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FREQUENCY DEPENDENCE OF FRACTURE STIFFNESS

Laura J. Pyrak-Nolte

Department of Earth & Atmospheric Sciences, Purdue University

David D. Nolte

Department of Physics, Purdue University

Abstract. The frequency dependence of fracture specific stiffness, for seismic or ultrasonic wave transmission, is shown to be a simple scaling property of fractures with spatially inhomogeneous distributions of stiffness. Different frequencies sample different subsets of the fracture geometry. Therefore the frequency dependence may be a simple consequence of fracture geometry and requires no additional dynamical mechanism. In this letter, the dynamic fracture stiffness is determined based on the displacement discontinuity model for wave transmission across a fracture. Local transmission coefficients are assumed to depend on the local static stiffness. For a fracture with a single value of static stiffness, the dynamic stiffness is frequency independent. A strongly inhomogeneous distribution of fracture stiffnesses produces a strong frequency dependence for the dynamic stiffness.

Introduction

Dynamic moduli of rock have been found to be larger in value than values of moduli determined by static stress-strain measurements [Jaeger & Cook, 1979]. The difference in magnitude of the static and dynamic moduli is often attributed to frequency-dependent dynamical effects, such as frictional slip along internal interfaces [Walsh, 1966]. Likewise in the case of the seismic characteristics of single fractures, laboratory experiments [Pyrak-Nolte et al., 1990] have found that dynamic fracture specific stiffness for natural fractures was greater in value than the statically determined values of fracture specific stiffness. The specific stiffness (units of force per volume) of a fracture is the ratio of an increment in stress (which produces elastic deformations of the asperities and voids in the fracture) to the resulting increment in far-field displacement. This definition is considered valid for both static and dynamic loading conditions. The displacement or closure of a fracture depends on the area of contact in a fracture and the aperture distribution. Much theoretical work has been performed to determine fracture displacement from the deformation of asperities between two opposing surfaces [Greenwood & Williamson, 1966; Walsh & Grosenbaugh, 1979; Brown & Scholz, 1985] and to determine fracture stiffness from both the asperity deformation and the deformation of the half-spaces defining the fracture [Hopkins et al., 1987 & 1990]. To experimentally determine static specific stiffness of a fracture, far-field displacement as a function of static stress is used [Goodman, 1976; Bandis et al., 1983]. To determine dynamic specific stiffness, the frequency-dependent transmission coefficient of the fracture is matched against experimentally measured transfer functions [Pyrak, 1988]. From work performed on single natural fractures in quartz-monzonite, dynamic stiffnesses were typically three times larger than static stiffnesses [Pyrak-Nolte et al., 1990]. As in the case of bulk moduli, this difference between static and dynamic fracture stiffness has been attributed to dynamical effects. Such explanations are potentially important because they may be used to make conclusions concerning physical properties of fractures at the microscopic scale.

In this paper, we present an alternate simple explanation for the larger values of dynamic stiffness compared to static stiffness, and further demonstrate that the general frequency dependence of fracture stiffness can be related directly to an inhomogeneous distribution of stiffness across the fracture. This mechanism for frequency-dependent fracture stiffness requires no additional dynamical effects, but is a simple consequence of the fracture void geometry. It is found that fracture stiffness is frequency dependent because different frequencies sample different subsets of fracture stiffness.

Model & Theory

The displacement-discontinuity theory [see Pyrak-Nolte et al., 1990, for literature review] for seismic wave propagation across a fracture models a fracture as a boundary condition at an interface between two half-spaces. The stress across the interface is continuous ($\tau_1 = \tau_2$, where the subscripts 1&2 refer to the half-spaces on either side of the fracture, and τ is stress), but the displacement is discontinuous ($\Delta u = u_1 - u_2 = \tau / \kappa$, where u is displacement, and κ is fracture specific stiffness). The discontinuity in displacement is inversely proportional to the specific stiffness of the fracture. For a given seismic impedance of the bulk rock represented by the half-spaces, the theory generates a transmission coefficient for a fracture with a uniform specific stiffness. The transmission coefficient is

$$T(\omega,\kappa) = \frac{1}{1 - \left(\frac{i\omega Z}{2\kappa}\right)} \tag{1}$$

where ω is the angular frequency of the wave, κ is the fracture specific stiffness, and Z is the impedance (density x phase velocity) of the bulk rock. The frequency dependence of the transmission coefficient arises from the inertia of the rock mass in response to the stress transmitted across the fracture through the fracture stiffness. If the fracture stiffness

