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Pinning mode resonances of new phases of 2D electron systems in high magnetic fields

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Abstract

A striking rf or microwave resonance is a generic feature of electron solid phases of two-dimensional electron systems. These resonances have served to identify and characterize such solids, in the insulator that terminates the series of fractional quantum Hall effects at high magnetic field, in the range of the integer quantum Hall effect, and in bubble phases in the first excited and higher Landau levels.

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1. Introduction

In low disorder two-dimensional electron systems (2DES) in high magnetic fields, B, electron–electron interactions are of central importance,since the kinetic energy of the electrons is effectively frozen out by Landau quantization. High B phases dominated by the electron–electron interaction include quantum liquids associated with the fractional quantum Hall effect (FQHE) [1], and electron solids related to the Wigner crystal. This paper is concerned with such electron solids, which are insulators owing to pinning by disorder. Unlike fractional quantum Hall states, which have the unmistakable signature of quantized Hall resistivity, dc transport studies of the solids cannot distinguish them readily from other insulators, most notably those due to single particle localization.

The insulating phases of 2DES at high B can be difficult to study by means of dc transport, since, particularly when at low temperature and far from transitions to liquids, they have extremely small conductivity. Broadband microwave spectroscopy has proven to be of great value in studying these insulators, because of a microwave or rf resonance that is generic to the spectrum of the insulating phases of sufficiently clean 2DES in high B. The resonance is identified as a pinning mode, a collective small oscillation of pieces of solid about their pinned positions. As we will discuss in this paper, the resonance can be regarded as a signature of electron solid formation.

The series of integer and fractional quantum Hall effects (IQHEs and FQHEs) terminates at low Landau level filling ν, or high B in an insulating phase. In this paper we refer to that insulator as the "low ν insulator" or LNI, to distinguish it from other insulating phases that can occur at much larger ν. Theories of 2DES without disorder have predicted [2] that the Wigner crystal would be the ground state for Landau level filling ν below about 1/6 at large areal density n. The development [3] of low disorder samples, in which a well developed 1/5 FQHE is observed, and in which the transition to the LNI occurs near that predicted value, created a great deal of interest [4] in the LNI as a pinned Wigner crystal. A recent result [5] from the present collaboration indicates that quantum correlations of the electrons (as measured by ν) play a crucial role in the melting of the LNI.

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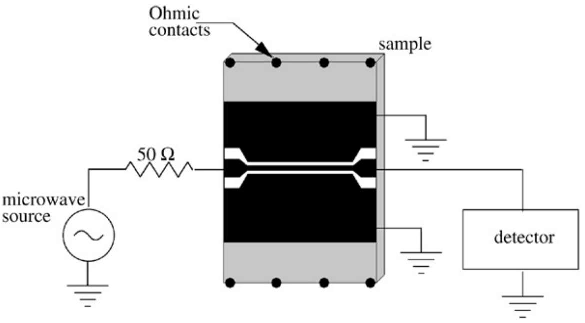

Fig. 1. Schematic of measurement of a transmission line. The highly conductive metal film that forms the CPW is shown as black. The broad side planes of the CPW are grounded and the center conductor is driven. The 2DES resides a fraction of a μm beneath the metal, and is capacitively coupled to the CPW.

Following a brief treatment of the measurement technique and a review of reasons for identifying the resonances as pinning modes of electron solids, the paper will present an overview of some recent advances that have been made by looking at samples of extremely low disorder. These include the discoveries of evidence [6] for distinct phases within the LNI of resonances [7,8] within the ranges of the integer quantum Hall effect, and of resonances [9–12] of more exotic solids known as "bubble phases", found in excited Landau levels.

2. Microwave and rf measurements of 2DES in high magnetic fields

Fig. 1 shows a schematic of the measurement system used to obtain all the data presented in this paper. On the top surface of the wafer, metal film is lithographed to define a standard type of transmission line, known as a coplanar waveguide (CPW). This transmission line is coupled capacitively to the 2DES a fraction of a μm beneath, and its measured loss is used to calculate the conductivity of the 2DES. The CPW has three elements, two ground planes at the sides and a narrow center conductor that is driven. The center conductor and side planes are separated by slots of width W, and the 2DES that is being measured is mainly under these slots. The effective size of the sample is then given by W, which has varied from 20 to 80 μm in our experiments.

