PHYSICS 149: Lecture 24

• Chapter 11: Waves
  – 11.8 Reflection and Refraction
  – 11.10 Standing Waves

• Chapter 12: Sound
  – 12.1 Sound Waves
  – 12.4 Standing Sound Waves
ILQ 1

A thick string and a thin string made of the same material have equal tensions. Which has the larger speed of waves on these strings?

A) thin string
B) thick string
C) waves have the same speed
ILQ 2

Of these properties of a wave, the one that is independent of the others is its

A) frequency
B) wavelength
C) amplitude
D) speed
Harmonic Waves

\[ y(x,t) = A \sin(\omega t - kx) \]

- \( A \): Amplitude = Maximum displacement of a point on the wave
- \( \lambda \): Wavelength: Distance between identical points on the wave
- \( T \): Period: Time for a point on the wave to undergo one complete oscillation.

- \( f = \frac{1}{T} \): Frequency
- \( \omega = \frac{2\pi}{T} \): Angular frequency
- \( k = \frac{2\pi}{\lambda} \): Wave number

\[ \nu = \frac{\lambda}{T} = \lambda f = \frac{\omega}{k} \]
Wave Speed

\[ y(x, t) = A \cos(\omega t - kx) \]

\[ \omega t - kx = \text{constant} \]

so \[ \omega \Delta t - k \Delta x = 0 \]

gives

\[ \frac{\Delta x}{\Delta t} = \frac{\omega}{k} = v \]

A wave \( y = A \cos(\omega t - kx) \) travels in +x direction

A wave \( y = A \cos(\omega t + kx) \) travels in -x direction
Superposition Principle

• When two or more waves pass through the same region the actual displacement is the sum of the separate displacements.

\[ y'(x, t) = y_1(x, t) + y_2(x, t) \]

• If two waves pass through the same region they continue to move independently.
Interference

- Interference describes what happens when two waves are travelling in the same direction through the same region of space.
- Superposition principle: the resulting displacement is the algebraic sum of their separate displacement
  \[ y'(x,t) = y_1(x,t) + y_2(x,t) \]
  - If the two waves arrive with the same amplitude (both waves have a crest): constructive interference and the resultant pulse is larger than either pulses.
  - If the two waves arrive with opposite amplitude (one has a crest, the other a valley): destructive interference the resulting pulse could be zero.
Interference and Superposition

Lecture 24

Purdue University, Physics 149
Reflection

- When a wave travels from one boundary to another, reflection occurs. Some of the wave travels backwards from the boundary
  - Traveling from fast to slow inverted
  - Traveling slow to fast upright
Reflection and Transmission

- When a wave strikes an obstacle or comes at the end of the medium it is travelling in, it is reflected (at least in part).
- If the end of the rope is fixed the reflected pulse is inverted.
- If the end of the rope is free the reflected pulse is not inverted.
Reflected Wave: Inverted or Not?

- The reflected wave will be inverted if it reflects from a medium with a higher mass density (that is, with a lower wave speed; recall $v = \sqrt{\frac{E}{\mu}}$).

- The reflected wave will not be inverted if it reflects from a medium with a lower mass density (that is, with a higher wave speed).

← This phenomenon can be explained by

(1) The principle of superposition at the fixed point at the end, or

(2) Newton’s third law for the forces between the string and the wall.
A transverse wave on a string is described by $y(x,t) = A \cos(\omega t + kx)$. It arrives at the point $x = 0$ where the string is fixed in place. Which function describes the reflected wave?

a) $A \cos(\omega t + kx)$

b) $A \cos(\omega t - kx)$

c) $-A \cos(\omega t - kx)$

d) $-A \sin(\omega t + kx)$
Example

You send a wave pulse (upright) down a lightweight rope, which is tied to a very heavy rope. At the boundary,

a) the reflected wave will be upright and the transmitted wave will be inverted.

b) the reflected wave will be inverted and the transmitted wave will be upright.

c) the reflected wave will be inverted and the transmitted wave will be inverted.

d) the reflected wave will be upright and the transmitted wave will be upright.
You send a wave pulse (upright) down a very heavy rope, which is tied to a lightweight rope. At the boundary,

a) the reflected wave will be upright and the transmitted wave will be inverted.

b) the reflected wave will be inverted and the transmitted wave will be upright.

c) the reflected wave will be inverted and the transmitted wave will be inverted.

d) the reflected wave will be upright and the transmitted wave will be upright.
Diffraction

Diffraction is the spreading of a wave around an obstacle in its path.

