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Effect of a Generalized Particle Momentum Distribution on Plasma Nuclear Fusion Rates

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We investigate the effect of a generalized particle momentum distribution derived by Galitskii and Yakimets (GY) on nuclear reaction rates in plasma. We derive an approximate semi-analytical formula for nuclear fusion reaction rate between nuclei in a plasma (quantum plasma nuclear fusion; or QPNF). The QPNF formula is applied to calculate deuteron–deuteron fusion rate in a plasma, and the results are compared with the results calculated with the conventional Maxwell–Bolzmann velocity distribution. As an application, we investigate the deuteron–deuteron fusion rate for mobile deuterons in a deuterated metal/alloy. The calculated deuteron–deuteron fusion rates at low energies are enormously enhanced due to the modified tail of the GY's generalized momentum distribution. Our preliminary estimates indicate also that the deuteron–lithium (D + Li) fusion rate and the proton–lithium (p + Li) fusion rate in a metal/alloy at ambient temperatures are also substantially enhanced.
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As shown by Galitskii and Yakimets (GY)[1] the quantum energy indeterminacy due to interactions between particles in a plasma leads to a generalized momentum distribution which has a high-energy momentum distribution tail diminishing as an inverse eighth power of the momentum, instead of the conventional Maxwell–Boltzmann (MB) distribution tail decaying exponentially. GY's generalized momentum distribution has been used by Coraddu *et al.*[2,3] in an analysis of anomalous cross-sections for $D(d,p)^3H$ observed from the low-energy deuteron beam experiments.[4–8] In this paper, we describe the effect of GY's generalized momentum distribution on the nuclear fusion rates in a plasma. The calculated results for deuteron–deuteron fusion rates are compared with the results of the conventional calculation with MB distribution. We investigate other nuclear fusion rates for (D + D), (D + Li), and (p + Li) reactions in metals/alloys.

There have been some indirect experimental evidences supporting the inverse eighth power of the momentum for the high-energy momentum distribution tails predicted by GY.[1] Results of high transverse momentum of charged-hadron production in 400-GeV/c proton–proton and proton–deuteron collisions and 800-GeV/c proton–proton collisions indicate that the momentum distribution tails decay as the inverse eighth power of the transverse momentum.[9] Similar inverse eighth power behavior has been also observed in antiproton–proton collisions at $\sqrt{s} = 1.8\,\text{TeV}$.[10]

The enhancement of $D(d,p)^3H$ reaction cross-sections at low energies as observed from the deuteron beam experiments on deuterated metal targets[4–7] may be due to GY's generalized momentum distribution.[2,3] More recent kinematic measurements for the D(d, p)T reaction in metal targets with the deuteron beam with deuteron energies $\sim$10 keV indicate that deuterons in the metal target are mobile.[8] The extracted values of 40 ± 5 eV[8] for deuterons in the metal target (PdO) cannot be explained by the conventional MB distribution of deuterons at the near ambient temperatures.

We start with a modified version of GY's generalized distribution function given by

$$f(E,\boldsymbol{p}) = n(E)\delta_\gamma(E-\varepsilon_\mathrm{p}) \tag{1}$$

where $n(E)$ is MB, Fermi–Dirac (FD), or Bose–Einstein (BE) distribution, modified by the quantum broadening of the momentum–energy dispersion ration, $\delta_\gamma(E-\varepsilon_\mathrm{p})$, due to particle interactions. $\delta_\gamma(E-\varepsilon_\mathrm{p})$ is given by

$$\delta_\gamma(E-\varepsilon_\mathrm{p}) = \frac{\gamma(E,\boldsymbol{p})}{\pi[(E-\varepsilon_\mathrm{p}-\Delta(E,\boldsymbol{p}))^2+\gamma^2(E,\boldsymbol{p})]} \tag{2}$$

where $\varepsilon_\mathrm{p} = \mathrm{p}^2/2\mu$ is the kinetic energy in the center of mass coordinate of an interacting pair of particles, μ is the reduced mass, $\Delta(E,\boldsymbol{p})$ is the energy shift due to the interaction (screening energy, etc.), and $\gamma(E,\boldsymbol{p})$ is the line width of the momentum–energy dispersion due to collision. $\gamma(E,\boldsymbol{p}) \approx \hbar\rho_\mathrm{c}\sigma_\mathrm{c}\sqrt{2E/\mu}$ where ρ_c is the number density of Coulomb scattering centers (nuclei), $\sigma_\mathrm{c} = \pi(Z_i^\mathrm{e}Z_j^\mathrm{e}\mathrm{e}^2)^2/\varepsilon_\mathrm{p}^2$ is the Coulomb scattering cross section, and $Z_i^\mathrm{e}\mathrm{e}$ is an effective charge which depends on ε_p.

