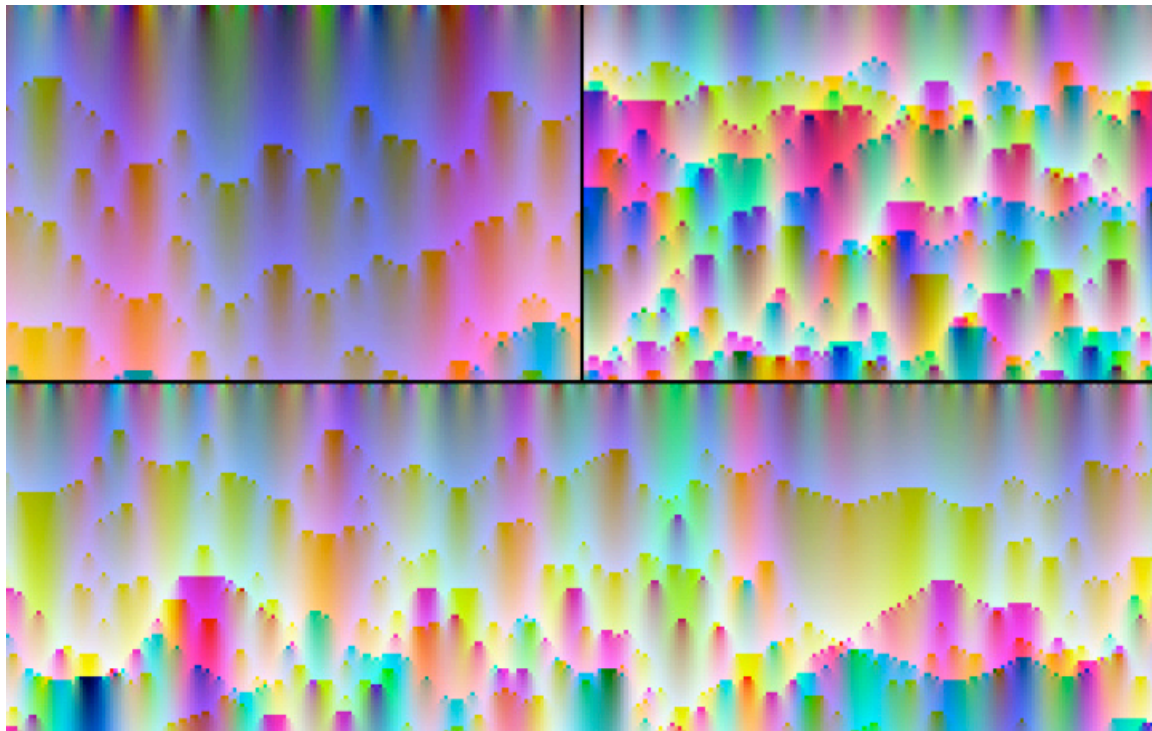


Introduction to Modern Dynamics

Chaos, Complex Networks and Warped Spaces



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