Heavy quarks diffusion in hydrodynamics
or
"How confident can we be in the Fokker-Planck coefficients we extract from / constrain with exp. data ?"

Heavy quarks production in heavy-ion collisions, W. Lafayette

P.B. Gossiaux
SUBATECH, UMR 6457
Université de Nantes, Ecole des Mines de Nantes, IN2P3/CNRS

With J. Aichelin, M. Bluhm, Th. Gousset, H. van Hees, R. Rapp and S. Vogel,

I. Understanding (partly) the present RHIC data on HQ E-loss; the Nantes viewpoint

II. Influence of the medium on our understanding

Based on

- *Heavy quarks thermalization in ultra relativistic heavy ion collisions: Elastic versus Radiative*, P.B. Gossiaux}, V. Guiho & J. Aichelin, Journal of Physics G **32** (2006) S359
- *Towards an understanding of the single electron data measured at the BNL Relativistic Heavy Ion Collider (RHIC)*, P.B. Gossiaux & J. Aichelin, Phys. Rev. C **78**, 014904 (2008); [arXiv:0802.2525]
- *Tomography of quark gluon plasma at energies available at the BNL Relativistic Heavy Ion Collider (RHIC) and the CERN Large Hadron Collider (LHC)*, P.B. Gossiaux, R. Bierkandt & J. Aichelin, Physical Review C **79** (2009) 044906; [arXiv:0901.0946]
- *Tomography of the Quark Gluon Plasma by Heavy Quarks*, P.-B. Gossiaux & J. Aichelin, J. Phys. G **36** (2009) 064028; [arXiv:0901.2462]
- *Energy Loss of Heavy Quarks in a QGP with a Running Coupling Constant Approach*, P.B. Gossiaux & J. Aichelin, Nucl. Phys. A **830** (2009), 203; [arXiv:0907.4329]
- *Competition of Heavy Quark Radiative and Collisional Energy Loss in Deconfined Matter*, P.B. Gossiaux, J. Aichelin, T. Gousset & V. Guiho, J. Phys. G: Nucl. Part. Phys. **37** (2010) 094019; [arXiv:1001.4166]
- *Gluon Radiation at small kT and Radiative Energy Loss of Heavy Quarks; I. The Bethe-Heitler Regime*, J. Aichelin, P.B. Gossiaux & Th Gousset (**In preparation)**

I. “Understanding” the RHIC HQ-data

What is the dominant E loss mechanism @ RHIC ?

What can we extract from experimental data ?

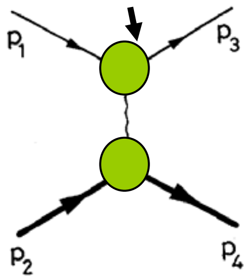
Collisional E loss : The Peshier – Gossiaux – Aichelin approach (2008)

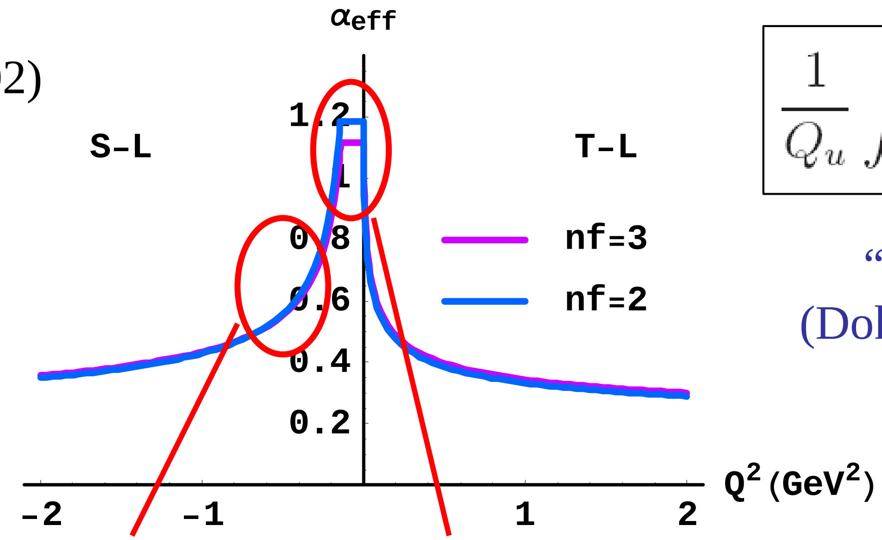
Motivation: Even a fast parton with the largest momentum P will undergo collisions with moderate q exchange and large $\alpha_s(Q^2)$. The running aspect of the coupling constant has been "forgotten/neglected" in most of approaches

Effective $\alpha_s(Q^2)$

(Dokshitzer 95, Brodsky 02)

$\alpha_{eff}(Q^2, T=0)$





$$\frac{1}{Q_u}\int_{|Q^2|\le Q_u^2} dQ\,\alpha_s(Q^2) \approx 0.5$$

"Universality constrain" (Dokshitzer 02) helps reducing uncertainties:

Large values for intermediate momentum-transfer => larger cross section

IR safe. The detailed form very close to $Q^2=0$ is not important does not contribute to the energy loss

A model; not a renormalizable theory

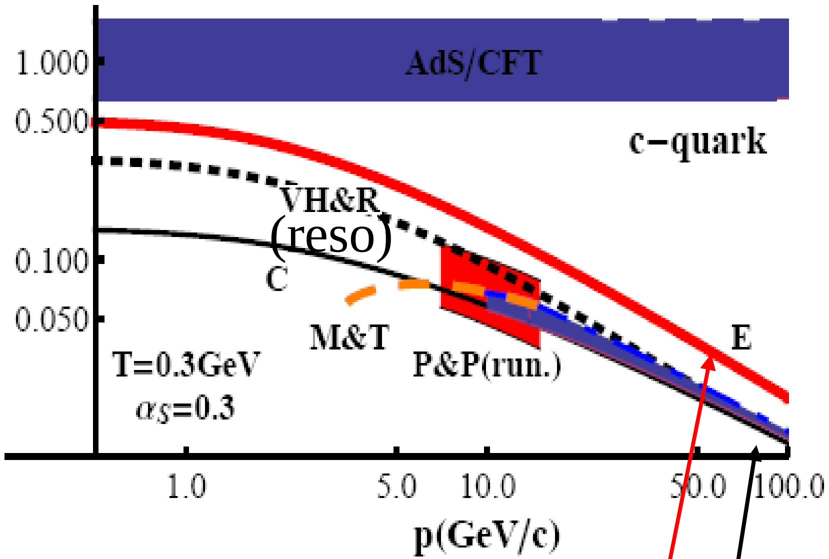
Running α_s : some Energy-Loss values for purely collisional processes

$$\frac{dE_{coll}(c/b)}{dx}$$

T(MeV) \ p(GeV/c)	10	20
200	**1 / 0.65**	**1.2 / 0.9**
400	**2.1 / 1.4**	**2.4 / 2**

