocity of the ball is zero. Is its acceleration at this point also zero? Explain.

No the acceleration is 9.8m/s^2 down

Q15 A ball rolls up an inclined plane, slows to a stop, and then rolls back down. Do you expect the acceleration to be constant during this process? Is the velocity constant?

The acceleration is constant the velocity is not

Q19 Is it possible for an object to have a horizontal component of velocity that is constant at the same time that the object is accelerating in the vertical direction? Explain by giving an example, if possible.

Yes a projectile

Ch 3 E4

Heart beat = 75 beats/minute

a) What is the time between pulses?

b) How far does an object fall in this time?

a) $t = \frac{60}{75} = 0.8 \text{ s}$

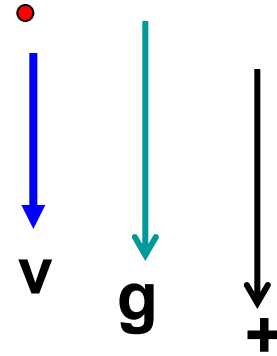
b) $d = v_0 t + \frac{1}{2} 9.8 t^2 = 3.136 \text{ m}$

9.8m/s²



Ch 3 E6

Ball is dropped
What is the change in velocity
between 1 and 4 seconds?



After 1 sec $v = 9.8$ $t = 9.8$ **m/s**

After 4 sec $v = 9.8 \times 4 = 39.2$ **m/s**

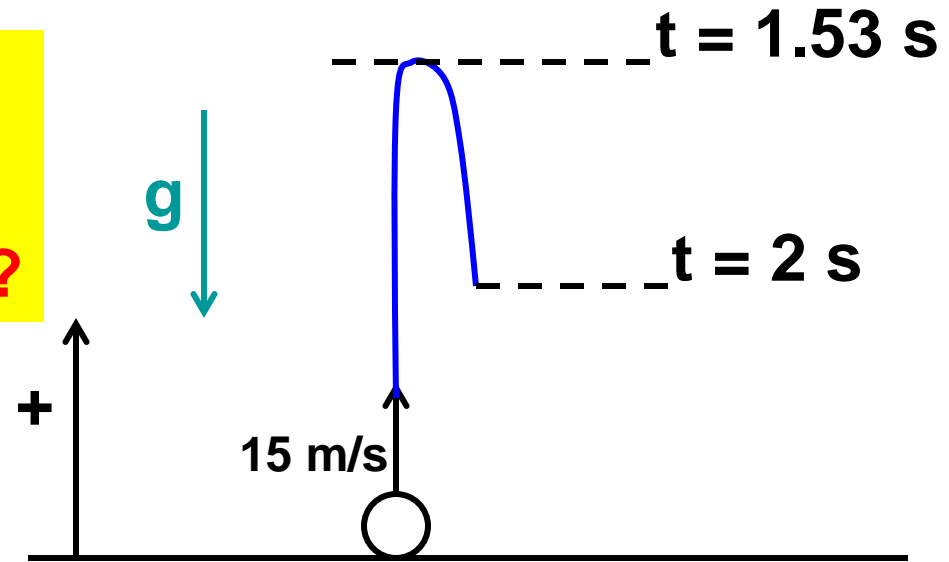
Change in velocity is 29.4 m/s ↓

Ch 3 E8

Ball thrown up at 15 m/s

a) How high after 1 second?

b) How high after 2 seconds?



After 1 sec $d = v_0 t + \frac{1}{2} a t^2 = 15 - 4.9 = 10.1$ m

After 2 sec $d = 15 \times 2 - \frac{1}{2} 9.8 \times 2^2 = 10.4$ m

Time to top $v = v_0 + a t$ $t = 15/9.8 = 1.53$ s

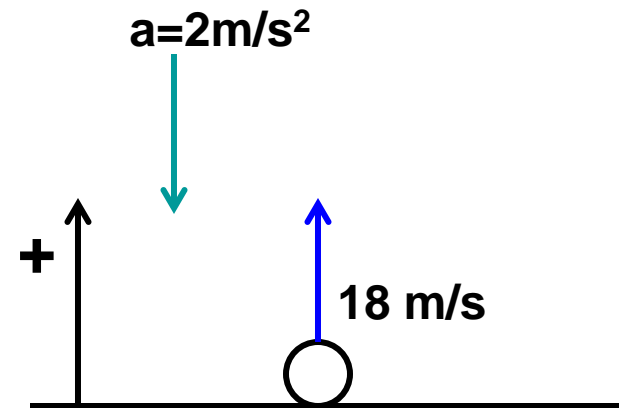
Height at top $d = 11.48$ m

Ch 3 E10

$$V_0 = 18 \text{ m/s} \quad a = -2 \text{ m/s}^2$$

a) What is v after 4 seconds?

b) What is time to top?



