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Wavelet analysis of velocity dispersion of elastic interface waves propagating along a fracture

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Abstract. A wavelet analysis is performed on seismic waveforms of elastic interface waves that propagate along a fracture. The wavelet analysis provides a direct quantitative measure of spectral content as a function of arrival time. We find that the spectral content of the interface wave signals is not stationary, but exhibits increasing frequency content for later arrival times, representing negative velocity dispersion. The dispersion increases from -11 m/sec/MHz to -116 m/sec/MHz as the stress on the fracture is increased from 3.5 kPa to 33 MPa. The negative velocity dispersion agrees with predictions from the displacement-discontinuity theory of the seismic response of fractures, and can be used to fit fracture stiffness.

Introduction

It has previously been demonstrated that seismic modes can propagate along macroscopic fractures in homogeneous solid media [*Pyrak-Nolte and Cook*, 1987; *Nagy*, 1991; *Pyrak-Nolte et al.*, 1992]. These interface waves may be regarded as propagating eigenmodes generated by the interaction of Rayleigh waves at the surfaces of two half-spaces, coupled by the specific stiffness of the fracture. The displacement-discontinuity boundary condition [*Kendall and Tabor*, 1971; *Schoenberg*, 1980; *Kitsunezaki*, 1983; *Pyrak-Nolte et al.*, 1990a and b], that describes the coupling of the two half-spaces, introduces a characteristic frequency into the dynamic response of the fracture. Therefore, the originally non-dispersive Rayleigh modes become dispersive when they are coupled in the interface modes.

Elastic interface waves have the potential for use in seismic characterization in the field, and may be especially useful for determining the stability of fractured rock masses. In this paper, we describe experimental measurements of elastic interface waves propagating along a synthetic fracture in aluminum. The velocity dispersion is obtained using wavelet analysis and is compared with theoretical values from the displacement-discontinuity theory for interface waves [*Pyrak-Nolte and Cook*, 1987].

Velocity Dispersion Theory

Velocity dispersion of interface waves can be determined theoretically using the displacement-discontinuity model to simulate the seismic response of a fracture [*Pyrak-Nolte and Cook*, 1987]. In this model, the stress across the fracture is assumed to be continuous, and the displacement across the fracture to be discontinuous. The discontinuity in displacement is inversely proportional to the specific stiffness of the fracture. The specific stiffness, κ, of a fracture depends on surface roughness and contact area of a fracture [*Yoshioka and Scholz*, 1989; *Brown and Scholz*, 1985 and 1986; *Hopkins et al.*, 1987 and 1990] and is defined as the ratio of the increment in stress to the resulting increment in displacement.

There are two distinct normal modes of the interface waves, designated as the "fast" and the "slow" interface wave. For the fast wave, the displacement-discontinuity is normal to the fracture plane (it is only sensitive to normal stiffness), while for the slow wave, the discontinuity is parallel to the fracture plane (it is only sensitive to shear stiffness). The slow interface wave has been found experimentally to have larger amplitudes [*Pyrak-Nolte et al.*, 1992], for reasons related to the energy partitioning between the two waves [*Gu*, 1994]. Therefore, in this paper, we focus on the dispersion properties of the slow interface wave, which dominates the seismic response of the fracture.

The phase and group velocities of the slow interface wave are obtained by finding the solutions for the normal modes [*Pyrak-Nolte and Çook*, 1987] in terms of the angular frequency ω and the wavenumber k. These velocities are given by

$$v_p = \frac{\omega}{k}$$

$$v_g = \frac{d\omega}{dk} = v_p + k\,\frac{dv_p}{dk} \tag{1}$$

and are shown in Figure 1 as functions of normalized frequency based on the physical parameters given in Table 1. The slow wave velocities have been normalized by the shear wave velocity of the intact material and are plotted against the frequency normalized by the characteristic frequency of the fracture $\omega_C=\kappa/Z$, where Z is the seismic impedance of the half spaces (i.e. Z = density * phase velocity). The phase and group velocities of the slow interface wave both exhibit strong frequency dispersion, varying from the bulk shear-wave velocity at low frequencies or high stiffnesses, to the Rayleigh wave velocity at high frequencies or low stiffnesses. At intermediate frequencies or stiffnesses, the phase and group velocities differ from one another and have maximum frequency dispersion.