This distribution, eq. (2), reduces to the δ-function in the limit of $\Delta \to 0$ and $\gamma \to 0$,

$$\delta_\gamma(E-\varepsilon_\mathrm{p}) = \delta(E-\varepsilon_\mathrm{p}) \tag{3}$$

The nuclear fusion rate for two nuclei is given by

$$\langle\sigma v_\mathrm{rel}\rangle = \int d\varepsilon_\mathrm{p} v_\mathrm{rel}\sigma(E_\mathrm{cm})f(\boldsymbol{p}), \tag{4}$$

where

$$f(\boldsymbol{p}) \approx N\int_0^\infty dE n(E)\delta_\gamma(E-\varepsilon_\mathrm{p}), \tag{5}$$

and the normalization N is given by

$$\int d\varepsilon_\mathrm{p} f(\boldsymbol{p}) = 1. \tag{6}$$

For a high energy region, $\varepsilon_\mathrm{p} \gg kT$, γ, and Δ, we obtain approximately

$$f(\boldsymbol{p}) \approx \frac{N}{\varepsilon_\mathrm{p}^2}\int n(E)\gamma(E,\mathrm{p})dE = N\hbar\sqrt{\frac{8kT}{\pi\mu}}\rho_\mathrm{c}\frac{(Z_i^\mathrm{e}Z_j^\mathrm{e}\mathrm{e}^2)^2}{\varepsilon_\mathrm{p}^4} \propto \frac{1}{\mathrm{p}^8} \tag{7}$$

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as shown by GY.[1] This is to be compared with the other conventional cases, $f(\boldsymbol{p}) \propto e^{-\varepsilon_p/kT}$.

We now derive an approximate analytical formula for quantum plasma nuclear fusion (QPNF) in order to obtain order-of-magnitude estimate for the nuclear fusion rate. The total nuclear fusion rate, R_{ij}, per unit volume (cm^{-3}) and per unit time (s^{-1}) is obtained from an expansion of eq. (2) in which the first term is $\delta(E - \varepsilon_p)$. R_{ij} is approximately given by

$$R_{ij} \approx R_{ij}^{C} + R_{ij}^{Q} \tag{8}$$

where R_{ij}^{C} is the conventional fusion rate calculated with the MB distribution and R_{ij}^{Q} is the QPNF contribution from the second term of the expansion of eq. (2) and is given by

$$R_{ij}^{Q} = \frac{\rho_i \rho_j}{1+\delta_{ij}} \langle \sigma v_{rel} \rangle \approx \frac{N}{1+\delta_{ij}} \frac{(\hbar c)^3}{\mu c} \alpha^2 \left(\frac{8(6!)}{\sqrt{\pi}} \right) S_{ij}(0)(Z_i^e Z_j^e)^2 (kT)^{1/2} \frac{\rho_c \rho_i \rho_j}{E_G^{7/2}} \tag{9}$$

where E_G is the Gamow energy, $E_G = (2\pi\alpha Z_i Z_j)^2 \mu c^2/2$, ρ_i is the number density of nuclei, and $S_{ij}(0)$ is the S-factor at zero energy for a fusion reaction between i and j nuclei.

We note that the spectral function $\delta_\gamma(E - \varepsilon_p)$ [eq. (2)] introduced by GY[1] needs to be modified to satisfy the following energy weighted sum rule (for fermions),

$$\int dE E \delta_\gamma(E - \varepsilon_p) = \frac{p^2}{2m} - \int d^3p' \frac{4\pi e^2}{|\boldsymbol{p} - \boldsymbol{p}'|} f(p'), \tag{10}$$

where $f(p)$ is the distribution function, given by eq. (5). Since the integral on the left-hand side of eq. (10) is a divergent integral, the GY parameterization of $\gamma(E, \boldsymbol{p})$ in $\delta_\gamma(E - \varepsilon_p)$ needed to be corrected for large $E \gg kT$ to satisfy the above sum rule. However, eqs. (4)–(9) are still valid since our results for the distribution function $f(p)$ do not depend on $\gamma(E, \boldsymbol{p})$ in the region where $E \gg kT$ due to the presence of $n(E) \propto e^{-E/kT}$ in eq. (5).

