Observation of a Pinning Mode in a Wigner Solid with $\nu = 1/3$ Fractional Quantum Hall Excitations

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We report the observation of a resonance in the microwave spectra of the real diagonal conductivities of a two-dimensional electron system within a range of $\sim \pm 0.015$ from filling factor $\nu = 1/3$. The resonance is remarkably similar to resonances previously observed near integer ν, and is interpreted as the collective pinning mode of a disorder-pinned Wigner solid phase of $e/3$-charged carriers.

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For two-dimensional electron systems (2DES) of low disorder in high magnetic field (B), interactions between carriers have astounding consequences. The most striking of these is the fractional quantum Hall effect (FQHE) [1], which is the signature of incompressible liquid states [2] at a rational fractional Landau level filling. The excitations of the fractional quantum Hall liquid carry fractions of the electronic charge [2] as verified in recent experiments [3–5]. Analogous to electrons, these fractionally charged excitations themselves may be localized by a disorder potential, or can interact to form other states, most notably other fractional quantum Hall liquids [6]. This Letter will present evidence for an entirely different interacting state of such quasiparticles: a Wigner solid. Owing to the inevitable pinning of the crystal by disorder, this state is insulating as well.

Wigner solids occur in 2DES for small Landau level fillings (ν) [7–9], when the kinetic energy of carriers is quenched by the magnetic field, so that the carriers become effectively dilute, with their typical spacing larger than the magnetic length $l_B = (\hbar/eB)^{1/2}$. At least for 2DES samples of extremely low disorder, Wigner solids can occur as well within the ranges of integer quantum Hall effect plateaus [10], in which there is a dilute population of electrons or holes in a partially filled Landau level, along with one or more completely filled Landau levels. Within the IQHE, Wigner solids are composed of carriers generated by moving ν away from its integer value; the insulating property of the pinned solid gives the quantized Hall plateau, which is concomitant with vanishing diagonal conductivity, its finite width.

The pinning of the Wigner solid by residual disorder [11], besides causing them to be insulators and producing finite correlation lengths for crystalline order, gives rise to a striking rf or microwave resonance in the spectrum [10,12–16]. This resonance is understood as a pinning mode, or a collective oscillation of solid domains within the disorder potential. Pinning modes [12] can be seen both in the low ν state [13–15], that terminates the FQHE series, and within IQHE's of samples with extremely low disorder [10,16].

This Letter reports the observation of a microwave resonance within the ν range of the $1/3$ FQHE. The resonance is interpreted as pinning mode of a Wigner solid of $1/3$ charged excitations, and this interpretation is supported by a comparison of the parameters of the resonance near $1/3$ with those of a pinning resonance observed near $\nu = 1$ in the same sample. Both the resonance near $\nu = 1/3$ and the resonance near $\nu = 1$ show an increase of peak frequency, f_{pk}, as quasiparticle charge density decreases, and the integrated absorptions of both resonances suggest that much of the quasiparticle charge density is participating in the mode. The resonance around $1/3$ occurs for fillings within ± 0.015 of $1/3$, much narrower than the ± 0.16 range of resonance within the IQHE around $\nu = 1$, and for the same quasiparticle number density, $\tilde{n}$, the resonance peak frequencies around 1 and $1/3$ are nearly the same.

The data presented are from a 2DES in a 50 nm wide GaAs/AlGaAs/GaAs quantum well (#7-20-99.1) with electron density $n = 1.1 \times 10^{11}/\text{cm}^2$, and low temperature mobility $\mu = 15 \times 10^6\ \text{cm}^2/\text{V s}$. The microwave spectroscopy technique is similar to those reported earlier [10,12–16]. As illustrated in Fig. 1(a), we deposited on the sample surface a meandering metal-film coplanar waveguide, with propagation length $l = 28$ mm and a slot of width $W = 30\ \mu$m separating a driven center conductor from broad, grounded, side planes. The microwaves couple capacitively to the 2DES underneath. We normalize the transmitted signal, P, taken at a given ν to that at a reference integer or rational fractional filling factor. In this Letter we choose this reference ν as ν_c for the particular QHE under consideration, to minimize effects of (i) drift as the superconducting magnet is swept, due to the variation of liquid He levels in the cryostat and to fluctuation in the temperature of the room, and (ii) background B dependence not due to the 2DES of the transmitted signal. Such background B dependence

