Pinning-Mode Resonance of a Skyrme Crystal near Landau-Level Filling Factor $\nu = 1$

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(Received 29 July 2009; published 1 June 2010)

Microwave pinning-mode resonances found around integer quantum Hall effects, are a signature of crystallized quasiparticles or holes. Application of in-plane magnetic field to these crystals, increasing the Zeeman energy, has negligible effect on the resonances just below Landau-level filling $\nu = 2$, but increases the pinning frequencies near $\nu = 1$, particularly for smaller quasiparticle or hole densities. The charge dynamics near $\nu = 1$, characteristic of a crystal order, are affected by spin, in a manner consistent with a Skyrme crystal.

DOI: 10.1103/PhysRevLett.104.226801 PACS numbers: 73.43.−f, 32.30.Bv, 73.21.−b

Spin textures, structures of spin rotating coherently in space, are of importance to a number of different classes of materials [1]. An example of such a spin texture with wide application in descriptions of magnetic order is the Skyrmion, a topological defect first used to describe baryons [2]. Skyrmions can form arrays which have been considered in a variety of magnetic systems, and which have been seen in bulk material [3] in magnetic field, in neutron scattering.

Skyrmions are particularly important in two-dimensional electron systems (2DESs) with an additional spin or pseudospin degree of freedom [4–7]. In these systems, the Skyrmion has a unit charge, bundled along with a texture containing multiple flipped spins (or pseudospins) which spread out in space to reduce exchange energy. The Skyrmion is also predicted [6] to have a spread-out charge distribution different, for example, from the Landau level (LL) orbitals that would characterize an isolated electron in 2DESs.

Skyrmions were identified in 2DESs as excitations near LL filling $\nu = 1$ in experiments [8–10] that measured the electron spin polarization vs ν, and the presence of skyrmions was also shown to affect the measured energy gaps in transport [11,12]. The wide applicability of skyrmions in 2DESs is attested to also by their presence in layer-index pseudospin of bilayers [13], valley pseudospin [14] in AlAs, or in predictions involving both valley pseudospin and spin in graphene [15].

Sufficiently dilute skyrmions are expected to crystallize, stabilized by Coulomb repulsion. This ‘‘Skyrme crystal’’ is a Wigner crystal in which the charged carriers at each site can be regarded as having the characteristics of Skyrmion spin textures. The predicted state has tightly interwoven spin and charge crystal order, and has been of great interest in 2DESs [16–28] and has been considered theoretically for graphene [29]. NMR [23–26] and heat capacity [27] experiments in the Skyrme crystal range near $\nu = 1$ in 2DESs have focused on the strong coupling of nuclear spins to electron spins, likely via a soft spin wave which was very recently unveiled in Raman scattering [28]. The soft spin mode has been interpreted as arising from XY-spin orientational order [17].

The main result of this Letter is that the *charge* dynamics near $\nu = 1$ show characteristics of a Skyrme crystal; we find these dynamics are affected by the Zeeman energy and by the charge density in a manner consistent with calculations [16,17] done for Skyrme crystals. Our ability to observe this demonstrates as well that the charge distribution of a Skyrmion in a crystal differs from that of an ordinary Landau quasiparticle. The experiments use broadband microwave conductivity spectroscopy, which has no direct spin sensitivity.

This microwave spectroscopy has some time ago revealed [30] pinned Wigner crystals of the quasiparticles or holes within integer quantum Hall effects. The microwave spectra of these ‘‘integer quantum Hall Wigner crystals,’’ like those of other pinned crystals [31] in 2DESs in high magnetic field, exhibit a striking resonance, which is understood as a pinning-mode resonance [32], due to the collective oscillations of the crystal within the disorder potential that pins it. Pinning modes are present within the integer quantum Hall effects around $\nu = 1$, 2, 3, and 4 in low-disorder 2DESs. When the exchange energy dominates the Zeeman energy, skyrmions are expected to be present around $\nu = 1$; it is well established [11,33] that near other integer ν under any experimentally accessible conditions in 2DESs, skyrmions do not form.