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$\kappa \to 0$, then $T \to 0$ and all of the wave is reflected off of the free surface. In the other extreme, as $\kappa \to \infty$, then $T \to 1$ and the fracture behaves as a welded interface. In general, the fracture acts as a low-pass filter, passing only those frequencies of the wave that are lower than the characteristic frequency $\omega_c = 2\kappa/Z$. Clearly, fractures with different stiffnesses will have different characteristic frequencies.

The results of the displacement-discontinuity theory assume a uniform stiffness for the fracture. Real fractures, however, are strongly inhomogeneous and have complicated geometries. Fractures consist of two rough surfaces in partial contact. The apertures, asperities, and points of contact in a fracture consist of many different sizes and are distributed inhomogeneously over the interface. The aperture of the voids and the spatial distribution of voids control the stiffness of the fracture. Rather than having a uniform stiffness, different regions of the fracture will have different stiffnesses. These local stiffnesses each define a different characteristic frequency ω_c. Some parts of the fracture will pass higher frequencies, while other parts will pass lower frequencies. This distribution of stiffnesses must be taken into account in the analysis of transmission of seismic waves across real fractures.

In order to average over the transmission coefficients of different parts of a fracture, the following approximations are made:

(1) the local transmission coefficient depends on the local static stiffness.

(2) regions of different stiffness transmit independently

The first approximation assumes that the local stiffness is defined as an average over length scales comparable to the wavelength. This assumption is valid if the asperity separation is smaller than the wavelength. Typical wavelengths for 10 MHz are 0.6 mm, compared to typical asperity separations of 0.5 mm [Brown et al., 1986] for some fractures. Therefore the first approximation is valid for frequencies below 10 MHz. Also implicit in the first approximation is a local stiffness that is varying slowly with respect to a wavelength. Otherwise, the transmitted signal will experience attenuation by scattering. This additional assumption is plausible because real fracture asperity distributions are spatially correlated, meaning that there exist extended regions of similar asperity heights. Scattering will be expected to play an increasingly important role at low frequencies when the wavelength exceeds the correlation length. However, if spatial distributions of stiffnesses are fractal, then the first approximation will be valid for arbitrary frequencies. The second approximation assumes that the total transmission of seismic wave amplitude is the sum of the transmitted amplitudes of individual regions. This is valid under the conditions of linearity. With this assumption, the forward scattering amplitude is calculated explicitly.

To calculate dynamic fracture stiffness, the transmission of seismic amplitudes is first averaged over the fracture surface. This averaging process is expressed in terms of an integral-kernel equation.

$$T(\kappa(\omega),\omega) = \int_0^L \int_0^L T(\kappa(x,y),\omega)\, dx\, dy \qquad (2)$$

where L is the length of a side of the fracture and the kernel T $(\kappa(x,y),\omega)$ is given by equation (1) with κ replaced by the spatially varying stiffness $\kappa(x,y)$. This expression implicitly defines a frequency-dependent fracture stiffness $\kappa(\omega)$ through

$$|T(\kappa(\omega),\omega)| = \left| \frac{1}{1 - i\,\dfrac{\omega Z}{2\,\kappa(\omega)}} \right|$$

and is easily inverted to yield

$$\kappa(\omega) = \frac{\omega Z |T|}{2\sqrt{1 - |T|^2}} \qquad (3)$$

where $/T/$ is the magnitude of the averaged transmission coefficient given by equation (2).