One advantage of the method is its ability to give quantitative $\mathrm{Re}(\sigma_{xx})$ in many cases. From P, the transmitted power normalized to unity for vanishing diagonal conductivity σ_{xx}, we calculate the real part of diagonal conductivity of the 2DES as

$$\mathrm{Re}(\sigma_{xx}) = -W|\ln(P)|/2Z_0L, \quad (1)$$

where L is the total length of the transmission line and $Z_0 = 50\ \Omega$ is the characteristic impedance of the CPW for $\sigma_{xx} = 0$. This formula is derived for the high f, low loss limit. We have checked the validity of the formula for our measuring conditions by performing calculations that take into account the distributed nature of the coupling between the 2DES and the transmission line.

The microwave electric field applied to the 2DES is spatially varying, with the wave vector of the lowest Fourier component $\sim \pi/W$. Eq. (1) neglects wave vector dependence in the 2DES conductivity. We will see below that under some conditions the spectra do show a sensitivity to W, and in these cases the values of $\mathrm{Re}(\sigma_{xx})$ plotted are understood to be calculated from P according to Eq. (1) and to be approximate.

3. Pinning mode as a signature for electron solid

The pinning mode interpretation of the resonance, and hence the identification of the resonance as a signature of an electron solid is now on firm ground. A resonance in the LNI has only been seen in samples that are of sufficient quality to exhibit the 1/3 FQHE. The frequency, f_{pk}, of the resonance is dependent on the sample quality as expected for a disorder-induced mode, with low f_{pk} appearing for samples of lower disorder.

To illustrate pinning mode spectra, and in particular the range of peak frequency f_{pk} that has been observed, Fig. 2 shows pinning modes exhibited by the high B insulator of three samples, which were grown in the same system, and have nearly the same density $n \approx 4.5 \times 10^{10}\ \mathrm{cm}^{-2}$. The spectra in the figure were taken at nearly the same B, and hence (with the matched n) nearly the same ν. The samples differ in their mobilities and in the way the 2DES is confined. The sample used to obtain the spectrum Fig. 2(a), with peak frequency $f_{pk} = 95$ MHz, is a quantum well of width $w = 65$ nm and mobility $\mu \approx 10^7\ \mathrm{cm}^2/\mathrm{V\,s}$. The $f_{pk} = 7$ GHz spectrum of Fig. 2(b) is from a $w = 15$ nm well with $\mu \approx 10^7\ \mathrm{cm}^2/\mathrm{V\,s}$, and the $f_{pk} = 1.6$ GHz spectrum in Fig. 2(c) from a single heterojunction with $\mu \approx 4 \times 10^6\ \mathrm{cm}^2/\mathrm{V\,s}$. The increase of f_{pk} with mobility, as expected for a pinning mode, is evident.

Fertig [13] considered various types of disorder present in heterostructures, and found that interface roughness disorder gave f_{pk} of the right order of magnitude for pinning mode resonances. The large increase of f_{pk} on decreasing quantum well width, seen on comparing the $f_{pk} = 95$ MHz and 7 GHz spectra in Fig. 2(a) and (b), is qualitatively consistent with pinning due to interface roughness. Interface roughness is known to result in lower mobility [14] as well width is decreased.

In the insulators that exhibit the resonance, the strongest indication that the insulation is due to pinning of a solid rather than to individual particle localization can be obtained by comparing hf_{pk}/k_B with the temperature range in which the resonance appears. For example, the resonance in Fig. 2(a) has $hf_{pk}/k_B \approx 4.5$ mK, but was observed at 75 mK, and for that n and B the resonance remains observable up to about 230 mK. The resonance cannot be characteristic of traps with individual electrons, since these would be ionized at the measuring temperature. For all the resonances we have studied, including those obtained in higher Landau levels (in the IQHE and bubble phases, to be covered in the next section) the temperature above which the resonance disappears exceeds hf_{pk}/k_B by a factor of least two, and by much larger factors particularly for higher participating carrier density and lower disorder.