≈ 10 % of HQ energy

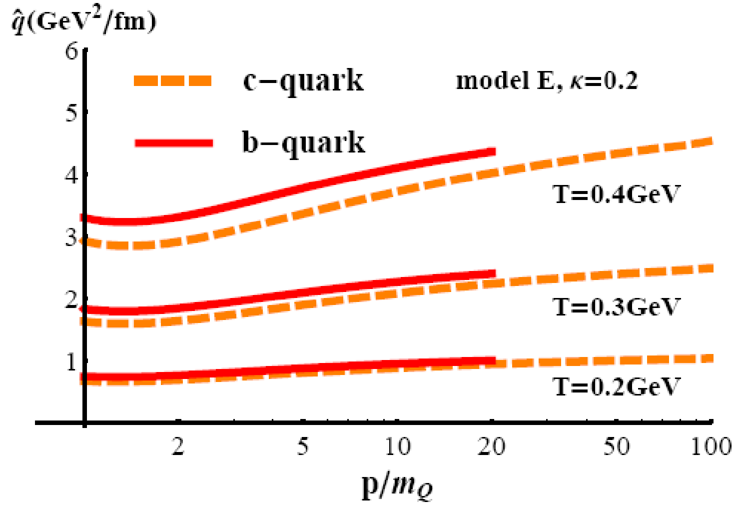
Drag coefficient



E: optimal μ, running α_{eff}

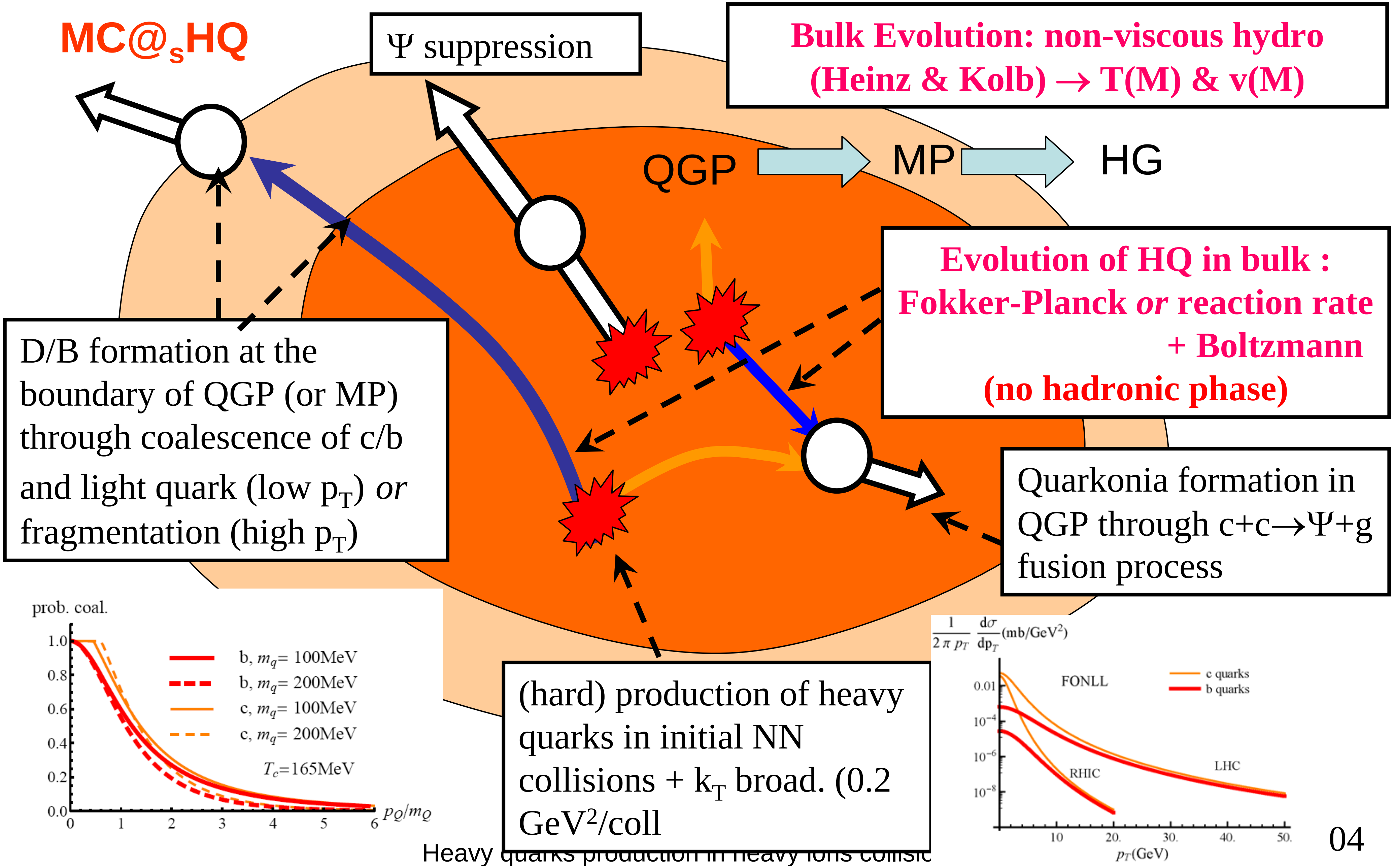
C: optimal μ, $\alpha_s(2\pi T)$

Transp. Coef …



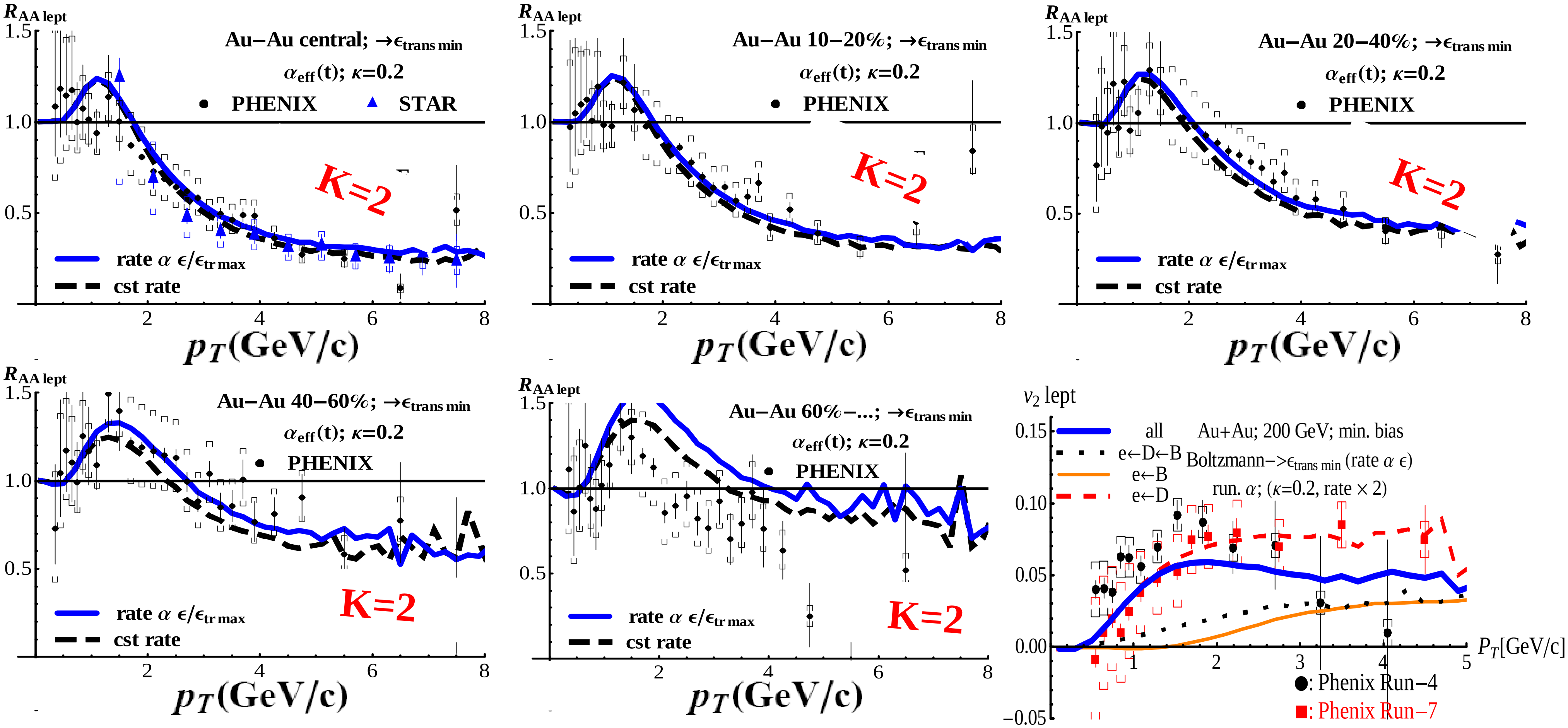
… of expected magnitude to reproduce the data (we "explain" the transport coeff. in a rather parameter free approach).