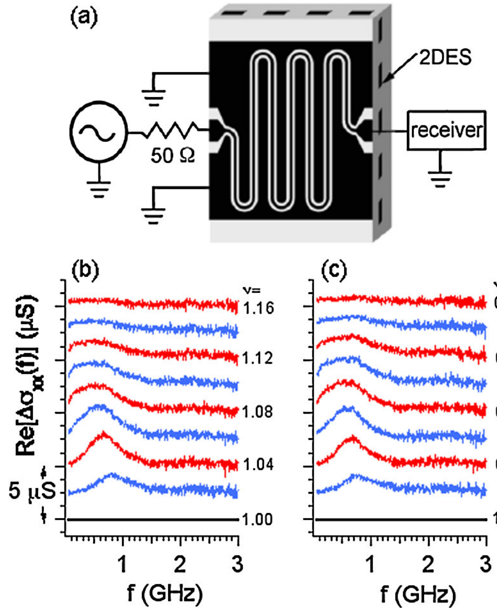

FIG. 1 (color online). (a) Schematic of the microwave measuring setup, and sample with coplanar waveguide transmission line, with metal film shown as black. (b) Real part of change in diagonal conductivity from that at Landau filling $\nu = 1$, $\mathrm{Re}[\Delta\sigma_{xx}(f)]$, vs frequency, f, for ν increasing from 1 (bottom trace) to 1.16 (top trace), in equal steps of 0.02. Successive traces are offset by 2 μS from each other for clarity. (c) $\mathrm{Re}[\Delta\sigma_{xx}(f)]$, vs f, for ν decreasing from 1 (bottom trace) to 0.84 (top trace), in equal steps of 0.02. Successive traces are offset by 2 μS from each other.

originates from magnetoresistance of the transmission line metal film, and from impurities in the GaAs substrate. From the measured P, we calculate changes in the real part of the 2DES diagonal conductivity from that at ν_c, the quantizing ν of the QHE being studied, as $\mathrm{Re}[\Delta\sigma_{xx}(\nu) - \sigma_{xx}(\nu_c)(f)] = (W/2lZ_0)\ln P(\nu)/P(\nu_c)$. This $Z_0 = 50\ \Omega$ is the characteristic impedance of the coplanar waveguide in the limit of zero 2DES conductivity. The sample temperature was at 40 mK for data in Fig. 1 and 50 mK for data in Fig. 2.

We will explicitly compare the resonances found near $\nu = 1/3$ with those near $\nu = 1$, measured in the same sample. Figure 1(b) shows a series of spectra, $\mathrm{Re}[\Delta\sigma_{xx}]$ vs f for ν between $\nu = 1.02$ and 1.16 in steps of 0.02. As in ref. [10], the spectra near $\nu = 1$ show clear resonances that we ascribe to pinning modes of the IQHWC. As ν increases from 1 the peak frequency decreases and the intensity of the resonance increases; as ν increases above 1.16, the resonance weakens and eventually vanishes. Spectra taken for $\nu < 1$, likewise in steps of 0.02, are shown in Fig. 1(c), and demonstrate that the resonance evolves symmetrically about $\nu = 1$, again vanishing for ν below about 0.84.