Diffraction: Bending of waves around obstacles
Speed of Waves Depends on Medium

When a wave reaches a boundary between two different media the speed and the wavelength change, but the frequency remains the same.

\[ f = \frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2} \]
Law of Refraction

\[ \sin(\theta_1) = \frac{v_1}{\sin(\theta_2)} = \frac{v_2}{\nu_2} \]

Angle of incidence and angle of refraction
Standing Waves

- Standing waves occur when a wave is reflected at a boundary in such a way that the wave appears to stand still.
- In a standing wave on a string, every point moves (as a whole) in simple harmonic motion (SHM) with the same frequency.
- Every point reaches its maximum amplitude simultaneously, and every point also reaches its minimum amplitude (namely, zero) simultaneously as well.
- Nodes are points of zero amplitude (that is, points that never move); antinodes are points of maximum amplitude. The distance between two adjacent nodes is $\frac{1}{2} \lambda$.
- Fixed end of a string is always a node, because it never moves.

$t_0 = 0$
$t_1 = \frac{1}{8} T$
$t_2 = \frac{2}{8} T$
$t_3 = \frac{3}{8} T$
$t_4 = \frac{4}{8} T$
Standing Waves

- Nodes: points of destructive interference where the cord remains still at all times. We have always nodes at fixed ends.
- Antinodes: points of constructive interference where the cords oscillates with maximum amplitude.
- The nodes and antinodes remain in a fixed position for a given frequency. Standing wave can occur at resonant frequencies.
- The lowest frequency that produces the standing wave yields a wave with one antinode. If you double the frequency there will be two anti-nodes etc.
How Do We Get Standing Waves?

• **Graphical Understanding**

  ![Graphical Understanding Diagram]

  Standing wave: superposition of incident and reflected waves

• **Mathematical Understanding**

  - Incident wave: \( A \cos(\omega t + kx) \) ← to the left
  - Reflected wave: \(-A \cos(\omega t – kx)\) ← to the right, inverted
  - According to the principle of superposition,
    \[
    y(x,t) = A \cos(\omega t + kx) – A \cos(\omega t – kx)
    = 2A \cos(\omega t) \sin(kx)
    \]
    Every point moves in SHM!
Possible Wavelengths and Frequencies

- \( \lambda_n = \frac{2L}{n} \) \((n = 1, 2, 3, \ldots)\)
- \( f_n = \frac{v}{\lambda_n} = \frac{nv}{2L} \) \((n = 1, 2, 3, \ldots)\)
- \( v = \sqrt{\frac{F}{\mu}} \)

- \( f_1 \) is called the fundamental frequency.
- \( f_1, 2f_1, 3f_1, \ldots, nf_1, \ldots \) (that is \( f_n \)) are called the natural frequencies or resonant frequencies.
- Resonance occurs when a system (like a bridge) is driven at one of its natural frequencies.

Don’t forget that fixed ends of the string are always nodes.
Standing Waves

- Shake the end of a string

\[ y(x, t) = A \sin(\omega t + kx) \]

- If the other end is fixed, a wave travels down and it is reflected back \(\Rightarrow\) interference

\[ y(x, t) = -A \sin(\omega t - kx) \]

- The two waves interfere \(\Rightarrow\) standing wave

\[ y(x, t) = 2A \cos(\omega t) \sin(kx) \]

- Nodes when \(\sin(n\pi) = 0\):

\[ x = \frac{n\pi}{k} = \frac{n\lambda}{2} \quad n = 0, 1, 2 \]

- Anti-nodes \(\sin(n\pi) = +/- 1\), half way between nodes

\[ L = \frac{3\lambda}{2} \]
Standing Waves

• The natural frequency is related to the length of the string $L$. The lowest frequency (first harmonic) has one antinode

$$L = \frac{1}{2} \lambda_1 \quad \lambda_1 = 2L$$

• The second harmonic has two antinodes

$$\lambda_2 = L$$

• The $n$-th harmonic

$$\lambda_n = \frac{2}{n} L$$
Standing Waves

- Once you know the wavelength you also know the frequency needed to have a standing wave. Recall:

\[ f = \frac{v}{\lambda} \quad \text{and} \quad f_n = \frac{v}{\lambda_n} = \frac{nv}{2L} = nf_1 \]

- Note that energy is not transmitted by a standing wave. Since the string is at rest at the nodes no energy flows.
ILQ

• A violin string of length $L$ is fixed at both ends. Which one of these is not a wavelength of a standing wave on the string?

  a) $L$
  b) $2L$
  c) $L/2$
  d) $L/3$
  e) $3L/2$
ILQ

- A violinist discovers while tuning her violin that her string is flat (has a lower frequency). She should

  a) tighten the string
  b) loosen the string
  c) play faster
A guitar’s E-string has a length of 65 cm and is stretched to a tension of 82N. If it vibrates with a fundamental frequency of 329.63 Hz, what is the mass of the string?