Schematic view of the global framework



Observables (Au-Au) vs (rescaled) Model

Best observable so far: R_{AA} for single non-photonic electrons



One reproduces R_{AA} *on all p_T range* with cranking K-factor ≈2 which permits to accommodate the "unknowns"

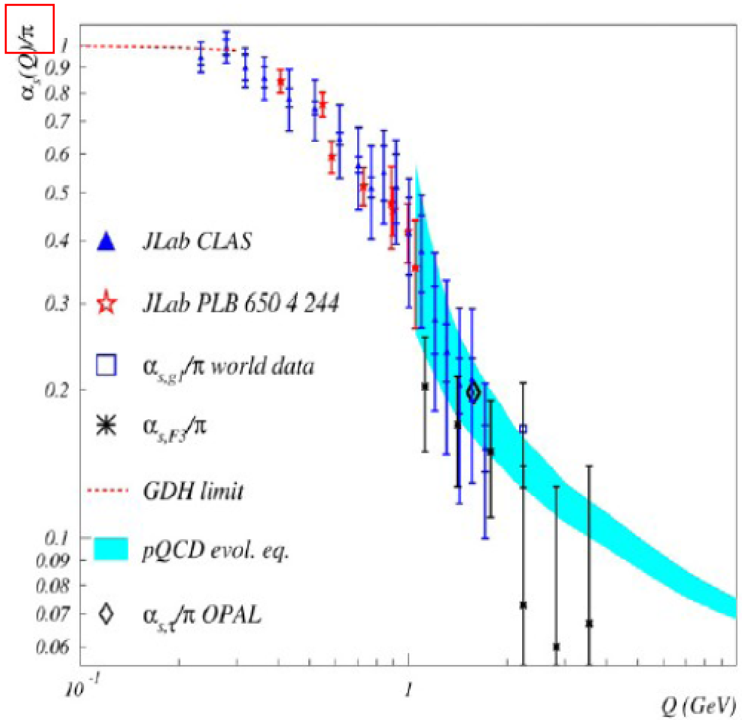
From P. Braun-Munzinger (INPC 2010)

the QCD strong coupling constant from sum rules and spin structure function data

new data from CLAS@JLab give first evidence for leveling off at low Q^2

Deur et al., Phys. Lett. B665 (2008) 349

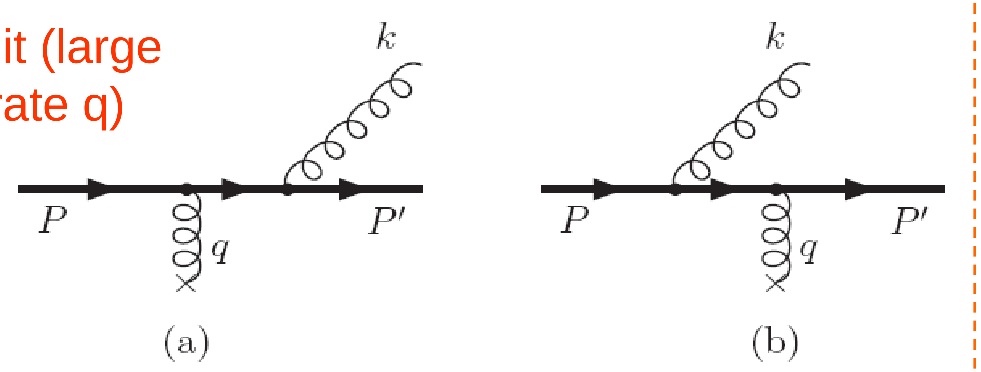
is QCD in the non-perturbative region a conformal field theory?

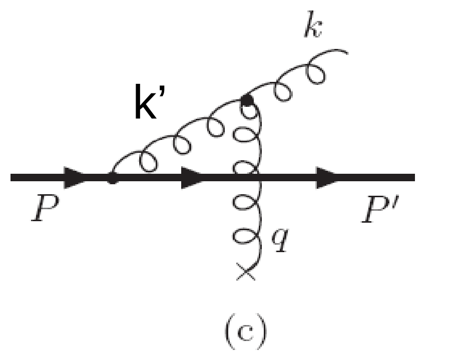


Basic (massive) Gunion-Bertsch

Radiation α deflection of current (semi-classical picture)

Eikonal limit (large E, moderate q)





Dominates as small x as one "just" has to scatter off the virtual gluon k'

$$\omega \frac{d^3\sigma_{\rm rad}^{x\ll 1}}{d\omega d^2 k_\perp dq_\perp^2} = \frac{N_c \alpha_s}{\pi^2}(1-x) \times \frac{J_{\rm QCD}^2}{\omega^2} \times \frac{d\sigma_{\rm el}^{Qq}}{dq_\perp^2}$$

with

$$\frac{J_{\rm QCD}^2}{\omega^2} = \left(\frac{\vec{k}_\perp}{k_\perp^2 + x^2 M^2 + \underbrace{(1-x)m_g^2}_{\text{Gluon thermal mass} \sim 2T}} - \frac{\vec{k}_\perp - \vec{q}_\perp}{\left(\vec{k}_\perp - \vec{q}_\perp\right)^2 + x^2 \underbrace{M^2}_{\text{Quark mass}} + (1-x)m_g^2} \right)^2$$

(phenomenological; not in BDMPS)