Results

The frequency dependence of fracture stiffness is explored for three different stiffness distributions $\kappa(x,y)$: (1) uniform, (2) bimodal, and (3) distribution from stratified continuum percolation [Nolte et al., 1989; Nolte and Pyrak-Nolte, 1991]. A uniform aperture distribution represents the simplest case. A bimodal distribution has been suggested [Hopkins et al., 1990] for some fractures in order to account for experimental measurements of static fracture stiffness. A stratified percolation is used to simulate fracture void geometries. The model is based on a fractal construction and produces spatially correlated aperture densities with an approximately log-normal size distribution [Nolte et al., 1989; Nolte and Pyrak-Nolte, 1991]. Several investigators [Tsang, 1984; Gale, 1987;

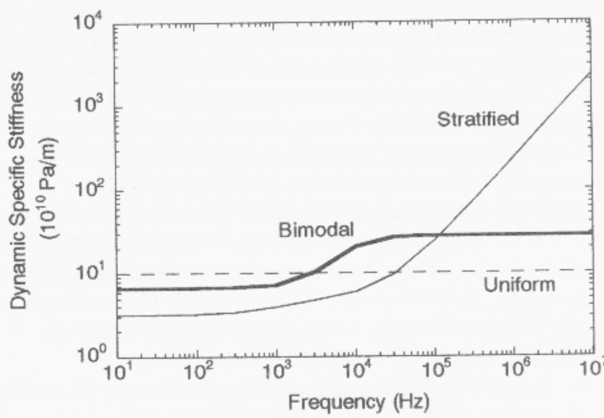

Fig. 1. Frequency-dependent specific fracture stiffness as functions of frequency. The results for three stiffness distributions are shown: 1) uniform with stiffness = 10 x 10^{11} Pa/m; 2) bimodal with a ratio of stiffnesses 18/2; and 3) from a stratified fracture model. The inhomogeneous stiffness distributions yield frequency dependent stiffnesses, with increasing stiffness for increasing frequency.

Hakami, 1989] have found that the apertures in many fractures have a log-normal distribution in size.

The dynamic fracture stiffness from equation (3) is shown in Figure 1 as a function of frequency for the different stiffness distributions given in Table 1. The dynamic fracture stiffness has no frequency dependence for the uniform distribution of stiffnesses. The bimodal stiffness distribution, where the stiffnesses have been assigned values with a ratio $\kappa_1/\kappa_2 = 18/2$, yields a dynamic fracture stiffness that depends weakly on frequency. For the stratified fracture model, the local stiffness is assumed to vary inversely with the height of the asperities b(x,y). This assumption is not crucial, but roughly reflects stress-displacement behavior for fractures. From the stratified model, a strong dependence of dynamic stiffness on frequency is observed. The dynamic stiffness is equal to static fracture stiffness at low frequencies, but increases with increasing frequency. The increase in dynamic stiffness with frequency for the stratified aperture distribution reflects the change in the subset of stiffnesses sampled. Larger stiffnesses are sampled for higher frequencies.

This model for frequency-dependent fracture stiffness is tested against experimental data. Pyrak-Nolte et al. [1990] made measurements of compressional wave transmissions across natural fractures in quartz-monzonite specimens. Compressional waves (source: 1 MHz resonant frequency) were propagated normal to the fracture plane. The experimental spectra in Figure 2 show the transmitted amplitude as a function of frequency for a fracture. To fit this data, we chose to simulate the fracture aperture distribution with the stratified percolation model [Nolte et al., 1989] because it has successfully described several properties of single fractures [Pyrak-Nolte et al., 1988]. Details of the stratified percolation model are described elsewhere [Nolte & Pyrak-Nolte, 1991]. In the model, fracture apertures at different stresses are modeled by patterns with 5 tiers with two adjustable parameters: the points per tier (PPT) and α. A PPT value of 8 was used for the data under 20 MPa stress, and a value of 9 was used for the data under 10 MPa. The apertures from the stratified percolation simulation are converted into specific stiffness values with $\alpha = 1.2 \times 10^{14}$ (for 10 MPa) and an $\alpha = 1.68 \times 10^{14}$ (for 20 MPa) used in Table 1. The fits to the data are shown as the solid lines in Figure 2. The fits are nearly perfect. In earlier work based on a uniform stiffness and the frequency of the maximum spectral amplitude, Pyrak-Nolte et al. [1990] fit a frequency-independent stiffness of 11×10^{12} Pa/m for a stress of 10 MPa and a stiffness of 24×10^{12} Pa/m for a stress of 20 MPa. The frequency-dependent stiffness provides a significantly better fit to the data. The resulting dynamic stiffnesses from