This Letter addresses the nature of the crystal around $\nu = 1$, and shows that the pinning mode near $\nu = 1$ is affected by Skyrmion formation. These effects become clear through systematic study of the dependence of the pinning mode on in-plane field, $B_{\|}$, which at fixed ν reduces the Skyrmion spin [4–7]. $B_{\|}$ has essentially no effect on the crystal just below $\nu = 2$, expected to be a Wigner crystal of single Landau quasiholes. In contrast, $B_{\|}$ increases the pinning frequency of the crystal near $\nu = 1$ just under conditions for which the skyrmions sizes are predicted [17] to be significant. In addition, the Skyrmion density (the same as the charge density) in the crystal increases for ν farther from exactly 1, and is also large for higher electron density 2DESs. We find $B_\parallel$ increases the pinning-mode frequency only for small enough Skyrmion density, or wide enough Skyrmion separation. This shows that spin size of skyrmions is affected by their mutual proximity [17], an illustration of the intertwined nature of charge and spin in this state.

We used 2DESs from two high quality wafers of AlGaAs/GaAs/AlGaAs quantum wells. Sample A is a 30 nm quantum well (no. 5-20-05.1), with the electron density $n = 2.7 \times 10^{11}/\text{cm}^2$, and a low temperature mobility $\mu = 27 \times 10^6\ \text{cm}^2/\text{V s}$. Sample B is a 50 nm quantum well (no. 7-20-99.1), with a lower $n = 1.1 \times 10^{11}/\text{cm}^2$ and $\mu = 15 \times 10^6\ \text{cm}^2/\text{V s}$. The microwave spectroscopy technique is similar to those reported earlier [30,31]. As illustrated in Fig. 1(a), we deposited on the sample a metal-film coplanar waveguide, with length $l = 4$–28 mm and slot width $W = 30$–$80\ \mu\text{m}$, impedance matched to $Z_0 = 50\ \Omega$. The microwaves couple capacitively to the 2DESs. From the absorption P of the signal by the 2DESs, we calculate the real part of its diagonal conductivity $\text{Re}[\sigma_{XX}(f)] = (W/2lZ_0)\ln P$.

To tilt the sample in magnetic field, we designed a rotator with low-reflection broadband microwave connections through flexible microstrips. The tilting angle θ is calculated from the total fields B_{tot} of prominent integer quantum Hall states; the perpendicular field component $B_\perp = B_{\text{tot}} \cos\theta$. We estimate the absolute error in determining ν at different tilt angles as about 0.01. The measurements were done at the low power limit and the sample temperature was 40 mK.

For sample A, Fig. 1 shows a set of conductivity spectra (b) around $\nu = 1$ and (c) just below $\nu = 2$. The resonances in the spectra, which flatten out for temperatures above 150 mK, are understood as the pinning modes of the

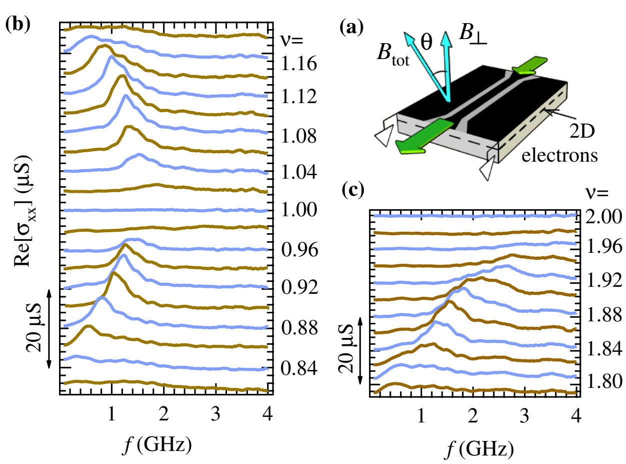

FIG. 1 (color online). (a) An illustration of the coplanar waveguide. (b) From sample A, at $\theta = 0°$, the real conductivity spectra at various ν, increasing from 0.82 to 1.18 in steps of 0.02. (c) Also taken at $\theta = 0°$, the real conductivity spectra at ν from 1.78 to 2.00, increasing in steps of 0.02.