Both cures the colinear divergences and influence the radiation spectra

Radiation spectra

$$\omega \frac{d^2\sigma^{x\ll 1}_{\text{rad}\,''\text{QCD}''}}{d\omega\, dq_\perp^2} \approx \frac{2N_c\alpha_s}{\pi} \ln\left(1 + \frac{q_\perp^2}{3\tilde{m}_g^2}\right) \times \frac{d\sigma^{Qq}_{\text{el}}}{dq_\perp^2}$$

… to convolute with your favorite elastic cross section

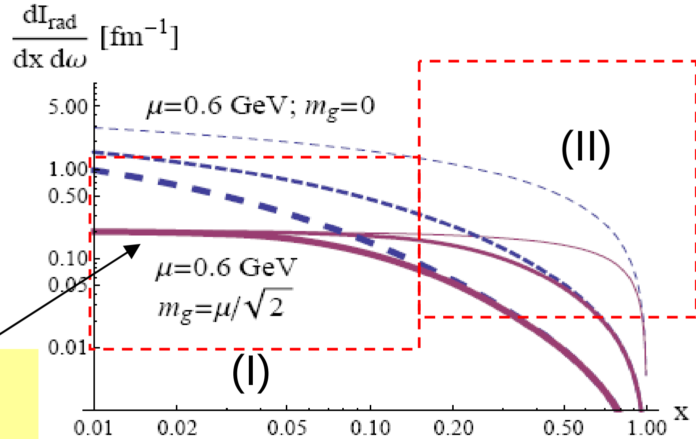
$$\tilde{m}_g^2 = (1-x)m_g^2 + x^2 M^2$$

For coulomb scattering:

Light quark

c-quark

b-quark



Strong dead cone effect for $x>m_g/M_Q$ (mass hierarchy)

Little mass dependence (especially from $q\to c$)

If typical $q_\perp \approx T$:

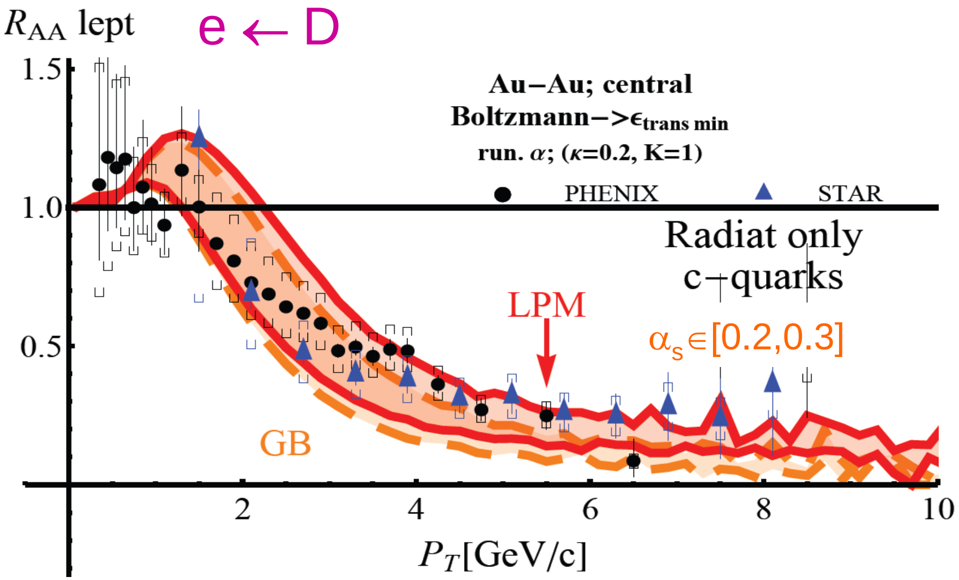
$$\frac{d^2 I^{x\ll 1}_{\text{GB}}}{dz\, d\omega} \sim \frac{2N_c\alpha_s}{3\pi} \times \frac{1}{m_g^2 + x^2M^2} \times \underbrace{\frac{\langle q_\perp^2\rangle}{\lambda}}_{\hat{q}}$$

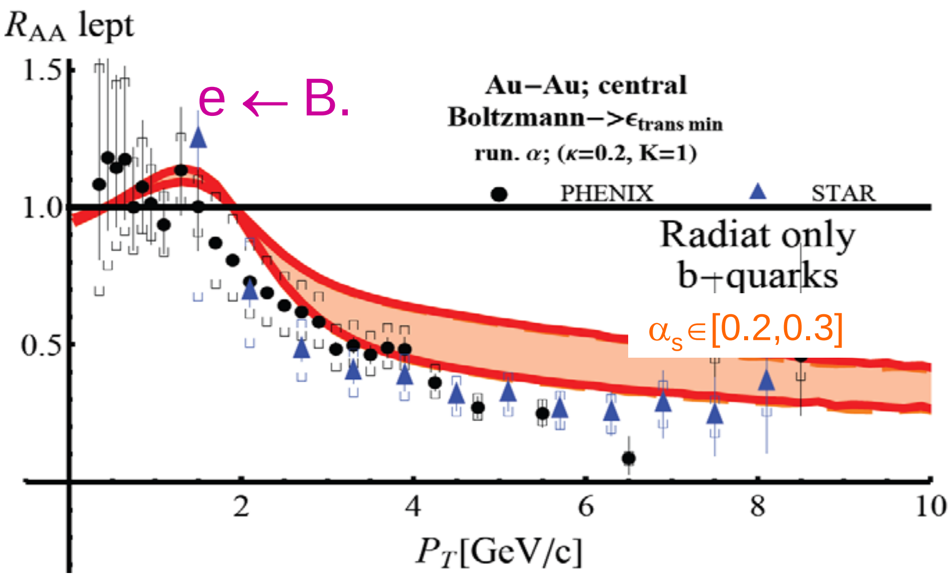
$$\Rightarrow \boxed{\frac{dE_{\text{GB}}(Q)}{dz} \approx \frac{4N_c\alpha_s}{\pi} \times \frac{0.8\mu}{M+\mu} \times \frac{E}{\lambda_Q}}$$

Strong mass effect in the average Eloss (mostly dominated by region II), similar to AdS/CFT

Interesting *per se*, but not much connected to the quenching or R_{AA}.