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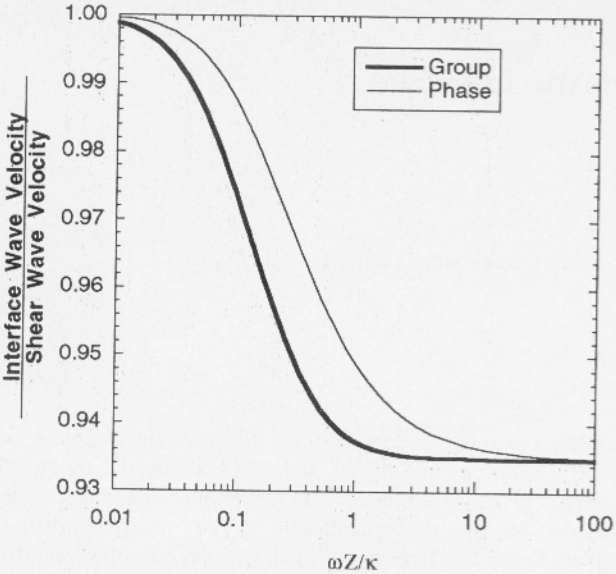

Figure 1. Theoretical values of phase and group velocity dispersion for the slow interface wave plotted as functions of frequency and fracture specific stiffness.

Wavelet Analysis

Wavelet analysis [*Combes et al.*, 1989], also known as multi-resolution analysis, uses a series of scaled and delayed oscillatory functions to decompose a time-varying signal into its nonstationary spectral components. The key advantage of wavelet analysis over traditional Fourier analysis is that the wavelet analysis retains information on how the spectral content varies with time delay. Wavelets also are advantageous over so-called windowed Fourier methods because with wavelets the relative accuracy of the delay and frequency remain constant over all of the delay-frequency parameter space.

In our wavelet analysis we use a non-orthonormal Morlet wavelet [*Morlet et al.*, 1982] mother function g(t) composed of a harmonic wave modulated by a Gaussian envelope

$$g(t) = \exp\left(-t^2/2\sigma^2\right)\exp\left(i\,2\pi t/T_0\right) \tag{2}$$

where T_0 is the period of oscillation. The mother wavelet attains a minimum uncertainty when the accuracy in the arrival time is equal to the accuracy in the period of oscillation. This condition is expressed as

$$\Delta t = \Delta T_0 \tag{3}$$

and fixes the relationship between σ and T_0 in eq.(2) as

$$\sigma = \frac{T_0}{\sqrt{2}\pi} \tag{4}$$

Table 1. Parameters used to determine theoretical values of group velocity in Figure 1. Compressional and shear wave velocity values are based on experimental measurements on aluminum.

Compressional Wave Velocity	6476.2 m/s
Shear Wave Velocity	3120.6 m/s
Density	2700 kg/m^3

The mother wavelet is scaled and delayed to produce a set of daughter wavelets according to

$$g((t-\tau)/\alpha) = \frac{2^{1/4}}{\sqrt{T_0}}\, e^{\pi} \exp\left(-\left((t-\tau)/\alpha\right)^2/2\sigma^2\right) \exp\left(i\,2\pi(t-\tau)/\alpha T_0\right) \tag{5}$$

where α is a scaling parameter, and τ is the delay. The prefactor ensures norm-squared normalization. The daughter wavelets retain the same uncertainty condition eq.(3) for all scales α and all delays τ.

The wavelet transform as a function of scale and delay is obtained by integrating a time varying signal S(t) over the daughter wavelets

$$W(\alpha,\tau) = \frac{1}{\sqrt{\alpha}}\int_{-\infty}^{\infty} g^*((t-\tau)/\alpha)\, S(t)\, dt \tag{6}$$

where g* is the complex-conjugate of the daughter wavelet, the prefactor is for normalization, and $W(\alpha,\tau)$ is the two-dimensional wavelet transform. The resulting transform is complex, with both amplitude and phase information. In our analysis, we consider only the amplitude of the wavelet transform. For our analysis, in which we are concerned with both the frequency content as well as the arrival time of the signal, the wavelet transform is superior to a moving windowed Fourier transform because the uncertainties in oscillation period and arrival time are equal for all frequencies and time delays, giving frequency and time information equal accuracy in the α-τ plane.

Experimental System

A synthetic fracture in aluminum was used to study velocity dispersion of interface waves. The isotropic aluminum eliminates dispersion that might occur in the bulk material. The synthetic fracture was made by cutting an aluminum cylinder in half, planing down the surfaces, and sandblasting the surfaces with 300 μm grit to roughen the surfaces. The sample is 0.293 m in diameter by 0.293 m in height. Shear piezoelectric crystals (1 MHz resonant frequency) were mounted on opposing faces straddling the fracture. The polarization of the shear-wave particle motion was perpendicular to the fracture. The transducers were excited with a 1000 Volt spike that was 0.3 μs in duration at a repetition rate of 100 cycles/sec. The received signal was sent to a digital oscilloscope and the data were collected by a computer. To change the specific stiffness of the fracture, the aluminum specimen was placed in a load frame and subjected to stresses from 3.5 kPa to 33 MPa, applied normal to the fracture plane.