Correlation lengths of Wigner crystalline order [15] can be estimated from f_{pk}. Though of interest, such estimates are necessarily indirect and model dependent. The models specify the nature of the correlation lengths that can be calculated. In particular, recent theories [13,16,17] take the length that can

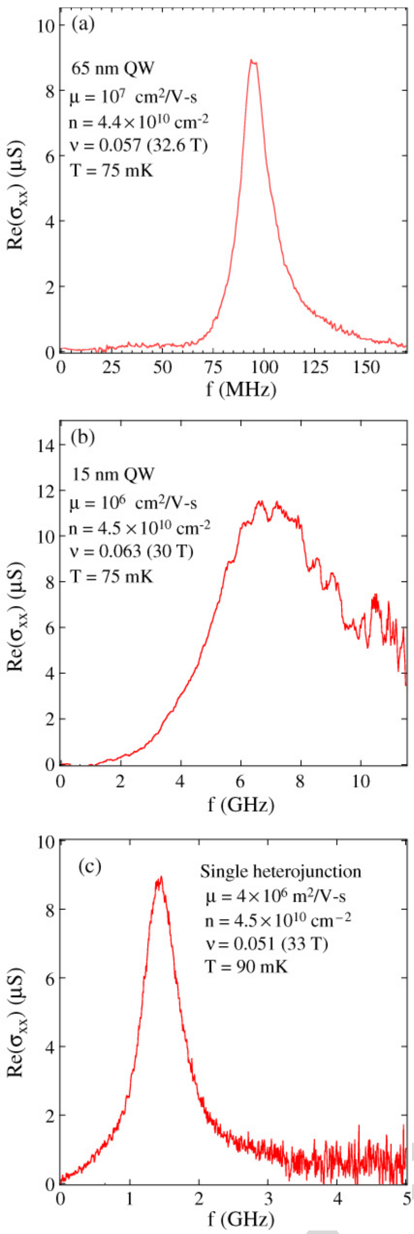

Fig. 2. Pinning mode resonances from two quantum well (QW) samples (a) and (b), and one single heterojunction (c), having nearly the same density and at nearly the same filling factor. Spectra are presented as the real part of diagonal conductivity, $\mathrm{Re}(\sigma_{xx})$ vs frequency f.

be calculated from f_{pk} to be the Larkin length (here denoted L), defined as the length over which deviations from crystal lattice positions reach the correlation length (here denoted ξ) of the effective disorder, which is determined from the disorder potential and the electron wavefunction.

The first step in the modeling is to use a simple "oscillator model" [18] to remove the hybridization of pinning and cyclotron modes. In high B, a piece of solid moving in a parabolic potential has two modes. The higher mode frequency is greater than the cyclotron frequency ω_c, and is far above the frequency range of interest here. ω_-, the frequency of the lower mode, is ω_0^2/ω_c, for $\omega_0 \ll \omega_c$, where ω_0 is the oscillation frequency in the same potential for $B = 0$. Identifying ω_- in this oscillator model as $2\pi f_{pk}$, we have $\omega_0 = (2\pi f_{pk}\omega_c)^{1/2}$. Next, L is found by matching [13,16–19] the pinning energy (from f_{pk} via ω_0) to the crystal shear deformation energy, which is estimated using the classical shear modulus $\mu_{t,cl} = 0.245e^2n^{3/2}/4\pi\epsilon_0\epsilon$ [20], where $\epsilon = 12.8$ is the GaAs dielectric constant. Taking $\omega_0 = c_t 2\pi/L$, where c_t is the $B = 0$ transverse phonon propagation velocity, $c_t = (\mu_t/nm^*)^{1/2}$, gives $L = (2\pi\mu_{t,cl}/neBf_{pk})^{1/2}$. From this formula, the $f_{pk} \approx$ 95 MHz and 7 GHz spectra in Fig. 2(a) and (b) respectively give L/a of 21 and 2.6, where a is the lattice constant of the crystal.

The Larkin length scale, which measures displacements from crystal positions against the effective disorder correlation length ξ, is to be contrasted with another crystalline order correlation length L_a, over which deviation from crystal lattice positions reaches the lattice constant a. L_a was used to characterize the pinned crystal in earlier theoretical work [18], which modeled the Wigner crystal with sinusoidal wavefunctions. Hence for a given pinned lattice, both L_a and L can be defined. In the case of interface roughness disorder, $\xi \ll a$ is a likely possibility, with interfacial features of 5 to 7 nm lateral size estimated in Ref. [14]. Statistical models [16] of the static response of the lattice to disorder relate the two length scales as $L_a/L \sim (a/\xi)^\beta$, with $\beta \sim 3$. Intriguingly, if ξ is less than a, even by a factor of a few times, Larkin lengths calculated from f_{pk} as in the previous paragraph would result in $L_a \gg a$, implying – indirectly and subject to the assumed models – highly ordered electron arrangements.