Results with (Coherent) Radiation Included

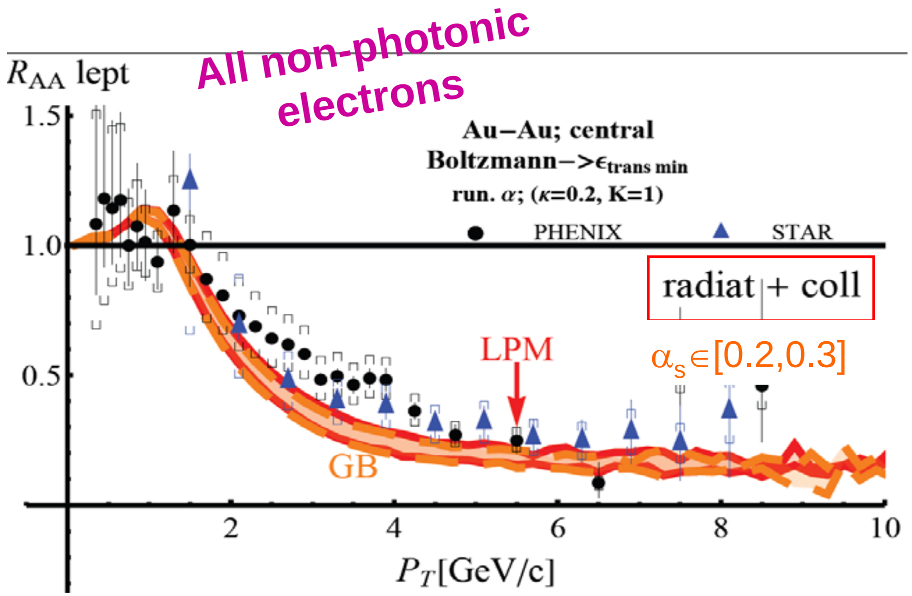




1. Coherence: Some moderate increase of R_{AA} for D at large p_T.
2. No effect seen for B

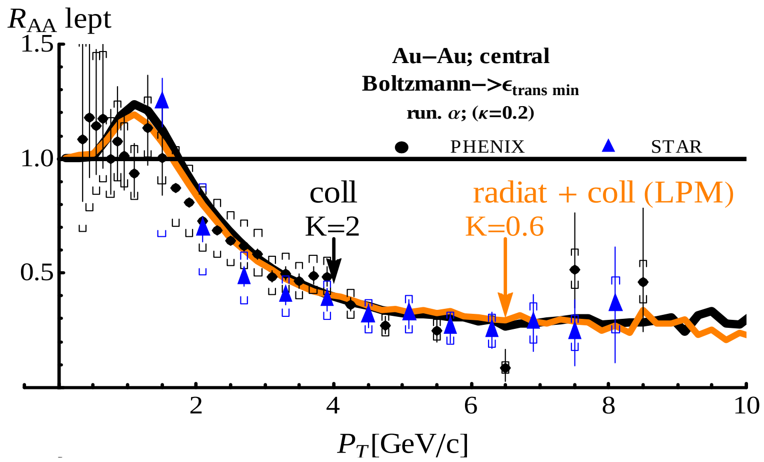
Conclusions can vary a bit depending on the value of the transport coefficient

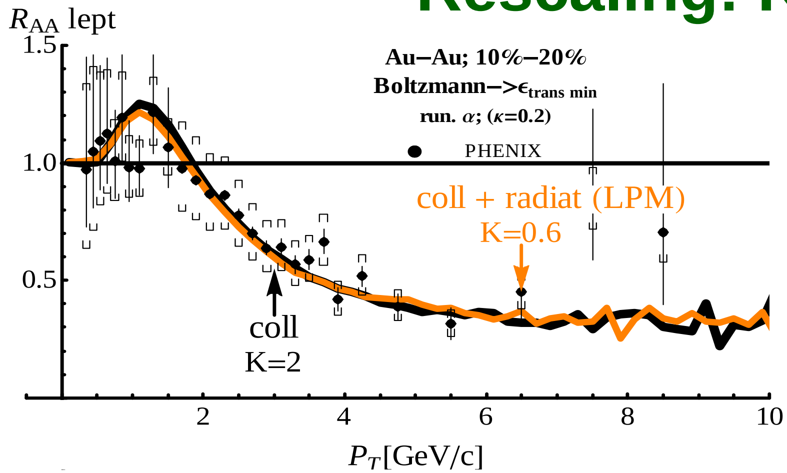
Indication that R_{AA} at RHIC is mostly the physics of rather numerous but small E losses, not very sensitive to coherence .

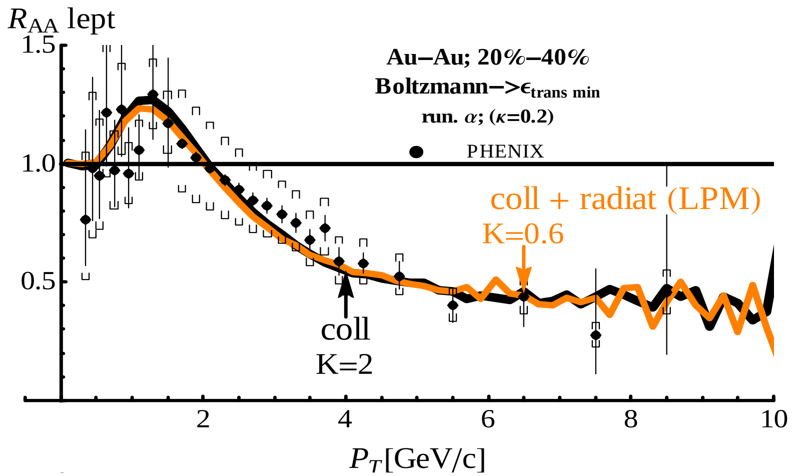


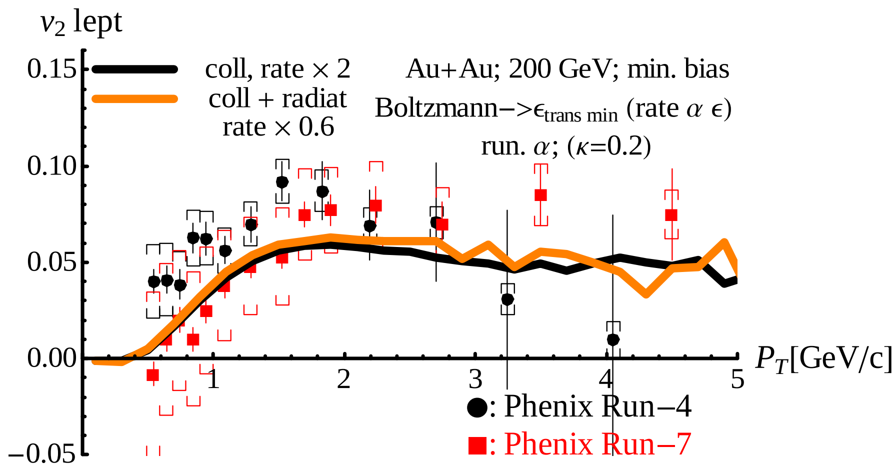
Collisional vs {Radiative + Coll}

Rescaling: K=0.6





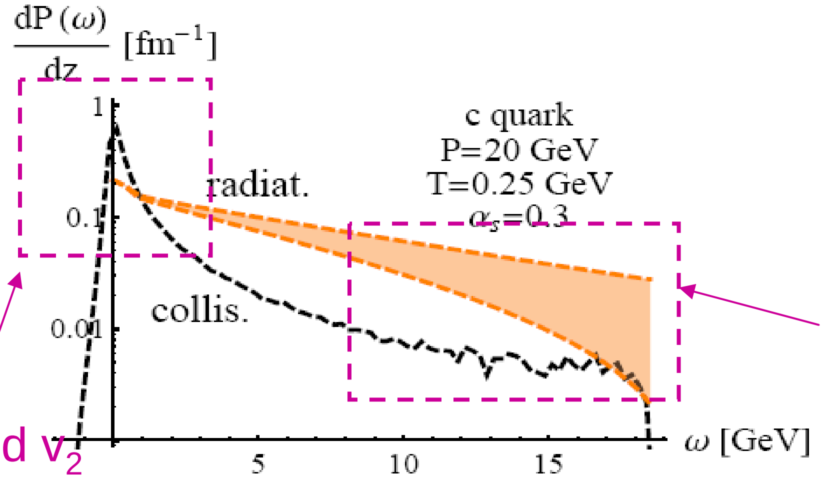




The present data cannot decipher between the 2 local microscopic E-loss scenarios

Interpretation

The heavy-quark physics at play for RHIC measured up to now (R_{AA} and v_2) is the one of small (relative) E-loss (and thus of the **Fokker-Planck equation**)… even at the largest p_T



R_{AA} and v_2 physics

Bad control on the theory

Explains why purely collisional models "work" so well

What we need

- D and B separately (in any case)
- tagged HQ jets and I_{AA} (and other correlations)

In our view, it is nevertheless more plausible to describe the physics in terms of a rather strong collisionnal energy loss supplied with an even stronger radiative energy loss (at least for γ ">>" 1).

