Solution - Problem of the Week 6

Jack and Jill are maneuvering a 3000 kg boat near a dock. Initially the boat’s position is \( <2, 0, 3> \) m and its speed is 1.3 m/s. As the boat moves to position \( <4, 0, 2> \) m, Jack exerts a force \( <-400, 0, 200> \) N and Jill exerts a force \( <150, 0, 300> \) N.

(a) How much work does Jack do?

We will apply the Energy Principle. System: the boat. Surroundings: Jack and Jill interacting with the boat. In general, the Energy Principle states that \( \Delta E_{sys} = W = \vec{F} \cdot \Delta \vec{r} \) where the last equality holds for a constant force acting over the distance \( \Delta \vec{r} \).

In what follows
\[
\Delta \vec{r} = \vec{r}_f - \vec{r}_i = (4, 0, 2)m - (2, 0, 3)m = (2, 0, -1)m
\]

The amount of work done by Jack is:
\[
\vec{F}_{Jack} \cdot \Delta \vec{r} = (-400, 0, 200)N \cdot (2, 0, -1)m = -800 - 200 = -1000 \ N \cdot \ m
\]

(b) How much work does Jill do?

\[
\vec{F}_{Jill} \cdot \Delta \vec{r} = (150, 0, 300)N \cdot (2, 0, -1)m = 300 - 300 = 0 \ N \cdot \ m
\]

(c) Assuming that we can neglect the work done by the water on the boat, what is the final speed of the boat?

Applying the Energy Principle:
\[
\Delta E_{sys} = \Delta KE = \frac{1}{2} M (v_f^2 - v_i^2) = \vec{F}_{net} \cdot \Delta \vec{r} = -1000 J
\]

Solving for the final speed, \( v_f = 1.01 \cdot m / s \)

(d) Without doing any geometrical calculations, say what is the angle between the (vector) force that Jill exerts and the (vector) velocity of the boat. Explain briefly how you know this.

The (vector) velocity of the boat is along the displacement \( (\Delta \vec{r}) \) direction. We have already found that Jill did no work in moving the boat from its initial position to its final position, yet she did exert a force. Therefore, the force Jill exerted is perpendicular to the boat’s velocity.
(e) What effect does Jill have on the boat’s motion? While Jill does no work on the boat, she has changed its direction of motion.