4. Evidence for new electron solid phases

This section will briefly highlight some recent results, in which pinning modes studied by microwave and rf spectroscopy have played a crucial role.

4.1. Evidence for distinct low ν insulating phases

In extremely low disorder samples, which exhibit the lowest resonance f_{pk}'s within the LNI, and hence are the most weakly pinned samples available, the spectrum evolves with ν quite differently than in other samples. Fig. 3 shows development of the spectra with decreasing ν, for a sample from the same $\mu \approx 10^7$ cm^2/V s wafer used to obtain Fig. 2(a), at density $n \approx 5.1 \times 10^{10}$ cm^{-2}. The sample exhibits a well-developed resonance in the reentrant range of insulator found [3] in high quality samples for ν between 2/9 and the 1/5 FQHE. The resonance reappears on the low ν side of the 1/5 FQHE. For ν below the range of the 1/5 FQHE, the evolution of the spectrum suggests a transition from one phase to another, since it exhibits, as ν is decreased, a crossover of intensity from the peak marked A in Fig. 3 to that marked B. Since there are insulators with resonances on both sides of such a transition, both the phases involved would be pinned electron solids. The crossover occurs roughly for ν between 0.18 and 0.12, where the spectrum appears to be complicated, with some additional peaks appearing. At elevated temperatures, this additional structure of the transition range disappears, and there is a simple crossover from peak A to peak B; at still higher temperatures, the resonances disappear.

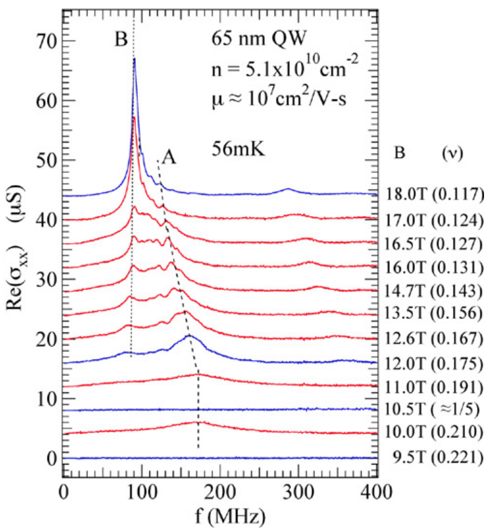

Fig. 3. Spectra, diagonal conductivity, $\mathrm{Re}(\sigma_{xx})$ vs frequency f, at several magnetic fields, B (Landau filling factors ν), from the same wafer used to obtain Fig. 2 for density $n \approx 5.1 \times 10^{10}$ cm^{-2}. Successive traces each offset by 4 μS, for clarity. A crossover of intensity between resonance "A" and resonance "B" is evident as the field increases. Fig. 2(a) presented peak B.

A number of observations support the picture of a phase transition. First, the crossover of intensity, complete with small additional features, is seen in just the same ν range in another extremely low disorder wafer, with $n \approx 10^{11}$ cm^{-2}, consistent with ν determining the transition point. At that density, about twice that of the sample of Fig. 3, the magnetic fields at which the crossover occurs are about twice those in Fig. 3. This means it is unlikely the crossover can be explained as interplay of the magnetic length $l_B = (\hbar/eB)^{1/2}$ and a disorder correlation length. Secondly, studies of samples with different CPW slot widths W indicate that there is a qualitative difference in the properties of the two phases. For peak A, f_{pk}, which is 160 MHz for $W = 30$ μm, is reduced to 125 MHz for $W = 60$ μm, while for peak B either W results in a 95 MHz f_{pk}. Only in the phase producing peak A, the W dependence is an indication that the mode can propagate over a length W, and that wave vector dependence of the 2DES response is being accessed at the small wave vector $\sim \pi/W$.

Theories have indicated a number of possibilities for identification of the solid phases. One is that composite fermions, exotic particles that can be regarded as containing an even number of flux with an electron, and which explain many aspects of the FQHE, may Wigner crystallize. Such composite fermion Wigner crystals are classed by the number of flux per electron, and theory has predicted [21–23] that a series of such crystals are the ground states within the LNI. Another approach appears in a recent theory [24] of disorder-free 2DES, which indicates that 2DES cannot undergo a direct phase transition from uniform liquid to Wigner crystal, but must pass through a series of intermediate phases. These intermediate phases can be thought of as composed of small regions of liquid and solid arranged in an ordered pattern, and appear instead of macroscopic phase coexistence, due to negative interface energy between the liquid and solid. Disorder-driven clustering of one phase within the other near a phase transition has also been predicted theoretically [25].