QGP properties: low momentum

As we reproduce experimental data with rescaled model:

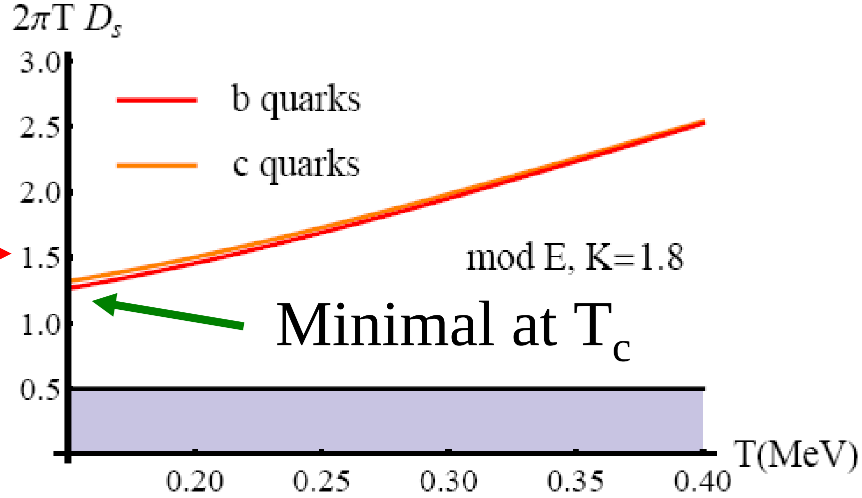
<u>Moore-Teaney:</u>

$$\frac{D}{\eta/(e+p)} \approx 6 \quad \Rightarrow \eta/s \approx DT/6$$

"robust"pQCD

$$\approx \frac{2\pi T\, D}{3 \times 4\pi}$$

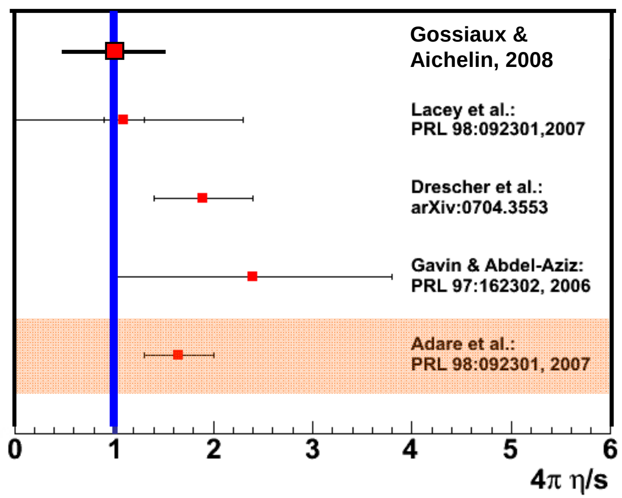
$$\boxed{\Rightarrow \eta/s \approx \frac{0.5}{4\pi}} \quad \text{at } T_c$$



<u>Strong coupling; AdS/CFT:</u>

$$\eta/s \approx DT/2 \approx \frac{2\pi T\, D}{4\pi}$$

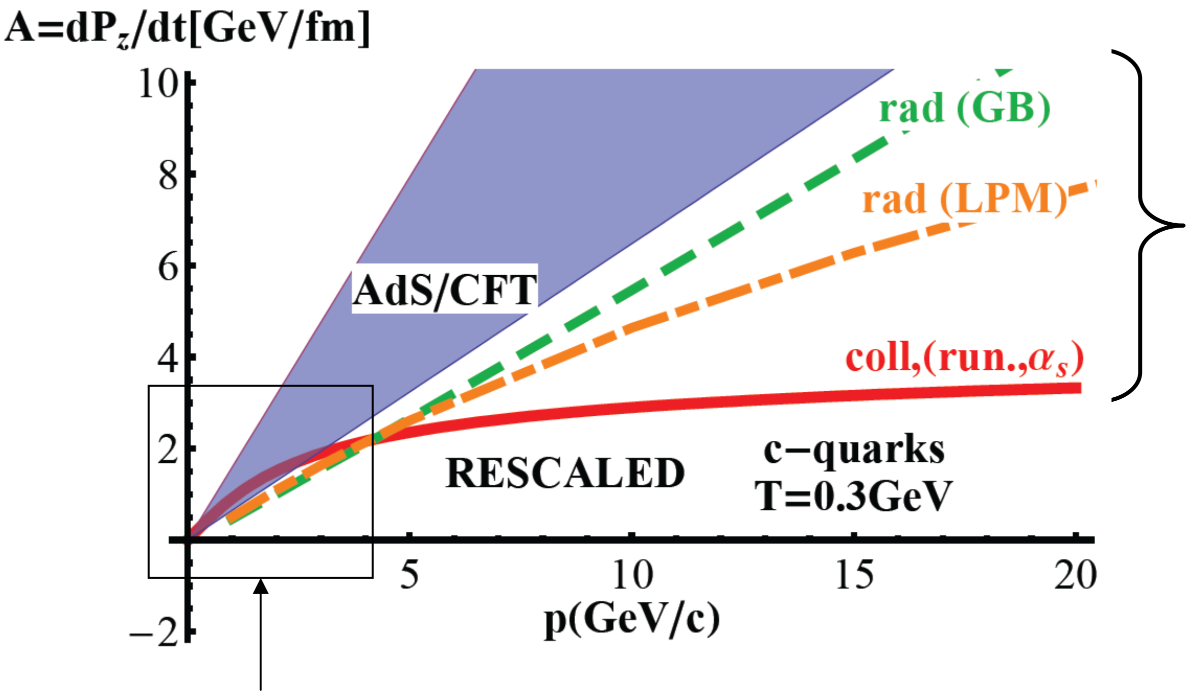
$$\boxed{\Rightarrow \eta/s \approx \frac{1.5}{4\pi}} \quad \text{at } T_c$$



But diffusion constant of heavy quark is already an interesting quantity in itself and could be evaluated on the lattice !!!