crystallized quasiparticles or holes, as in Ref. [30]. For brevity, we shall refer to ν just below 2 showing resonance as $\nu = 2^-$, and the resonant ν ranges just above (below) 1 as $\nu = 1^+$ (1^-). In both Figs. 1(b) and 1(c), the resonance peak frequency f_{pk} decreases with increasing partial filling factor $\nu^* = |\nu\text{-nearest integer}|$. This is consistent with the picture of a weakly pinned crystal [32], because higher ν^* means denser quasiparticles or holes and stronger interaction, and thus a weaker role of disorder. Comparing Figs. 1(b) and 1(c), we notice that, close to integer ν, the $\nu = 2^-$ resonances have a stronger ν^* dependence of f_{pk} than the $\nu = 1^{+,-}$ resonances.

Still for sample A, we repeat the measurement in a tilted magnetic field. We fix $B_\perp$ and hence ν, and increase B_{tot} by a factor of $1/\cos\theta$. Figure 2(a) shows the spectra at $\nu = 1.05$ and 1.91, each taken at $\theta = 0°$ and 31°. At these two ν, the quasiparticles or holes are within the same orbital LL ($N = 0$), and have equal density $n^* \sim 1.3 \times 10^{10}/\text{cm}^2$, calculated as $(\nu^*/\nu)n$. At $\theta = 0°$, the $\nu = 1.05$ resonance is at about half the frequency of the $\nu = 1.91$ resonance. Tilting to 31° has negligible effect on the $\nu = 1.91$ resonance, but shifts the $\nu = 1.05$ resonance to higher frequency. The $\nu = 0.95$ resonance, not shown here, behaves almost identically to the $\nu = 1.05$ resonance.

Figures 2(b) and 2(c) present the effect of $B_\parallel$ for $\nu = 1^+$, 1^- and 2^-. Figure 2(b) uses the $\theta = 0°$ data in Fig. 1, and plots f_{pk} vs n^*, for all three crystal phases: $\nu = 2^-$, $\nu = 1^+$, and $\nu = 1^-$. All of the three curves show f_{pk} decreases as n^* increases, as we have seen in Fig. 1. The $\nu = 1^+$ and $\nu = 1^-$ curves agree well, indicating good symmetry be-

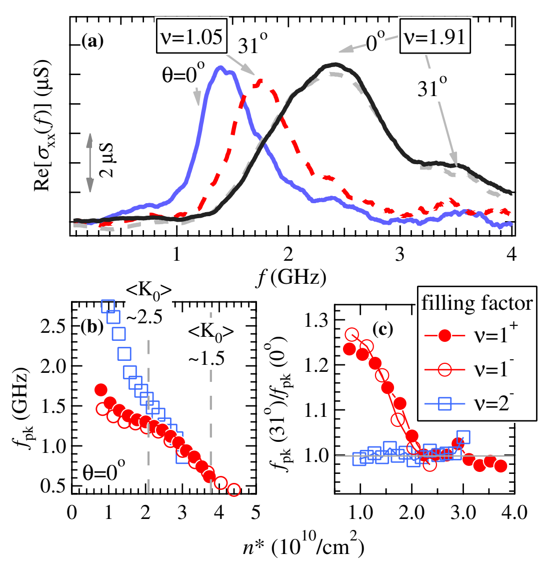

FIG. 2 (color online). (a) For sample A, the real conductivity spectra at $\nu = 1.05$ and $\nu = 1.91$, measured at $\theta = 0°$ (solid) and 31° (dash). (b) For sample A at $\theta = 0°$, the resonance peak frequencies f_{pk} as a function of quasiparticle density n^* for the $\nu = 2^-$ (open squares), $\nu = 1^+$ (solid circles), and $\nu = 1^-$ (open circles) crystals. The Skyrmion sizes, $\langle K_0 \rangle$, estimated by Ref. [17], are marked in the graph. (c) The relative change in f_{pk} from $\theta = 0°$ to 31°.

