A Monte Carlo Approach to Uncertainty Analysis in Cosmogenic Nuclide Dating

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The Model

- In this talk we'll consider a model for cosmogenic production of ^{36}Cl from Gosse and Phillips (2001).
- The model incorporates production of ^{36}Cl by spallation reactions, absorption of thermal and epithermal neutrons, and muogenic production.
- The model depends on a number of sample specific parameters, including bulk density, water content, sample thickness, chemical composition, surface erosion, and ^{36}Cl ratio.
- The model depends on a number of physical parameters, including decay rates, production rates, and attenuation rates.
- The model incorporates scaling factors for elevation and latitude and terrain shielding.

The Model

- Given nominal values of the sample and physical parameters and a time period t, it's relatively simple to compute the cosmogenic production of ^{36}Cl.
- We can also compute the amount of nucleogenic ^{36}Cl.
- We can then find the age that matches the measured ^{36}Cl to the predicted ^{36}Cl.
- In the Monte Carlo procedure, we generate many random values of the sample and physical parameters and then compute the age of the sample for each set of parameters.
- We then compute the mean, standard deviation, and histogram of the resulting ages.
- We'll present results for a particular sample collected by Phillips at Bishop Creek.

Extensions

- An obvious extension is to include other cosmogenic nuclides, such as ^{3}He, ^{10}Be, ^{14}C, ^{21}Ne, and ^{26}Al.
- Another obvious extension is to include time varying production rates in the model.
- Are there other important aspects of the problem that are not included in our current model?

Sample Parameters

Parameter	Nominal	Uncertainty	Description
ϵ	5 mm/ka	$\pm 25\%$	Erosion Rate
θ	0.10	$\pm 30\%$	Water Content
ρ_β	1.62 g/cm^3	$\pm 20\%$	Bulk Density
l_s	3.0 cm	$\pm 10\%$	Sample Thickness
R_{36Cl}	1.149e-12	$\pm 5\%$	^{36}Cl/Cl Ratio
C_{Cl}	46 ppm	$\pm 5\%$	Cl Concentration
C_K	3.65% wt	$\pm$ 0.1	K Concentration
C_{Ca}	2.90% wt	$\pm$ 0.1	Ca Concentration
C_U	4 ppm	$\pm 5\%$	U Concentration
C_{Th}	6 ppm	$\pm 5\%$	Th Concentration

Sample Parameters

- Are there any other sample specific parameters that should be included?
- What do you think of the uncertainties in the previous table?

Physical Parameters

Parameter	Nominal	Uncertainty	Description
λ_{36Cl}	2.3028e-6 /yr	$\pm 1\%$	Decay rate of ^{36}Cl
Λ_f	170 g/cm^2	$\pm 10\%$	attenuation length
Λ_μ	1500 g/cm^2	$\pm 10\%$	attenuation length
$P_{s,Ca}(0)$	4.4496e-21 /yr	$\pm 6.8\%$	spallation from Ca
$P_{s,K}(0)$	8.8982e-21 /yr	$\pm 6.8\%$	spallation from K
$P_f(0)$	626.0 atms/(g yr)	$\pm 6.8\%$	fast neutrons

Physical Parameters

- Other important parameters include A_i, ξ_i, $\sigma_{sc,i}$, $\sigma_{th,i}$, $I_{a,i}$, S_i, $Y^u_{n,i}$, $Y^{th}_{n,i}$, and K_m. We use the values from Table 2 of Gosse and Phillips (2001). Uncertainties in these parameters do need to be considered.
- There is also a collection of parameters related to the atmosphere, including the average atomic weight, etc. We've used values from Gosse and Phillips.
- Are there other important physical parameters that need to be considered?
- What do you think of the uncertainties in the previous table?