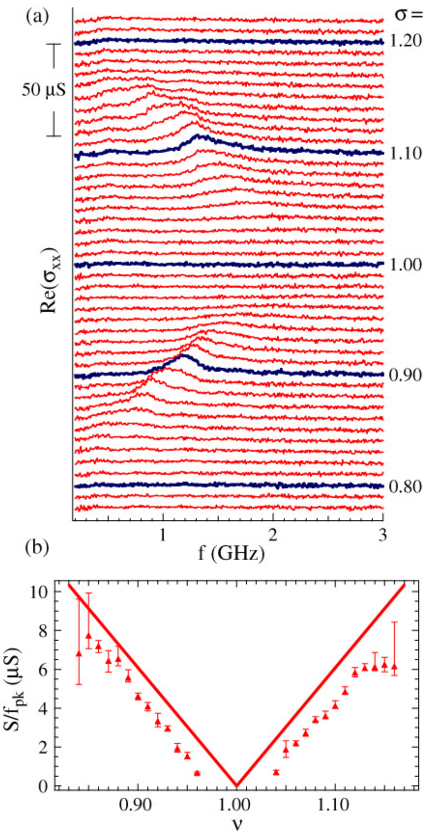

Fig. 4. (a) Spectra, $\mathrm{Re}(\sigma_{xx})$ vs frequency, f, near Landau filling $\nu = 1$, which show the development of a resonance within the IQHE. (b) S, the integrated $\mathrm{Re}(\sigma_{xx})$ vs f, divided by the peak frequency, vs ν for the data in panel (a). The solid lines show the S/f_{pk} expected from the sum rule, as discussed in the text, and the points are experimental data.

4.2. Role of electron solids in the IQHE

A recent achievement of microwave spectroscopy is finding evidence for electron solids in low disorder samples even for ν greater than or nearly equal to one, far above the range of the LNI. These solids of higher Landau levels can be regarded as made of electrons or holes from the "top" (partially occupied) Landau level along with one or more completely filled and nearly inert Landau levels. The signature of the IQHE in dc is quantization of the Hall conductivity to e^2/Jh, where J is an integer, accompanied by vanishing diagonal conductivity. The Hall conductivity is quantized due to the filled Landau levels. The experiments [7,8,11,12] reviewed in this subsection and the next indicate that there are conditions when the vanishing of the diagonal conductivity is due to the pinning of an electron solid composed of carriers from the top, partially filled Landau level.

In low disorder samples we have found that spectra taken within IQHE minima of dc σ_{xx} – but not precisely at integer filling – show striking resonances. A series of spectra taken within the range of the IQHE around $\nu = 1$ appears in Fig. 4. The data are from a 30 nm quantum well, with typical density about 3×10^{11} cm^{-2}/V s, and ultrahigh mobility

2.4×10^7 cm^2/V s. Resonances appear in bands on either side of $\nu = 1$, and increase in f_{pk} as the integer filling is approached. The same sample [8,10] has also shown resonances in the IQHE around integer fillings $J = 1, 2, 3$ and 4. We have also seen the resonances within the $J = 1, 2$ and 3 IQHE ranges in a sample with much lower density, $n \approx 10^{11}$ cm^{-2}, and high mobility $\mu \approx 10^7$ cm^2/V s.

The resonances on either side of integer filling J are interpreted as due to pinning modes of a Wigner crystal of "top" (partly occupied) Landau level particles, ($\nu > J$) or holes ($\nu < J$); we refer to this solid as the integer quantum Hall Wigner crystal (IQHWC). The density of carriers available to form the IQHWC in the integer J IQHE is $n^* = n\nu^*/\nu$, where $\nu^* = |\nu - J|$ is the partial filling of the relevant Landau level, hence the vanishing of the resonance just at $\nu = J$. The increase of f_{pk} as ν approaches J is understood as an effect of the reduction of n^*, analogous to that seen [15,26] in lowest Landau level Wigner solids on reducing overall density by applying a backgate bias. In both cases, the reduction of the crystal density has the effect of reducing the electron–electron interaction, which causes the electrons to adjust their positions to better minimize the electron–disorder interaction energy. When this happens the electrons are more strongly pinned, increasing f_{pk}.