tween quasiparticles and quasiholes. The $\nu = 1^{+,-}$ curves also overlap with the $\nu = 2^-$ curve for high n^*. But for low n^* ($<2.5 \times 10^{10}/\mathrm{cm}^2$), the $\nu = 1^{+,-}$ resonances have lower f_{pk} than the $\nu = 2^-$ resonance for the same n^*.

Figure 2(c) plots the ratio, $f_{\mathrm{pk}}(31°)/f_{\mathrm{pk}}(0°)$, of $\theta = 31°$ and $0°$ f_{pk}. f_{pk} increases with tilting only for the $\nu = 1^{+,-}$ resonances with low n^* ($n^* < 2.3 \pm 0.3 \times 10^{10}/\mathrm{cm}^2$, with the error arising mainly from uncertainty in ν of data at different angles). The two curves of f_{pk} vs n^* for $\nu = 1^+$ and 1^- agree with each other, indicating a still good particle-hole symmetry. Tilting has negligible effect on the $\nu = 2^-$ resonances, or the $\nu = 1^{+,-}$ resonances with high n^*. Further tilting of sample A, up to $\theta = 51°$, does not affect the $\nu = 2^-$ resonances, and does not further alter f_{pk} vs n^*, for $\nu = 1^{+,-}$ at low n^*.

Sample B has lower n than sample A, and as discussed below, is predicted to be capable of supporting larger skyrmions. Near integer ν, sample B also displays resonances in the real conductivity spectra. Figure 3 shows the resonances at three typical ν: (a) 0.96, (b) 1.04, and (c) 1.90, each taken at $\theta = 0°$ and $\theta = 63°$. Tilting shifts the $\nu = 0.96$ and $\nu = 1.04$ resonances to higher frequencies, in clear contrast to the negligible effect it has on the $\nu = 1.90$ resonance.

For sample B, at $\theta = 0°$, $50°$ and $63°$, Fig. 4 shows f_{pk} vs n^* for $\nu = 1^+$, 1^- and 2^-. f_{pk} decreases as n^* increases, consistent with a weak pinning picture, and $\nu = 1^+$ and 1^- resonances show good agreement with each other consistent with particle-hole symmetry. The $\nu = 2^-$ resonances show no change as the sample is tilted. The $\nu = 1$ curves lie below the 2^- curves, more for lower n^*. As θ increases, the $\nu = 1^{+,-}$ curves move upward toward the 2^- curve, and for $63°$, the curves are close together except at the smallest n^*.

By analogy to the Wigner crystal pinning modes found at very low ν ($\lesssim 1/5$) [31], the observed resonances at ν just away from 1 and other integers are clear signatures of crystallized quasiparticles or holes [30]. Here we show that the dependence on $B_\parallel$ indicates a distinct difference of the crystals of $\nu = 1^{+,-}$ from that of $\nu = 2^-$, and provides evidence for a Skyrme crystal near $\nu = 1$.