QGP properties: stopping power

Gathering all *rescaled* models (*coll. and radiative*):



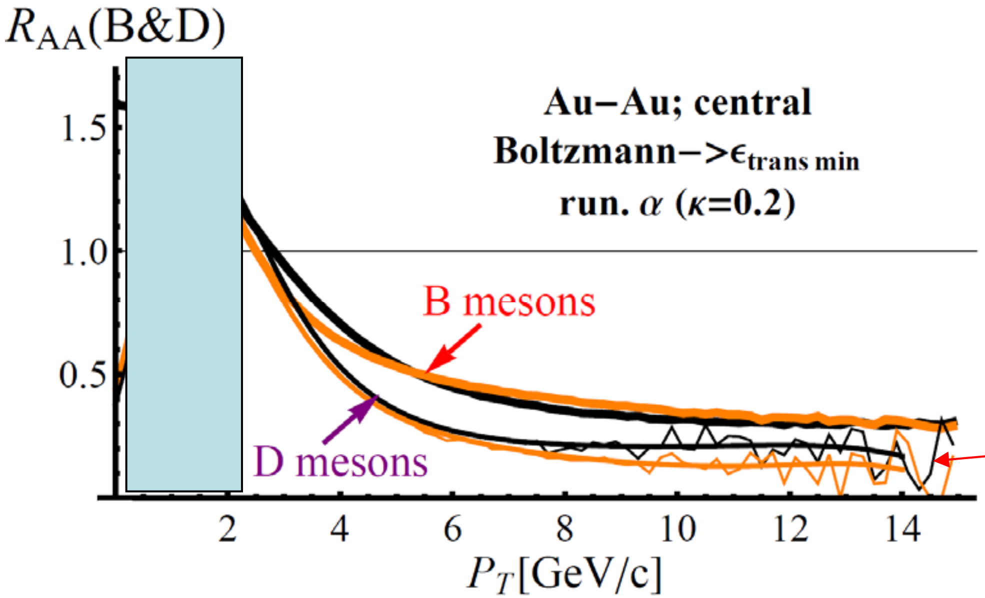
Seems "under control"

Challenge

Exp. cannot resolve between those various trends

quite consistent as the drag coefficient reflects the average momentum loss (per unit time) => large weight on x ~ 1

D & B meson: RHIC II (radiat + collisional)



Collis., rate x 2

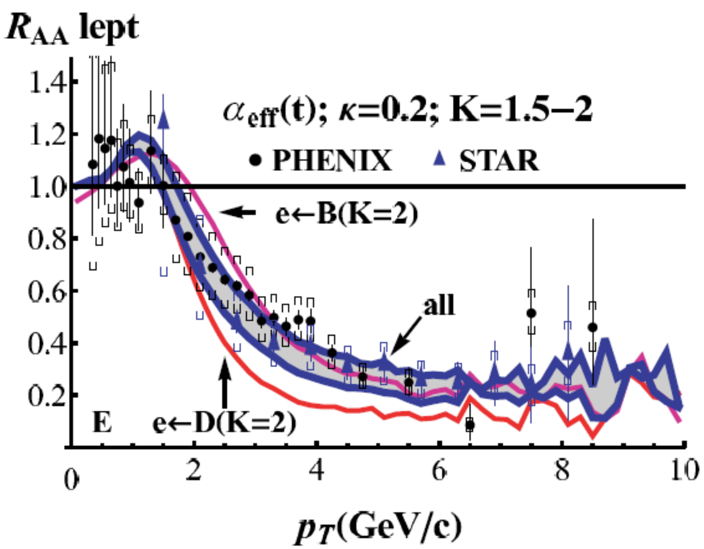
Collis. + Rad (LPM), rate x 0.6

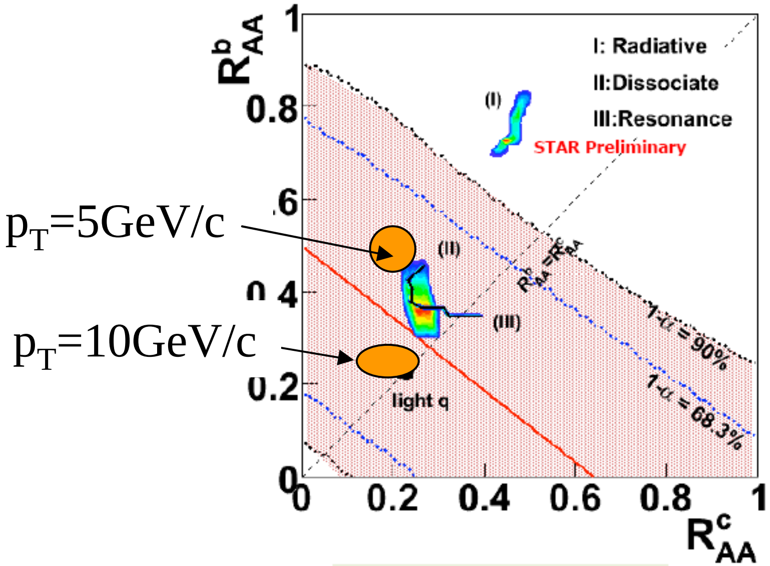
α_s(rad)=0.3

… some small deviations for D spectra at large p_T

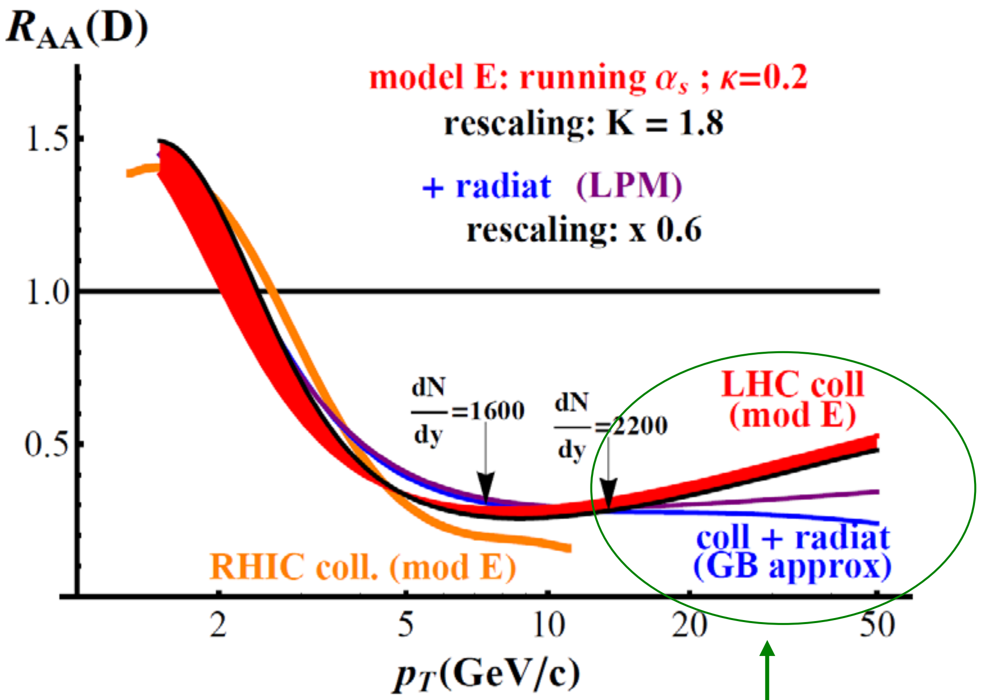
Z. Xu (sqm08)