The spectra in Fig. 4(a) were analyzed to obtain Fig. 4(b), which shows S/f_{pk} vs ν, where S is the integrated Re(σ_{xx}) vs f. S/f_{pk} is plotted because it is proportional to the density of carriers participating in the resonance according to a sum rule originally due to Fukuyama and Lee [18], $S/f_{pk} = n^* e\pi/2B = \nu^* e^2 \pi/2h$. The lines on the figure show S/f_{pk} calculated from the sum rule and the known n^*, and there is reasonable agreement between the predicted and measured S/f_{pk}.

Around the $J = 1$ IQHE only, there is evidence, particularly from NMR [27], that skyrmions, charge carrying spin textures composed of a number of spins, can be the favored excitations. At low temperature, the skyrmions would then crystallize [28–30] within the $J = 1$ IQHE. In both samples that exhibit resonances in the IQHEs, these resonances appear to be quite similar for $J = 1$ and for $J > 1$, though skyrmions are expected only for $J = 1$. Hence if a skyrmion crystal is present in the $J = 1$ IQHE, it gives a resonance that is at least superficially similar to that of the ordinary IQHWC lattice of single electrons or holes. It may be that the smooth, large scale spin modulation typical of skyrmions may apply to carriers arranged on a Wigner lattice with little effect on the pinning.

4.3. Bubble phases in excited Landau levels

In higher Landau levels, the top Landau level carriers can form a variety of electron solids depending on ν^*, according to theories [31–34] and dc transport experiments [35,36] on extremely low disorder 2DES. As ν^* increases above the range of the IQHWC, the top Landau level electrons form one or more "bubble phases", lattices of clusters (bubbles) of multiple electron guiding centers. In dc transport experiments, the ν regions roughly where the bubble phases had been predicted exhibit the [35,36] "reentrant integer quantum Hall effect" (RIQHE), in which diagonal resistance vanishes and Hall resistance is quantized at the same value as a neighboring IQHE. In a range of ν^* around 1/2, theories [31–34] had predicted a striped, unidirectional charge density wave phase, and anisotropic dc transport was found in experiments [35,36].

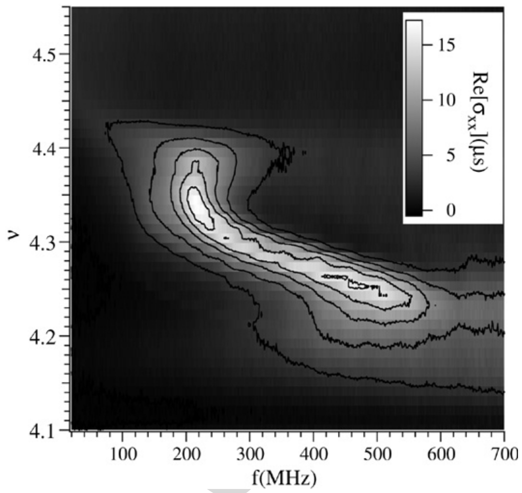

Fig. 5. Grayscale image of spectra, Re(σ_{xx}) vs frequency, f, in the bubble phase. Contour lines at intervals of 1 μS (starting at 3 μS) are superimposed.

The bubble phases exhibit striking rf resonances. Fig. 5 shows the development of these resonances with ν. The data shown in this figure and in Fig. 6 below are from the same wafer used to obtain the data in Fig. 4. A theory [37] describing the evolution of the peak with ν^* is qualitatively consistent with the data, and the ν^* range of occurrence of the resonance is in good agreement with the range predicted for the bubble phase [32,33]. As with the IQHWC resonance, f_{pk} decreases as ν^* increases, and S/f_{pk} is in reasonable agreement with that expected from n^*. We have observed bubble phase resonances at ν as high as $\approx 8 + 3/4$.