To explain the effect of $B_\parallel$, the spin degree of freedom must be invoked. This is clear from the negligible effect of $B_\parallel$, except for the crystals with low n^* near $\nu = 1$. The presence of an effect for the resonances near $\nu = 1$ that is completely absent near $\nu = 2$ is an indicator that Skyrmion formation is involved, since these are well known to be absent around $\nu = 2$. Most strikingly, the $\nu = 2^-$ resonance is completely unaffected by tilt, out to $51°$ ($B_\parallel = 6.9$ T) for sample A and $67°$ ($B_\parallel = 5.35$ T) for sample B. The same $B_\parallel$ values near $\nu = 1$ (achieved for smaller θ around that filling) have clear effect on the $\nu = 1$ resonances. The crystals for $\nu = 2^-$, 1^+ and 1^- all belong to the lowest orbital LL. If the effect of $B_\parallel$ were due to the orbital wave function, it would be unlikely that the same effect would be completely absent for the $\nu = 2^-$ crystal and for the $\nu = 1^{+,-}$ crystals with high n^*.

The effect of $B_\parallel$ near $\nu = 1$ finds a natural explanation in the predicted crystal of skyrmions [16–22]. The Skyrmion size, $\langle K_0 \rangle$, denotes the number of flipped spins (relative to the maximally spin-polarized state with the same charge) [6,17]. Two key parameters control $\langle K_0 \rangle$. One is the Zeeman to Coulomb energy ratio [6], $\tilde{g} = g\mu_B B_{\mathrm{tot}}/(e^2/4\pi\epsilon_0\epsilon l_0)$, where $|g| \approx 0.44$ is the g factor in GaAs, μ_B the Bohr magneton, $\epsilon \approx 13$ the dielectric constant of GaAs, and $l_0 = \sqrt{\hbar/eB_\perp}$ the magnetic length. Larger $\tilde{g}$ makes flipping spins more costly, and favors smaller $\langle K_0 \rangle$. The second parameter is n^*: increasing n^* by tuning ν away from 1 brings skyrmions closer and limits $\langle K_0 \rangle$. Reference [17] calculated $\langle K_0 \rangle$, predicting the reduction of $\langle K_0 \rangle$ on increasing $\tilde{g}$ or n^*.

For our samples, A has $\tilde{g} \sim 0.019$ at $\nu = 1$ in perpendicular field, and B with a lower n has a smaller $\tilde{g} \sim 0.012$, because $\tilde{g} \sim \sqrt{n}$. Upon tilting, $\tilde{g}$ increases by a factor of $1/\cos\theta$, the same as B_{tot}. Based on Ref. [17], the calculated

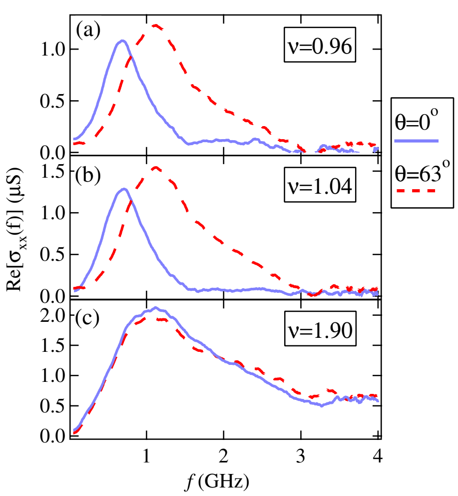

FIG. 3 (color online). The conductivity spectra of sample B, taken with $\theta = 0°$ (solid curve) and $\theta = 63°$ (dashed curve), at (a) $\nu = 0.96$, (b) $\nu = 1.04$, and (c) $\nu = 1.90$.

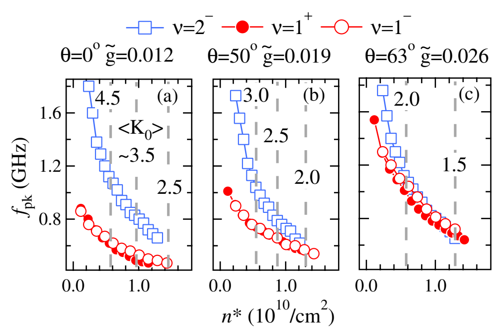

FIG. 4 (color online). From sample B, f_{pk} as a function of n^* for the $\nu = 2^-$ (open squares), $\nu = 1^+$ (solid circles), and $\nu = 1^-$ (open circles) resonances, at (a) $\theta = 0°$, (b) $\theta = 50°$, and (c) $\theta = 63°$. The Skyrmion sizes, $\langle K_0 \rangle$, estimated by Ref. [17], are marked in the graph.