Wavelet Transforms of Seismic Data

The results of the wavelet analysis of seismic waveforms are presented in Figure 2. The raw waveforms received during the experiments are included in the Figure for comparison. The wavelet transform is presented in two-dimensions with color representing the strength of the transform (red-high; blue low). Figure 2a shows the wavelet transform for intact aluminum. Figures 2b and 2c show the wavelet transforms for the fracture at low stress (3.5 kPa) and high stress (33 MPa). The two-dimensional representation of the wavelet transform gives a direct measure of the frequency content of a seismic signal as a function of group arrival time. Velocity dispersion is observed as a finite slope. The source waveform can also exhibit a change in frequency as a function of time, called chirp, which also appears as a finite slope. It is therefore necessary to quantify the chirp of the ultrasonic transducers. The transducer chirp in this experimental configuration was 3% per MHz. This frequency chirp on the source must be subtracted from the experimental waveforms received on the fracture to obtain the velocity dispersion of the interface waves. It is interesting to note that the wave amplitudes in Figure 2b and 2c decrease with increasing stress. This behavior is unintuitive and is a consequence of energy partitioning between body waves and interface waves. Gu [1994] has used boundary integral methods to show that the shape and amplitude of the waveforms varies with frequency and fracture specific stiffness. (See, for example, Fig. 4.4 in Gu [1994]).

Several striking features are apparent in the transforms, comparing the intact transform with the transforms of the seismic signals in the fractured specimen. The waveform on the intact specimen is a single wavepacket without additional structure. The waveforms on the fracture have complicated structure, with energy arriving significantly later than for the intact case and with lower frequency content. The lower frequency content for the interface waves relative to the frequency content of the intact sample is consistent with the interface behaving as a displacement discontinuity, i.e. the interface acts as a low pass filter [*Pyrak-Nolte et al.*, 1990]. The velocity of the interface wave increases with increasing

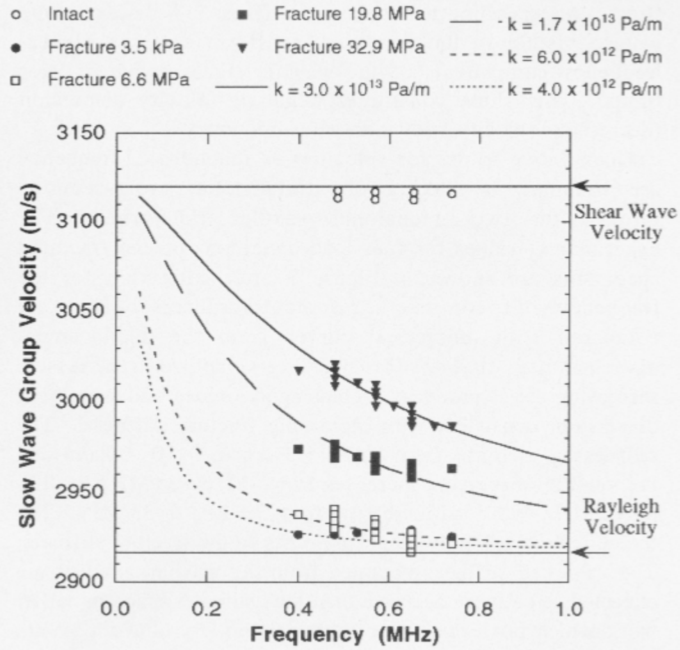

Figure 3 Comparison of theoretical dispersion curves and the experimental velocities computed from the wavelet transform of the slow interface wave. The velocity dispersion increases from -11 m/sec/MHz to -116 m/sec/MHz with increasing stress applied normal to the fracture.