Scale Factors

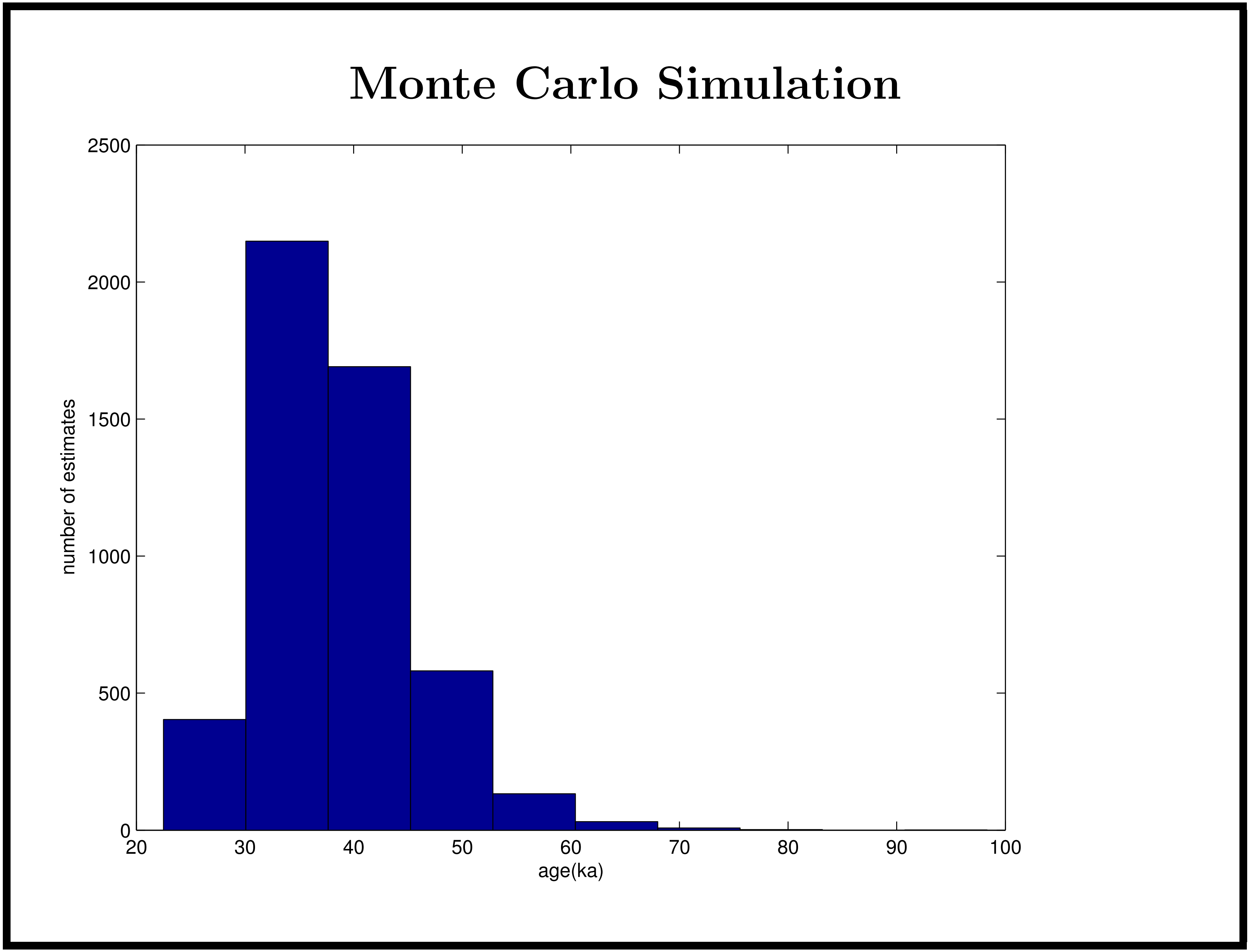
Parameter	Nominal	Uncertainty	Description
S_{el}	3.32	$\pm 15\%$	Elevation/Latitude
S_{μ}	1.96	$\pm 15\%$	Elevation/Latitude
S_T	0.99	$\pm 10\%$ of $(1 - S_T)$	Terrain Shielding
S_L	1.00	?????	Neutron Leakage