At low ν^* there is a transition from the bubble phase to the IQHWC. Fig. 6 shows the development of spectra in a narrow range of ν at that transition. Two resonances are present and are identified from their evolution with ν as being from the IQHWC or the bubble phase. In agreement with theories [32,33,38] the transition point (at which the resonances have equal S/f_{pk}) is estimated at $\nu = 4.22 \pm 0.02$. The simultaneous presence of the two resonances is good evidence of phase coexistence, consistent with (albeit not establishing) a first order transition, as theories [33,34] predicted. Consistent with the data as well would be ordered intermediate phases [24,34] or disorder-driven clustering [25] of one phase within the other; these possibilities were mentioned in Section 4.1, in conjunction with the transition from fractional quantum Hall liquid to electron solid.

S/f_{pk} of the bubble phase resonance is related to the participating charge density just as for the IQHWC resonance, so does not give a particular signature of the charge of the bubbles. However, for the same ν^*, and the same disorder potential, a theory [32] has predicted the bubble phase f_{pk} to be smaller by a factor of $M^{1/2}$, where M is the number of carriers per bubble, because of the differing shear moduli of the IQHWC and bubble phases. The f_{pk} ratio of the bubble

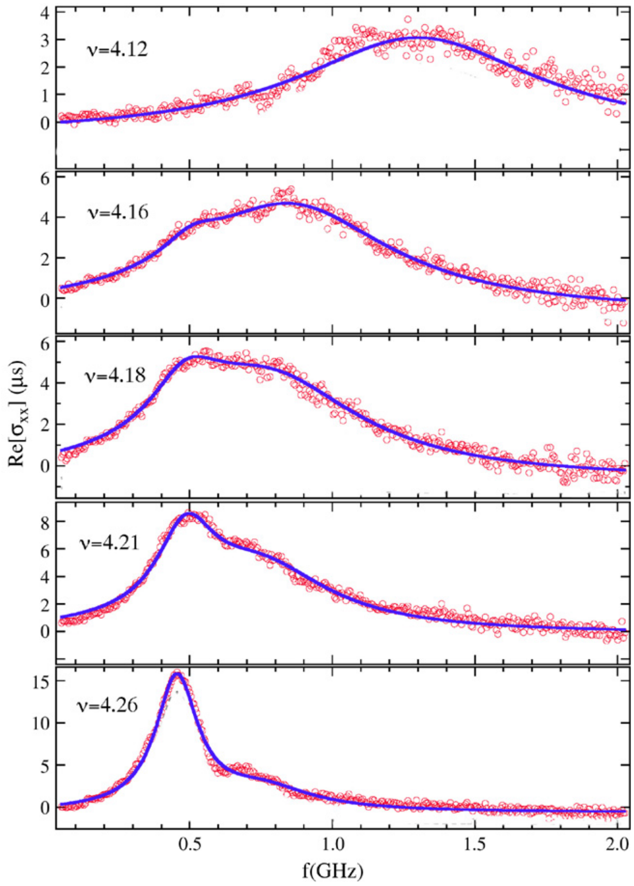

Fig. 6. Spectra, Re(σ_{xx}) vs f at several filling factors in the range of the transition between IQHWC and bubble phases [10]. The circles are data, and the blue lines are fits to a pair of lorentzians. Lines from the IQHWC (higher f_{pk}) and from the bubble phase (lower f_{pk}) are both present. The intensity crosses over from IQHWC line to the bubble phase line as ν is increased. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

phase and IQHWC peaks is consistent in this picture with an $M = 2$ bubble phase. At high ν^* the bubble phase undergoes a transition to an anisotropic stripe phase that was seen in dc transport [35,36]. Recent data shows an anisotropic resonant rf response in the stripe regime, and will be discussed in a future publication [39].

RIQHEs have been seen in dc transport in the first excited Landau level [40,41] in pairs of bands on either side of the 7/2 and 5/2 FQHE states. These RIQHE states are particularly challenging to study owing to the low temperatures required. Theories [32,42] have predicted bubble phases in these regions as well. Consistent with this interpretation, microwave study [12] has revealed a resonance in one band, centered around $\nu = 2.58$; the absence of resonances in the other bands is likely due to the temperature of about 35 mK of that microwave study, which is well above those used in the dc transport work.

5. Conclusion

Electron solid phases in 2DES at high B exhibit a generic pinning mode resonance in the rf or microwave range. Spectroscopic studies of extremely high quality 2DES have revealed a rich variety of electron solids, including multiple phases within the LNI, Wigner crystals whose pinning is important in producing the finite width of IQHE plateaus, and the bubble phases. The pinning mode will continue to be a valuable phenomenon for identification and in-depth study of the electron solids.

Acknowledgements

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