Figure 2 presents the main results of this Letter, the spectra of the resonance within the range of the 1/3 FQHE. Figure 2(a) shows traces of $\mathrm{Re}[\Delta\sigma_{xx}]$ vs f for ν decreasing from 0.331 to 0.319, and Fig. 2(b) shows traces for ν increasing from 0.335 to 0.353. For both panels of the figure, $\nu_{\mathrm{ref}} = 1/3$, and the spectra are taken at ν separated by steps of only 0.002. To facilitate the discussion, we denote $\tilde{\nu} = \nu - \nu_c$, where $\nu_c = 1$ or $1/3$ is the quantizing filling of the quantum Hall effect. The dependence on $\tilde{\nu}$ of the resonance near $\nu = 1/3$ strongly resembles that seen near $\nu = 1$ in Fig. 1(b) and 1(c): as $|\tilde{\nu}|$ increases from zero the resonance near 1/3 initially increases in intensity, and decreases in peak frequency. As $|\tilde{\nu}|$ increases further, to around $|\tilde{\nu}| \sim 0.015$, the resonant peak in $\mathrm{Re}[\Delta\sigma_{xx}(f)]$ vs f disappears, and the resonance near 1/3 merges into a monotonically decreasing curve. Different from the spectra near $\nu = 1$, the range of ν around 1/3 for which a resonance is observed is much narrower, and the peak conductivities of the spectra are much smaller, necessitating the above-described steps to suppress instrumental drift and background absorption.

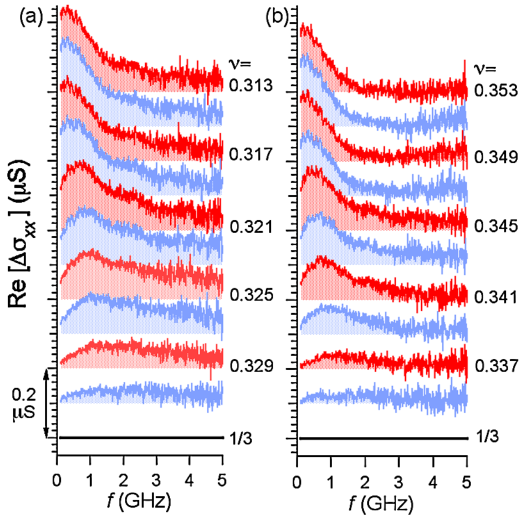

FIG. 2 (color online). $\mathrm{Re}[\Delta\sigma_{xx}(f)]$, here the change in real part of the diagonal conductivity from that at Landau filling $\nu = 1/3$, vs frequency f Successive traces were taken at intervals of 0.002 in ν, and are offset by 0.1 μS from each other for clarity. (a) ν from 1/3 (bottom trace), to 0.313 (top trace) (b) ν from 1/3 (bottom trace), to 0.353 (top trace).

Parameters of the resonances near $\nu = 1$ and 1/3 are summarized in Fig. 3. Within the picture of the resonances as pinning modes, the common features of the the resonances in the two different ν ranges can be readily interpreted. The first common feature of the resonances is that f_{pk} increases as $|\tilde{\nu}|$ (and hence the quasiparticle charge density) decreases, as shown for $\nu = 1$ in panel (a), and for $\nu = 1/3$ in panel (b). This behavior has been observed [13] in the low ν Wigner crystal (whose density is equivalent to the total density n of the sample) on tuning n with a backgate. Predicted in weak pinning theories [17–20], the increase of f_{pk} is due to the reduction in the carrier-carrier interaction (which results in softening of the crystal) that occurs as the intercarrier spacing is increased.