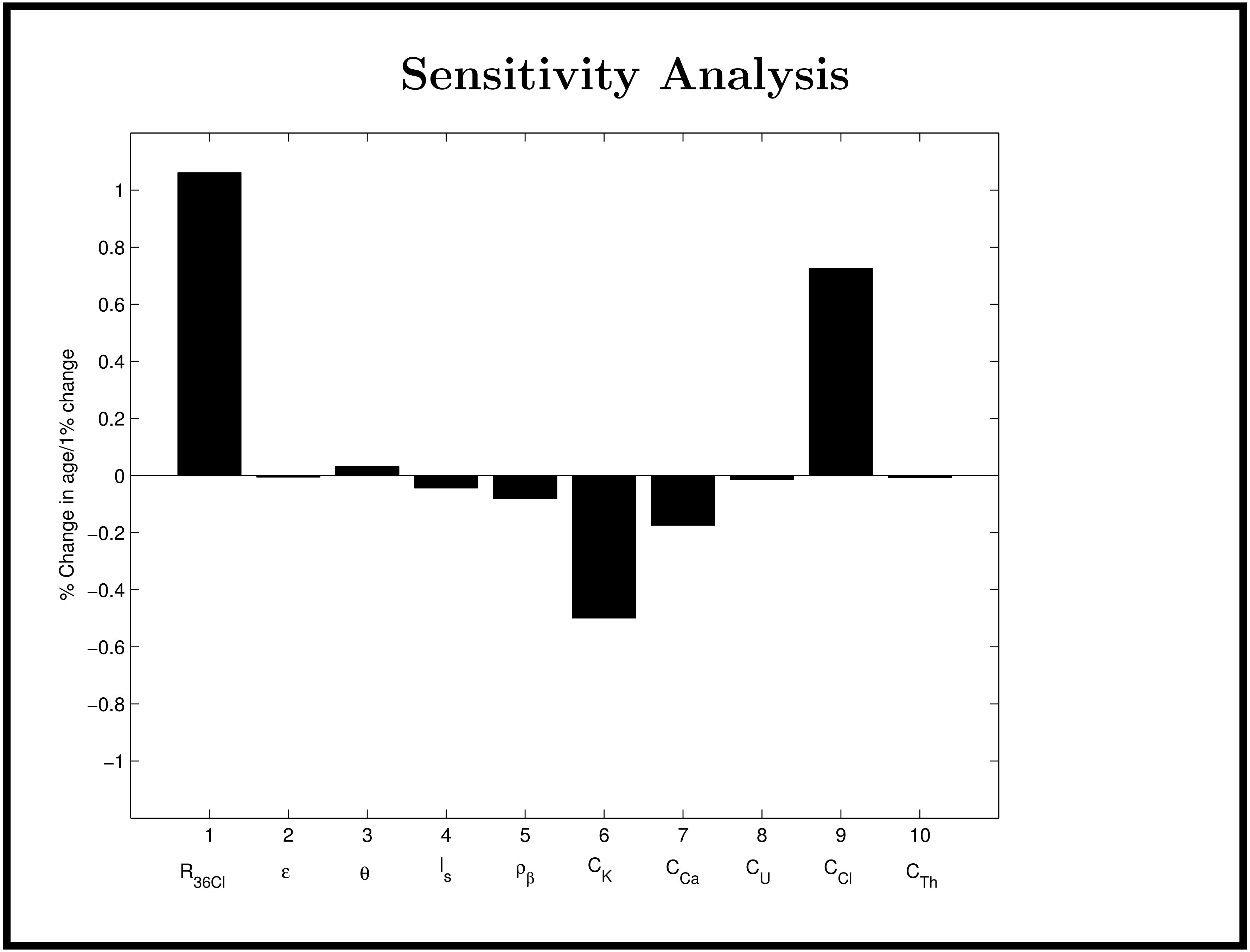
Scale Factors

- For the elevation and latitude factor, S_{el}, we've used the Lal (1991) model. Lal suggests an uncertainty of $\pm$ 10-20%, but this isn't firmly quantified.
- What other models for S_{el} should we include? What about S_{μ}?
- For terrain shielding, the uncertainty in actual measurements of the slope and other physical features of the site will contribute to the uncertainty in S_T in a way that could be quantified.
- I'm not aware of any attempts to quantify uncertainty in estimates of S_L.
- If you had to subjectively assign uncertainties to these parameters today, what would they be?