In Fig. 1, the calculated fusion reactivities, $\langle \sigma v \rangle$, for (D + D) fusion are plotted as a function of temperature kT. Figure 1 shows that quantum effects are important at low temperatures even for the Debye–Hückel plasma, $\rho \gg (kT/e^2)^3$.

The mobility of protons and deuterons in Pd and other metals has been experimentally demonstrated.[11,12] However, other heavier nuclei (Li, B, etc.) are most likely to have much less mobility and most of them are stationary in metal/alloy lattices. Because of the deuteron mobility in metals, (D + D) fusion rates in metals were investigated using the MB velocity distribution for deuterons with a hope that the high-energy tail of the MB distribution may increase the (D + D) fusion rates in metals.[13] However, the calculated results for the (D + D) fusion rates with the MB distribution were found to be extremely small at ambient temperatures.[13]

For the case of (D + D) fusion reaction, order-of-magnitude estimates for R_{DD}^{Q} are shown as a function of the mobile deuteron density ρ in Table I calculated from eq. (9) with $S_{ij}(0) = 110$ keV-barn for both $D(d,p)^3H$ and $D(d,n)^3He$ reactions combined. The mobile deuteron density of $\rho = 6 \times 10^{22}\ cm^{-3}$ is probably an upper limit of the maximum density achievable in deuterated metals/alloys.

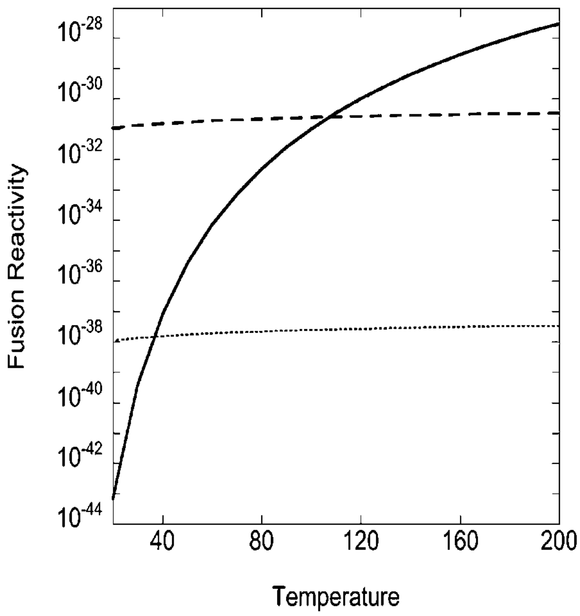

Fig. 1. Fusion reactivity, $\langle \sigma v \rangle$, in units of cm^3/s as a function of temperature kT in units of eV. The solid line, the dashed line, and the dotted line corresponds the results calculated using the MB distribution, the GY distribution with $\rho = 10^{22}\ cm^{-3}$, and the GY distribution with $\rho = 10^{15}\ cm^{-3}$, respectively.

Table I. Order-of-magnitude estimates for DD fusion rate, R_{DD}, in units of $cm^{-3}\ s^{-1}$. The particle (mobile deuterons) number density, ρ, is in units of $6 \times 10^{22}\ cm^{-3}$. $Z_i^e = Z_i = 1$, $Z_j^e = Z_j = 1$, $\rho_c = \rho_i = \rho_j = \rho$, and $kT = 0.02$ eV are assumed.

ρ ($6 \times 10^{22}\ cm^{-3}$)	R_{DD}^{Q} ($cm^{-3}\ s^{-1}$)	Power (W/cm^3)
10^{-4}	0.19×10^{3}	0.1×10^{-9}
10^{-3}	0.33×10^{6}	0.20×10^{-6}
10^{-2}	0.39×10^{9}	0.23×10^{-3}
10^{-1}	0.42×10^{12}	0.25
1	0.43×10^{15}	0.25×10^{3}

To grasp the significance of the results for R_{DD}^{Q} in Table I, we can compare the DD fusion rate, R_{DD}^{C}, calculated with the MB distribution.[13] The calculated values[13] of R_{DD}^{C} with $\rho = 6 \times 10^{22}\ cm^{-3}$ are $\sim 10^{-73}\ cm^{-3}\ s^{-1}$ with the electron screening energy of $E_s = e^2/a_o \approx 27.17$ eV (a_o is the Bohr radius) and $\sim 10^{-31}\ cm^{-3}\ s^{-1}$ with $E_s = 4e^2/a_o \approx 108.7$ eV, compared with $R_{DD} \approx 0.43 \times 10^{15}\ cm^{-3}\ s^{-1}$ given in Table I.