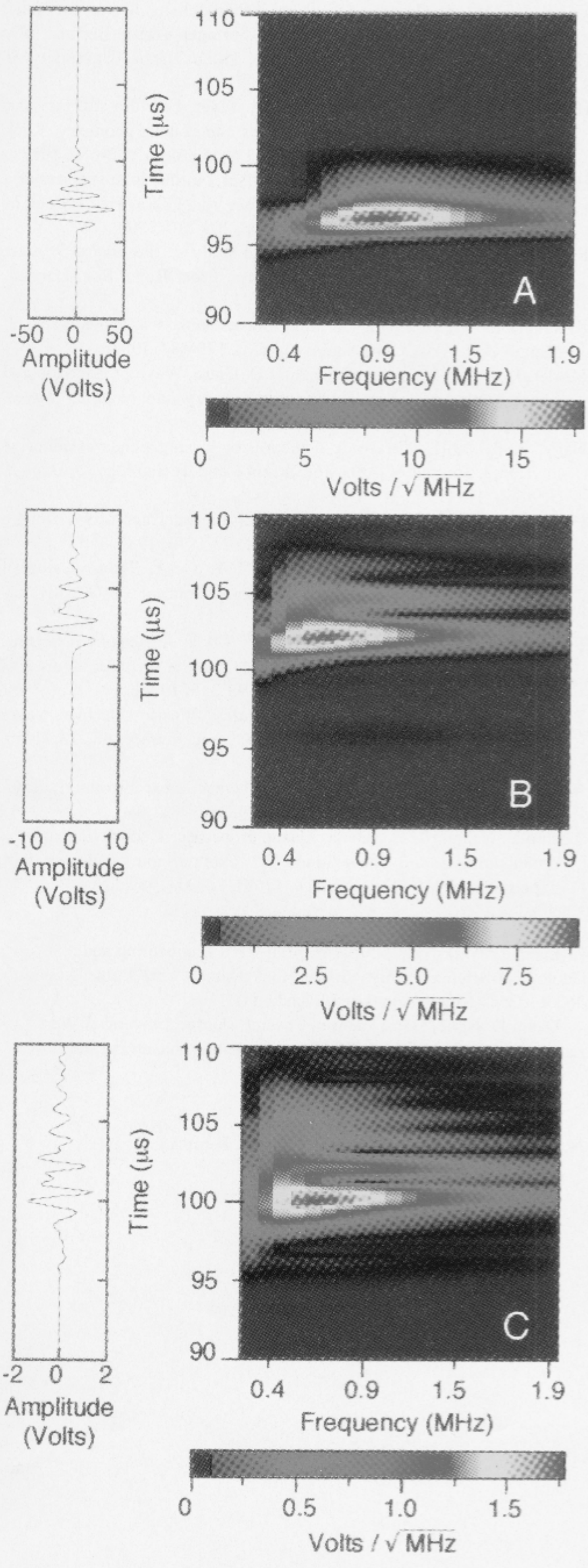

Figure 2. Wavelet transforms of received seismic waveform for (a) intact, (b) fractured sample - 3.5 kPa, (c) fractured sample - 33 MPa. Red represents the largest amplitudes while violet represents the lowest amplitudes. The horizontal axis gives the frequency content of the waveform, and the vertical axis gives the arrival time. The original waveforms are shown for comparison.

stress, corresponding to increasing stiffness. A distinct slope is also visible in the fractured transform data, as higher-frequency components of the seismic signal arrive at later times. This slope constitutes negative velocity dispersion (decreasing velocity with increasing frequency).

Quantitative values for velocities as functions of frequency are obtained by taking one-dimensional cross-sections through the two-dimensional wavelet transforms. The experimental values for four load conditions on the fractured specimens are shown in Figure 3 after correction for the frequency chirp on the ultrasonic transducers. They are compared with theoretical curves from the displacement discontinuity theory for varying fracture stiffnesses. Increasing stress produces increasing velocities and increasing dispersion, consistent with increasing fracture stiffness. The stiffnesses increase from 4×10^{12} Pa/m to 3×10^{13} Pa/m and the velocity dispersion increases from -11 m/sec/MHz to -116 m/sec/MHz with load increasing from 3.5 kPa to 33 MPa. The theoretical curves provide accurate fits to the fracture stiffness. The values of stiffness obtained from the wavelet analysis are expected to be more accurate than the estimates made in earlier work which neglected velocity dispersion [*Pyrak-Nolte et al.*, 1992].

Summary

In summary, we have performed a wavelet analysis on the seismic signals of elastic interface waves propagating along a synthetic fracture in aluminum. The wavelet analysis allows the frequency content of the seismic waveform to be obtained as a function of arrival time. The interface wave exhibits negative velocity dispersion and is consistent with theoretical values predicted from the displacement-discontinuity theory of the seismic response of fractures. The interface waves can be understood as coupled Rayleigh waves, which by themselves are dispersionless waves. However, the displacement-discontinuity boundary condition introduces a characteristic frequency ω_c to the seismic response of a fracture, making the interface waves dispersive. The wave velocity changes from the Rayleigh wave velocity at frequencies greater than ω_c, to the bulk shear wave velocity at frequencies less than ω_c. Because the velocity dispersion is sensitive to fracture stiffness, and hence to stress conditions, these interface waves may provide a means of detecting fractures in the field and assessing fracture stability.

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