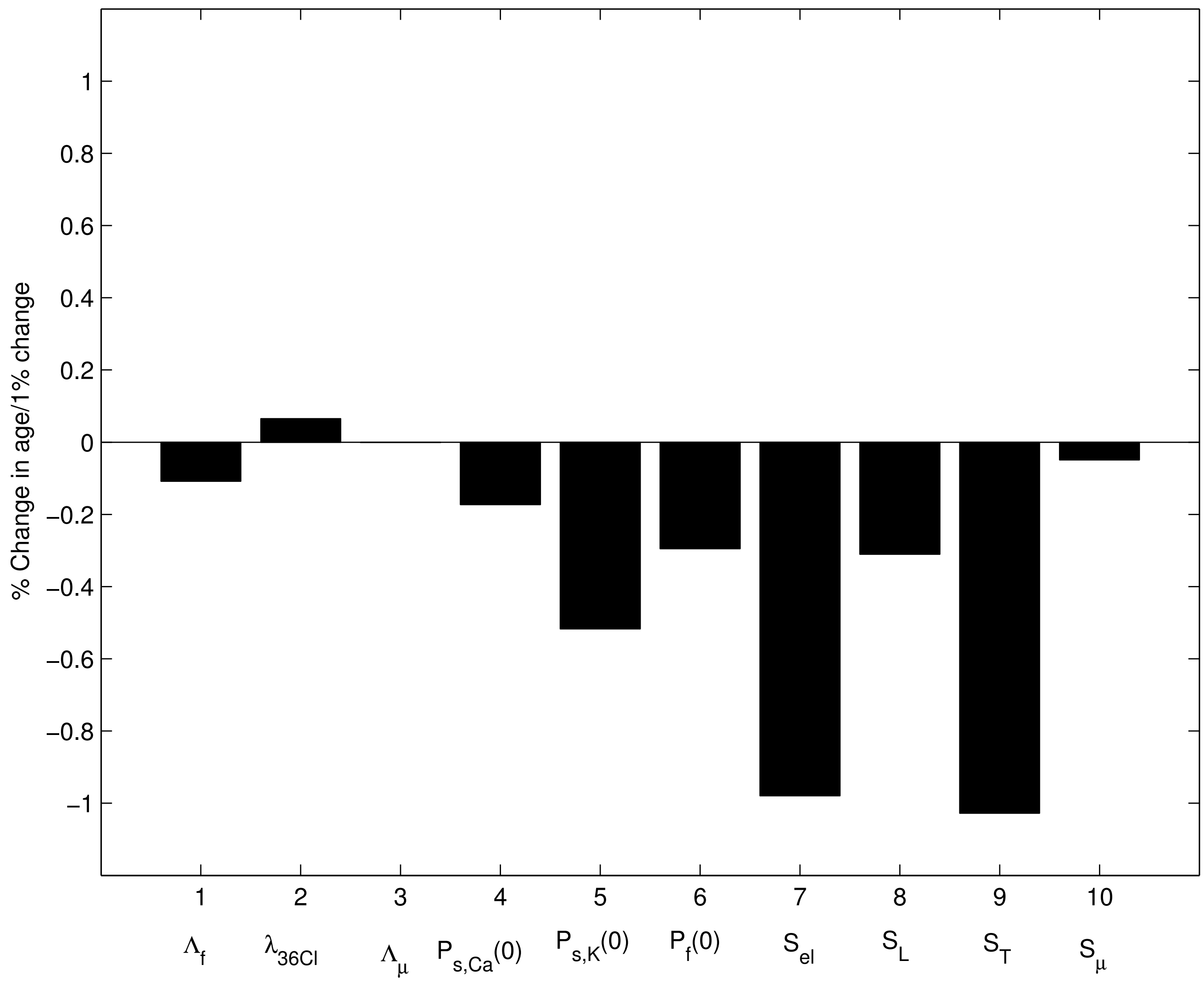
Monte Carlo Simulation

- Using the nominal values of the parameters, we obtained an age of 37.2 ka.
- 5,000 random sets of parameters were generated, using independent and normal distributions centered around the nominal values.
- Computing ages for the 5,000 random sets of parameters took about 20 seconds on a PC.
- The mean estimate of the age was 38.5 ka.
- The standard deviation of the ages was 7.0 ka.
- This is an uncertainty of $\pm 18\%$.
- If we treat the scale factors as exact, this drops to $\pm 8.5\%$.
- The following histogram shows the distribution of the age estimates.


Monte Carlo Simulation
number of estimates
0
500
1000
1500
2000
2500
20
30
40
50
60
70
80
90
100
age(ka)


Sensitivity Analysis
% Change in age/1% change
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
1
2
3
4
5
6
7
8
9
10
R_{36Cl}
ε
θ
I_s
ρ_β
C_K
C_{Ca}
C_U
C_{Cl}
C_{Th}

Sensitivity Analysis



Discussion

- The estimated age is relatively insensitive to many of the parameters.
- The estimated age is sensitive to the concentrations of Ca, K, and Cl, as well as to the ^{36}Cl ratio.
- The estimated age is sensitive to the production rates $P_{s,Ca}(0)$, $P_{s,K}(0)$, and $P_f(0)$.
- The estimated age is also very sensitive to the scaling factors S_L, S_{el} and S_T.

Future Work

- The current implementation uses hard coded values of the parameters. This needs to be replaced by a graphical user interface that allows the user to interactively assign values and uncertainties to the parameters.
- The code needs to include support for "spike calculations" in cases where a Cl spike was added to the sample.
- Alternate models for S_{el} and S_{mu} need to be supported.