Recent results of cross-section measurements from deuron beam experiments with metal targets by Kasagi *et al.*,[4,5] Rolfs *et al.*,[6,7] and Kasagi[8] indicate that the modified high-energy distribution tail may be occurring. Recently, Rolfs *et al.*[6,7] have investigated the electron screening effect in the $D(d,p)^3H$ reaction with a low energy (center-of-mass energies between ~4 and ~15 keV) deuteron beam on deuterated targets (32 metals, 3 insulators, 3 semiconductors, 3 groups 3 and 4 elements, 13 lanthanides). They have found that all deuterated metals yield large extracted values of the screening energy U_e ranging from $U_e = 180 \pm 40$ eV (Be) to $U_e = 800 \pm 90$ eV (Pd), while all deuterated non-metal targets yield smaller values of $U_e \leq 80$ eV. If we interpret the anomalous values of U_e for metal targets in terms of the modified tail ($\propto p^{-8}$), wide variations of 32 different values of U_e ranging from $U_e = 180 \pm 40\,\text{eV}$ (Be) to $U_e = 800 \pm 90\,\text{eV}$ (Pd) may be correlated with the number density of mobile deuterons in metal targets which in turn may be related to deuteron loading ratios, deuteron diffusion coefficients in metals, external stimulations, etc., such as applied electromagnetic fields.

R_{ij}^{Q} given by eq. (9) covers (p + D), (D + Li), (p + Li), and (p + B) reactions in addition to (D + D) reactions. Given similar conditions as for the DD fusion, the fusion rate for the D^6Li fusion reaction is estimated to be ~10% of the DD fusion rate, $R_{D^6Li}^{Q} \approx 0.1 R_{DD}^{Q}$.

The DD fusion reactions create neutrons and radioactive tritium. A candidate aneutronic and non-radioactive nuclear fusion reaction is $^6\text{Li}(\text{p}, {}^3\text{He})^4\text{He}$ with Q = 4.02 MeV. Given the same conditions as for the DD fusion, the fusion rate for the p^6Li fusion is estimated to be ~30% of the DD fusion rate, $R_{p^6Li}^{Q} \approx 0.3 R_{DD}^{Q}$. The other aneutronic and non-radioactive reaction, $^7\text{Li}(\text{p}, {}^4\text{He})^4\text{He}$, has a much lower fusion rate, $R_{p^7Li}^{Q} \approx 0.5 \times 10^{-2} R_{DD}^{Q}$. We plan to investigate nuclear fusion rates for (p + ^{6}Li) and (p + ^{11}B) reactions in hydrogenated metals/alloys. We plan also to calculate the fusion rate for $^{11}\text{B}(\text{p}, \alpha)2^4\text{He}$ (Q = 8.69 MeV). If we could achieve sufficiently high fusion rates for $^6\text{Li}(\text{p}, {}^3\text{He})^4\text{He}$ and $^{11}\text{B}(\text{p}, \alpha)2^4\text{He}$ fusion reactions in hydrogenated metals/alloys, they could become attractive alternative methods for generating clean nuclear fusion energy.

In summary we have investigated the quantum corrections to the equilibrium rate of nuclear fusion rate in a plasma. Using a generalized particle momentum distribution given by Galitskii and Yakimets,[1)] we have constructed an approximate semi-analytical formula for the nuclear fusion reaction rate between nuclei in a plasma. The calculated results show that the quantum corrections due to QPNF lead to a dramatic increase of the fusion rate for mobile deutrons in deuterated metal/alloy at ambient temperatures. Our preliminary estimates indicate also that the deuteron–lithium (D + Li) fusion rate and the proton–lithium (p + Li) fusion rate in a metal/alloy at ambient temperatures are also substantially enhanced due to the modified high-energy tail ($\propto p^{-8}$) of the momentum distribution.

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