$\langle K_0 \rangle$ for different $\tilde{g}$ and n^* are marked in Fig. 2(b) and Fig. 4.

Consistently for two samples [34], increasing $\tilde{g}$ beyond that needed to bring $\langle K_0 \rangle$ (obtained from n^* and θ according to Ref. [17]) down to around 2.0, produces no change on the $1^{+,-}$ resonances. We interpret this as due to the skyrmions becoming small enough that their resonance cannot be distinguished from that of ordinary Landau quasiparticles. This is particularly clear in sample B, in which the f_{pk} vs n^* curves for $1^{+,-}$ at large θ or large n^* match the curves for 2^-. The curves come together for $\langle K_0 \rangle$ below about 2, even though this condition occurs at different n^* when $\tilde{g}$ changes. In sample A, for $n^* = 2.3 \times 10^{10}$ cm^{-2}, the largest n^* for which we observe tilt-induced change in f_{pk} that we ascribe to Skyrmion formation, we estimate $\langle K_0 \rangle \approx 2.4$ at 0°, and $\langle K_0 \rangle \approx 2.1$ at 31°.

At fixed n^*, decreasing $B_\parallel$ produces larger predicted $\langle K_0 \rangle$, and we find a lower f_{pk}. This suggests a crystal formed by larger skyrmions is more weakly pinned. This effect can be due to two possible mechanisms which may both be operational: one from Skyrmion-disorder interaction and the other from Skyrmion-Skyrmion interaction. Larger Skyrmions average disorder over a larger area, resulting in an effectively weaker disorder, and hence a lower f_{pk}. f_{pk} is also sensitive to the shear modulus, which is determined by the interaction between skyrmions. When the skyrmions grow bigger and begin to overlap, the inter-Skyrmion repulsion is expected to get stronger and to increase the shear modulus of the crystal [35]. And within the weak pinning picture [32], a stiffer crystal has lower f_{pk}, because it is more difficult for the crystal to deform to take advantage of the disorder landscape.

As ν departs from 1, a phase transition is predicted, from a triangular-lattice ferromagnet of spin helicity to a square-lattice antiferromagnet [16–22], in favor of lower exchange energy at a cost in the Madelung energy. Calculations in Ref. [17] predict this to happen roughly at $\langle K_0 \rangle \sim 2.5$ in Fig. 2(b), $\sim$5 in Fig. 4(a), and $\sim$1.5 in Fig. 4(c). In our measurement we do not see any abrupt jump in the pinning frequencies. One possible explanation is that sample disorder smears the phase boundary.

To summarize, the predicted Skyrmion sizes affect the frequencies of pinning modes near $\nu = 1$, in a manner indicating the crystal around that filling is formed of skyrmions at least for small n^* and small $\tilde{g}$. As $B_\parallel$ causes the predicted skyrmions to shrink, we observe upshift of the pinning frequency, which saturates as skyrmions approach single spin flips. The dependence of the pinning on the spin is a consequence of the coupled charge and spin degrees of freedom for crystallized skyrmions.

We thank R. Côté, H. A. Fertig, and Kun Yang for helpful discussions, and we thank G. Jones, J. Park, T. Murphy, and E. Palm for experimental assistance. The work at NHMFL is supported by DOE Grant No. DE-FG02-05-ER46212. At Princeton, support was provided by DOE Grant No. DE-FG21-98-ER45683, and by NSF MRSEC MR-0819860. NHMFL is supported by NSF Cooperative Agreement No. DMR-0084173, the State of Florida and the DOE.

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