\[ f_1 = \text{fundamental frequency} \ (\text{lowest possible}) \]

\[ L = \frac{\lambda}{2} \]

\[ f = \frac{v}{\lambda} \] tells us \( v \) if we know \( f \) and \( \lambda \)

\[ v = \sqrt{\frac{F}{\mu}} \]

\[ v = \lambda f \]

\[ = 2 \ (0.65 \text{ m}) \ (329.63 \text{ s}^{-1}) \]

\[ = 428.5 \text{ m/s} \]

\[ v^2 = \frac{F}{\mu} \]

\[ \mu = \frac{F}{v^2} \]

\[ m = \frac{F}{v^2} \]

\[ = 82 \ (0.65) / (428.5)^2 \]

\[ = 2.9 \times 10^{-4} \text{ kg} \]
Sound Waves

- Sound is a longitudinal wave, with compressions and rarefactions of air.
- The wave can be described by the gauge pressure $p$ (a measure of the compression and rarefaction of the air) as a function of time.
- The wave can also be described by the displacement $s (= \Delta x)$ of an element of the air, which will oscillate around an equilibrium position (in the direction of energy transport). The restoring force is caused by the air pressure.

When $p$ is zero (node), $s$ is maximum or minimum (antinode)!
# Speed of Sound

## Table 12.1

### Speed of Sound in Various Materials (at 0°C and 1 atm unless otherwise noted)

<table>
<thead>
<tr>
<th>Medium</th>
<th>Speed (m/s)</th>
<th>Medium</th>
<th>Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon dioxide</td>
<td>259</td>
<td>Blood (37°C)</td>
<td>1570</td>
</tr>
<tr>
<td>Air</td>
<td>331</td>
<td>Muscle (37°C)</td>
<td>1580</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>334</td>
<td>Lead</td>
<td>1322</td>
</tr>
<tr>
<td>Air (20°C)</td>
<td>343</td>
<td>Concrete</td>
<td>3100</td>
</tr>
<tr>
<td>Helium</td>
<td>972</td>
<td>Copper</td>
<td>3560</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>1284</td>
<td>Bone (37°C)</td>
<td>4000</td>
</tr>
<tr>
<td>Mercury (25°C)</td>
<td>1450</td>
<td>Pyrex glass</td>
<td>5640</td>
</tr>
<tr>
<td>Fat (37°C)</td>
<td>1450</td>
<td>Aluminum</td>
<td>5100</td>
</tr>
<tr>
<td>Water (25°C)</td>
<td>1493</td>
<td>Steel</td>
<td>5790</td>
</tr>
<tr>
<td>Seawater (25°C)</td>
<td>1533</td>
<td>Granite</td>
<td>6500</td>
</tr>
</tbody>
</table>
Frequencies of Sound Waves

- Humans with excellent hearing can hear frequencies from 20 Hz to 20 kHz (audible range).

- The terms infrasound and ultrasound are used to describe sound waves with frequencies below 20 Hz and above 20 kHz, respectively.

- Some animals (such as dogs and dolphins) can hear ultrasound, while some others (such as elephants) can hear infrasound.
Velocity ILQ

A sound wave having frequency $f_0$, speed $v_0$ and wavelength $\lambda_0$, is traveling through air when it encounters a large helium-filled balloon. Inside the balloon the frequency of the wave is $f_1$, its speed is $v_1$, and its wavelength is $\lambda_1$.

Compare the speed of the sound wave inside and outside the balloon

A) $v_1 < v_0$
B) $v_1 = v_0$
C) $v_1 > v_0$

$V_0 = 343 \text{ m/s}$
$V_1 = 965 \text{ m/s}$
Frequency ILQ

A sound wave having frequency $f_0$, speed $v_0$ and wavelength $\lambda_0$, is traveling through air when it encounters a large helium-filled balloon. Inside the balloon the frequency of the wave is $f_1$, its speed is $v_1$, and its wavelength is $\lambda_1$.

Compare the frequency of the sound wave inside and outside the balloon

A) $f_1 < f_0$
B) $f_1 = f_0$
C) $f_1 > f_0$

Time between wave peaks does not change!
Wavelength ILQ

A sound wave having frequency $f_0$, speed $v_0$ and wavelength $\lambda_0$, is traveling through air when it encounters a large helium-filled balloon. Inside the balloon the frequency of the wave is $f_1$, its speed is $v_1$, and its wavelength is $\lambda_1$. Compare the wavelength of the sound wave inside and outside the balloon

A) $\lambda_1 < \lambda_0$
B) $\lambda_1 = \lambda_0$
C) $\lambda_1 > \lambda_0$

\[ \lambda = \frac{v}{f} \]