- Terrain shielding computations with associated uncertainties need to be included in the code.
- Ultimately, this kind of uncertainty computation should appear in the CRONUS project's web based calculator.

Future Work

- The basic physical parameters should not vary from sample to sample or location to location. By examining several samples from each of several locations, we should be able to better estimate these parameters and factor out the location dependent and sample specific effects.
- By considering multiple samples from the same location, we can eliminate variations in the scaling factors and reduce the problem to uncertainties in the sample specific and basic physical parameters. This provides a way to validate the Monte Carlo simulations.
- Improving our estimates of the elevation and latitude scale factors will require experiments with samples from a number of locations with variations in altitude and latitude.

Conclusions

- We have demonstrated a Monte Carlo procedure for ^{36}Cl dating that produces an uncertainty together with the estimated age.
- The greatest sensitivity in the estimated age is to the concentrations (C_{Cl}, C_{Ca}, C_K), production rates ($P_{s,Ca}(0)$, $P_{s,K}(0)$, $P_f(0)$), and scaling factors (S_L, S_{el}, S_T.)
- We've been fairly conservative in putting uncertainties on the parameters. This is more a reflection of our lack of knowledge (epistemic uncertainty) than of random variation. This is particularly true of the scaling factors.
- Reducing the uncertainty in the age estimate to $\pm 5\%$ will require substantial reductions in the uncertainties of all of the parameters mentioned here.