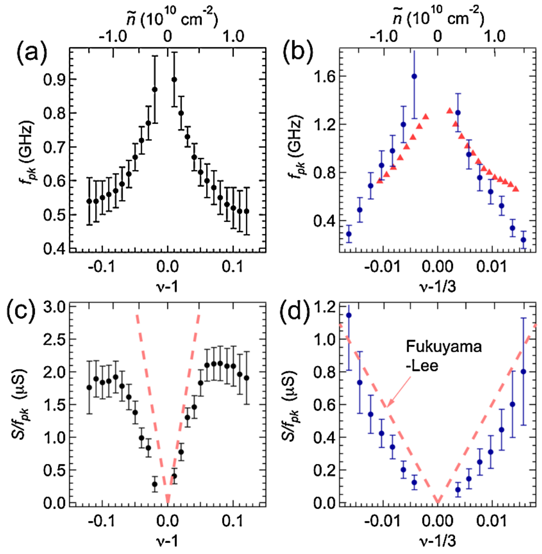

FIG. 3 (color online). (a) The resonance peak frequencies f_{pk} near filling factor $\nu = 1$, vs ν (lower axis) and quasiparticle-hole density, $\tilde{n}$ (upper axis). (b) Solid circles: f_{pk} near $\nu = 1/3$ vs ν (lower axis) and $\tilde{n}$ (upper axis); triangles: f_{pk} vs $\tilde{n}$ on upper axis only, near $\nu = 1$, and with the sample tilted to 63°. (c) S/f_{pk} vs ν, near $\nu = 1$, where S is integrated spectrum $\mathrm{Re}(\sigma_{xx})$ vs f. (d) S/f_{pk} vs ν, near $\nu = 1/3$. Dashed lines in (c) and (d) are predicted S/f_{pk} calculated from quasiparticle charge density as described in the text.

This reduction in the carrier-carrier interaction allows the carriers to distribute themselves closer to minima in the disorder potential, raising the pinning energy per carrier and thus f_{pk}.

The integrated conductivity of the resonances near $\nu = 1$ and $1/3$ is in rough agreement with predictions based on the quasiparticle charge density, $ne\tilde{\nu}/\nu$. Within an oscillator model [17] of the pinning mode, the integrated $\mathrm{Re}[\sigma_{xx}(f)]$ of a resonance is $S = ne(|\tilde{\nu}|/\nu)\pi f_{\mathrm{pk}}/2B$, so S/f_{pk} is nearly proportional to $|\tilde{\nu}|$. Figures 3(c) and 3(d) show S/f_{pk} vs ν for the resonance near $\nu = 1$ and $1/3$, with S obtained by integrating over the experimental frequency range, up to 3 GHz. For the $1/3$ data, the spectra merge into a decreasing background for larger $\tilde{\nu}$, as seen in Fig. 2; in Fig. 3(d) we present data on S/f_{pk} for $|\tilde{\nu}|$ up to the largest value for which we resolve a peak in the spectrum, about $|\tilde{\nu}| = 0.015$. Dashed lines in the plots show S/f_{pk} calculated from the oscillator model without any variable parameter. For both quantum Hall effects, the experimentally determined S/f_{pk} vs ν roughly follow or lie below the calculated results; differences between the calculation and the data can be due to missed amplitude in the integration of the spectra, and to existence of some the quasiparticles not in the crystal: for example, some carriers might be strongly bound to disorder for small $|\tilde{\nu}|$, or beginning to form liquid puddles at larger $|\tilde{\nu}|$. For large $|\tilde{\nu}|$, S/f_{pk} vs ν flattens in the data taken around $\nu = 1$ [Fig. 3(c)] but continues to increase with $|\tilde{\nu}|$ around $\nu = 1/3$ [Fig. 3(d)]. The reason for this difference is unclear; it may be that liquid states to take up the carriers without contributing to the resonance are better developed near $\nu = 1$.