a) $v = v_0 + at$

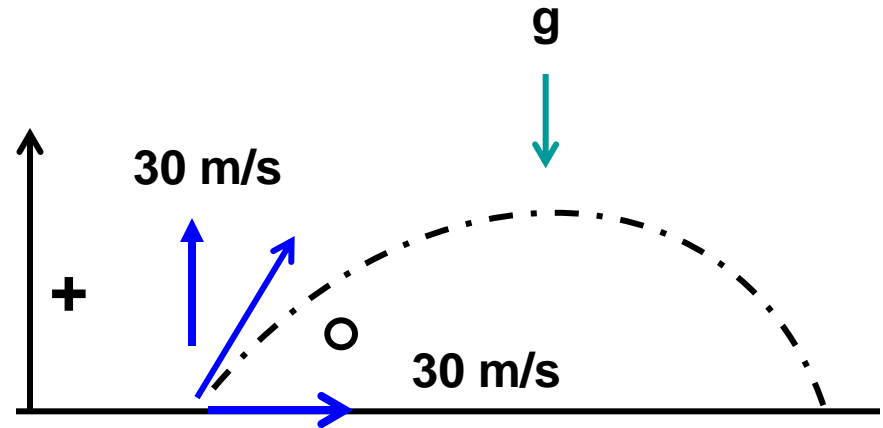
$$= 18 - 2 \times 4 = 10 \text{ m/s}$$

b) $v = 0 \quad t = \frac{18}{2} = 9 \text{ s}$

Ch 3 E16

$$V_{0v} = 30 \text{ m/s} \quad V_{0H} = 30 \text{ m/s}$$
$$g = -9.8 \text{ m/s}^2$$

- a) What is time to top?
b) What is the range?



a) $v = v_0 + at$ $t = 30/9.8 = 3.06\text{s}$ $t_R = 6.12\text{s}$

b) $d = 30 \times t_R = 183.6\text{m}$

Ch 3 CP2

$$V_{01} = 0 \text{ m/s} \quad V_{02} = 12 \text{ m/s}$$

- a) What are the velocities after 1.5s?
- b) How far has each ball dropped in 1.5s?
- c) Does the velocity difference change?

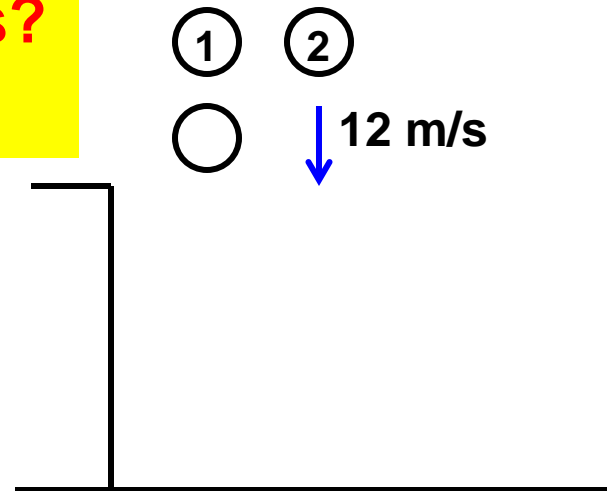
a) $v_1 = at = 9.8 \times 1.5 = 14.7 \text{ m/s}$

$$v_2 = 12 + 9.8 \times 1.5 = 26.7 \text{ m/s}$$

b) $d_1 = \frac{1}{2}at^2 = 11.03 \text{ m}$

$$d_2 = v_2t + \frac{1}{2}at^2 = 29.03 \text{ m}$$

c) **No**



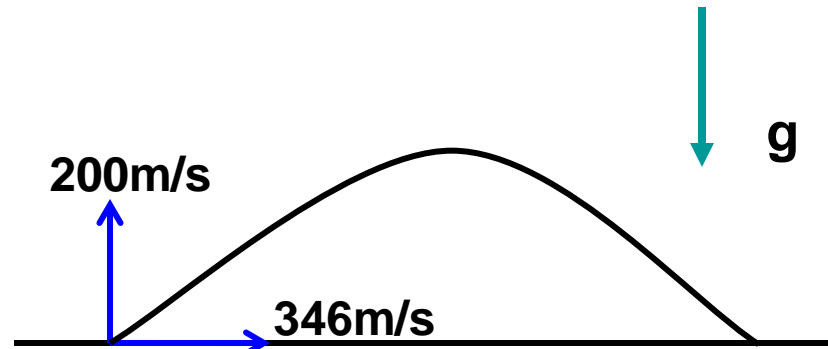
Ch 3 CP4

$$V_{0v} = 200\text{m/s} \quad v_{0H} = 346\text{m/s}$$

a) How long in the air?

b) How far?

$$\text{c) } v_{0v} = 346 \quad v_{0H} = 200$$



$$\text{a) } v = v_0 + at \quad \text{time to top} = \frac{200}{9.8} = 20.4\text{s}$$

$$\text{time to range} = \frac{400}{9.8} = 40.8\text{s}$$

$$\text{b) } d = 346 \times 40.8 = 14120\text{m}$$

$$\text{c) } \uparrow 346 \rightarrow 200 \quad \text{time} = \frac{692}{9.8} = 70.6\text{s}$$

$$d = 200 \times 70.6 = 14120$$