TABLE 1. Stiffness Distributions

Uniform	$\kappa = 1.0 \times 10^{11}$ Pa/m
Bimodal	$\kappa_1 = 5.0 \times 10^{11}$ Pa/m $\kappa_2 = 5.6 \times 10^{10}$ Pa/m
Stratified	$\kappa(x,y) = \alpha/b(x,y)$ $\alpha = 1.0 \times 10^{12}$

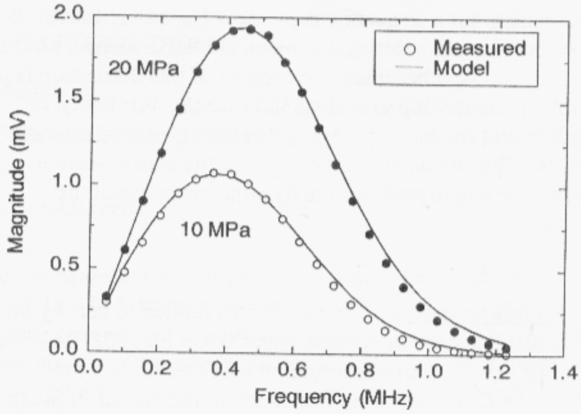

Fig. 2. Transmitted compressional wave amplitudes as functions of frequency. The data points represent compressional wave spectral data from experiments on a fracture in granite subjected to stresses of 10 and 20 MPa (Pyrak-Nolte et al., 1990). The solid curves represent fits using the stratified model and the frequency-dependent stiffness.

equation (3) are shown in Figure 3 as functions of frequency. The values of uniform stiffness from Pyrak-Nolte et al [1990] are the average of the stiffnesses predicted with the dynamic frequency model. We should point out that the use of the stratified percolation model was motivated by previous success of the model. However, we cannot rule out other possible aperture distributions based on the fit. The point here was simply to demonstrate a plausible origin of the frequency-dependent fracture stiffness.

Summary

From this study, we have found that the frequency-dependent nature of fracture specific stiffness can be a simple consequence of the subset of apertures sampled by a given frequency. No additional dynamical effects must be invoked. However, it must be expressed that this is only one possible

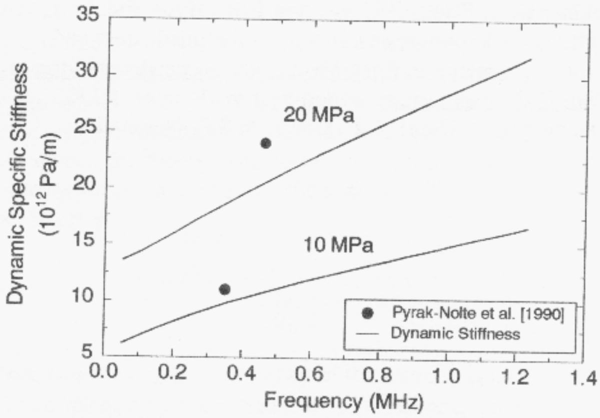

Fig. 3. The dynamic stiffnesses (solid curves) as functions of frequency used to fit the data in Figure 2. Solid dots represent the uniform stiffness and single frequency used by Pyrak-Nolte et al. (1990) to fit data.

mechanism for a frequency-dependent fracture stiffness. It does not rule out additional dynamical effects such as locking or friction. Another interesting aspect of this discussion is the implicit relationship to scaling and fractals. For fractal or multifractal fracture topologies, there are no intrinsic length scales. This allows the wavelength of the elastic wave to define the length scale for the dynamic response of the system.

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L. J. Pyrak-Nolte, Department of Earth & Atmospheric Sciences, 1397 Civil Engineering Building, Purdue University, West Lafayette, IN 47907-1397.

D.D. Nolte, Department of Physics, Purdue University, West Lafayette, IN 47907-1396.

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