The ν range of the resonance around $\nu = 1/3$ is much narrower than the range of the resonance around $\nu = 1$, but can be understood in terms of composite fermion theory [21,22], supporting the interpretation of 1/3 FQH resonance as an analogous pinning mode of FQHE quasiparticles. CFs have their own effective filling factor, $\nu_{\mathrm{CF}} = \nu/(1 - 2\nu)$, for CFs composed of two flux quanta plus an electron, and the 1/3 FQHE is an IQHE of such CFs at $\nu_{\mathrm{CF}} = 1$. A Wigner solid in the 1/3 FQHE can be construed as composed of CFs within their $\nu_{\mathrm{CF}} = 1$ IQHE. and as such is evidence for CF interaction. FQHEs of interacting CFs have been observed [6] as well, and CF Wigner solids have been considered theoretically [23–25] for low electron Landau fillings. Changing $\nu = 1/3 \pm 0.015$ to CF filling results in $\nu_{\mathrm{CF}} = 1 \pm 0.15$. This is in agreement with the range of the pinning mode around $\nu = 1$, which we observe out to 0.16 away from $\nu = 1$. Furthermore, this ν is reasonably close to theoretical predictions of $\sim 1/7$ [8,9] for the upper ν of a Wigner solid in the lowest Landau level, and to experimental results in low disorder n type samples, with insulating behavior [26] and pinning modes [15] seen for ν as high as about 0.218.

Comparison of peak frequencies of the resonances near $\nu = 1$ and near $\nu = 1/3$ strongly implies that the two resonances are of similar origin. If spin effects on the $\nu = 1$ resonance are suppressed, the f_{pk} for the two ranges of ν agree for the same quasiparticle number density $\tilde{n} = ne\tilde{\nu}/\nu\tilde{e}$, where the quasiparticle charge $\tilde{e} = e$ or $e/3$. The resonances around $\nu = 1$ in samples of this density have been shown [16] to be sensitive to tilting the sample in the magnetic field, which has been interpreted [16] as due to suppression of Skyrmion formation [27,28], due to the larger Zeeman energy produced by the larger total magnetic field for a tilted sample near $\nu = 1$. In addition to f_{pk} vs $\tilde{n}$ and ν near 1/3, Fig. 3(b) shows f_{pk} near $\nu = 1$ vs $\tilde{n}$ on the upper axis, for the sample tilted 63° from perpendicular to the magnetic field. That angle [29], according to ref. [16], is sufficient to fully suppress the skyrmion effects. There is apparent agreement between f_{pk} vs $\tilde{n}$ in the IQHE and FQHE. We stress that while the agreement is much improved for the tilted-field data, there is agreement to within a factor of 2 even without tilting.

The agreement between f_{pk} vs $\tilde{n}$ in the IQHE and FQHE can be interpreted within the weak pinning picture, if we take the solid near $\nu = 1/3$ to be formed by $e/3$-charged carriers with Coulomb interaction. f_{pk} has been calculated [18] for a Coluomb-interacting solid subject to weak, randomly distributed disorder centers with potential strength V_0, as $f_{\mathrm{pk}} \sim (V_0^2/\mu\tilde{e}B\xi^6)$, where $\tilde{e}$ is the charge of a carrier, μ is the shear modulus of the solid, and ξ is an effective disorder correlation length, which is the larger of the carrier wave function size (approximately the magnetic length in the low $|\tilde{\nu}|$ limit) or the size of the disorder centers. The disorder, which has been proposed [19] to be due to interface roughness, is electrostatic, so V_0 must be proportional to $\tilde{e}$, and the classically calculated $\mu \sim (\tilde{n})^{3/2}\tilde{e}^2$ [30], so V_0^2/μ is $\tilde{e}$ independent. The factor $\tilde{e}B$ should also be the same for the $\nu = 1/3$ and 1 effects. The interpretation in terms of the weak pinning theory is then that the effective disorder correlation length ξ is the same in the solids around $\nu = 1$ and $1/3$.

To summarize, for ν within a range of ± 0.015 about $1/3$, we found that the real diagonal conductivity spectra of a 2DES exhibit microwave resonances. By analogy to similar resonances previously found near integer ν, we interpret the resonances found near $\nu = 1/3$ as the pinning modes of a disorder-pinned solid phase of CFs, or quasiparticles of the $1/3$ FQHE.

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