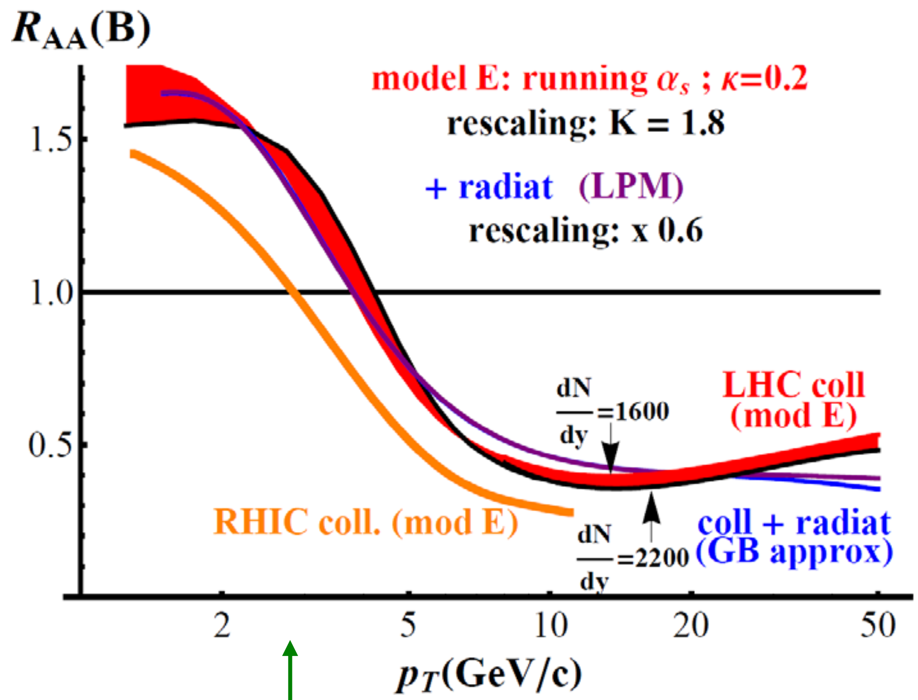
PRC 78 (fig 9)





D & B meson: LHC (radiat + collisional)





D spectra in Pb-Pb (5.5 TeV): Some window to decipher between the various Energy-loss models, for $p_T > 20$ GeV/c

B spectra in Pb-Pb (5.5 TeV): Pretty independent of E-loss model (properly calibrated w.r.t. RHIC data)

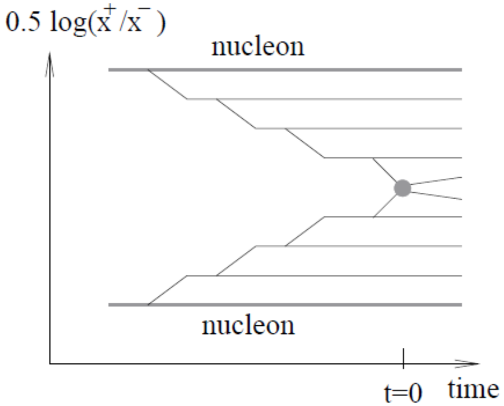
QGV in pp at LHC ?

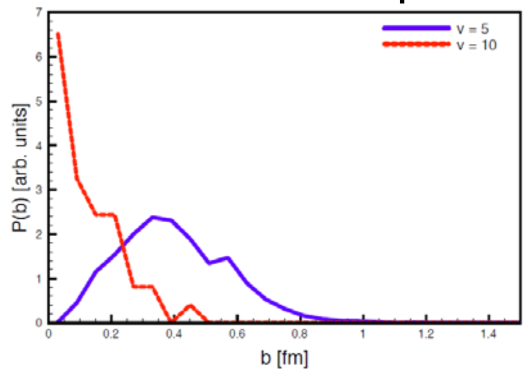
arXiv:1012.0764

Motivation: initial energy density ε_0 in pp (LHC) ~ ε_0 in CuCu (RHIC) $\Rightarrow$
possible quenching of (heavy flavours) jets

Combining MC@sHQ with EPOS

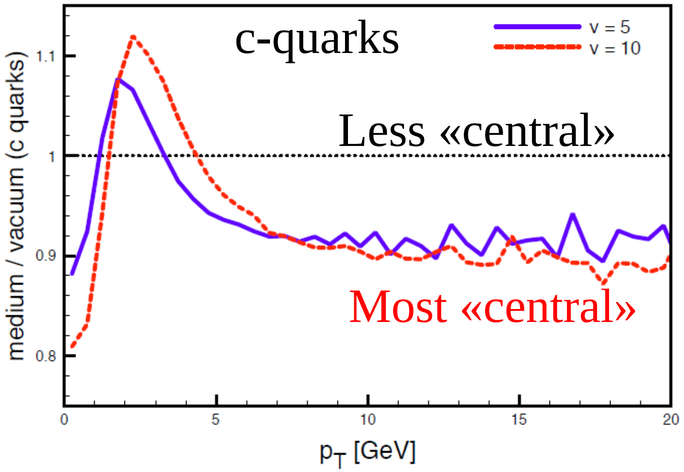
Centrality ≡ number ν of initial individual parton-parton collisions



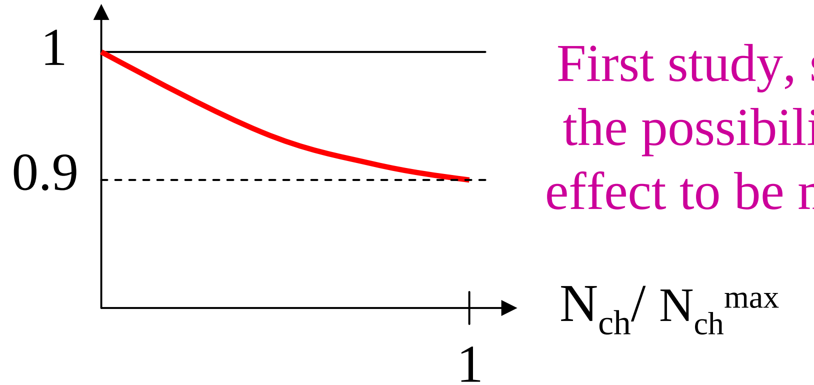


In EPOS: $N_{ch} \propto \nu$

Quenching



$$\frac{\frac{N_D(p_T>7\mathrm{GeV})}{N_D(p_T>?^{inc}}(N_{ch})}{\frac{N_D(p_T>7\mathrm{GeV})}{N_D(p_T>?^{inc}}(N_{ch}=0)}$$



First study, showing the possibility of an effect to be measured

Experimentally: no base-line for pp $\Rightarrow$?

II. Influence of the medium on heavy quarks phenomenology

Motivations :

I. We want to use experimental data to constrain the transport coefficient as much as possible

II. 2 groups (Texas AM & Nantes) have proposed **models** compatible with the non-photonic single-electron data, although the drag coefficients differ by a significant amount… How can this be possible ?

Role of the other ingredients ?

Models $\equiv$ microscopic model of HQ-QGP interactions (transport coefficients) + medium + initial distributions + hadronization + kinetic equation + …

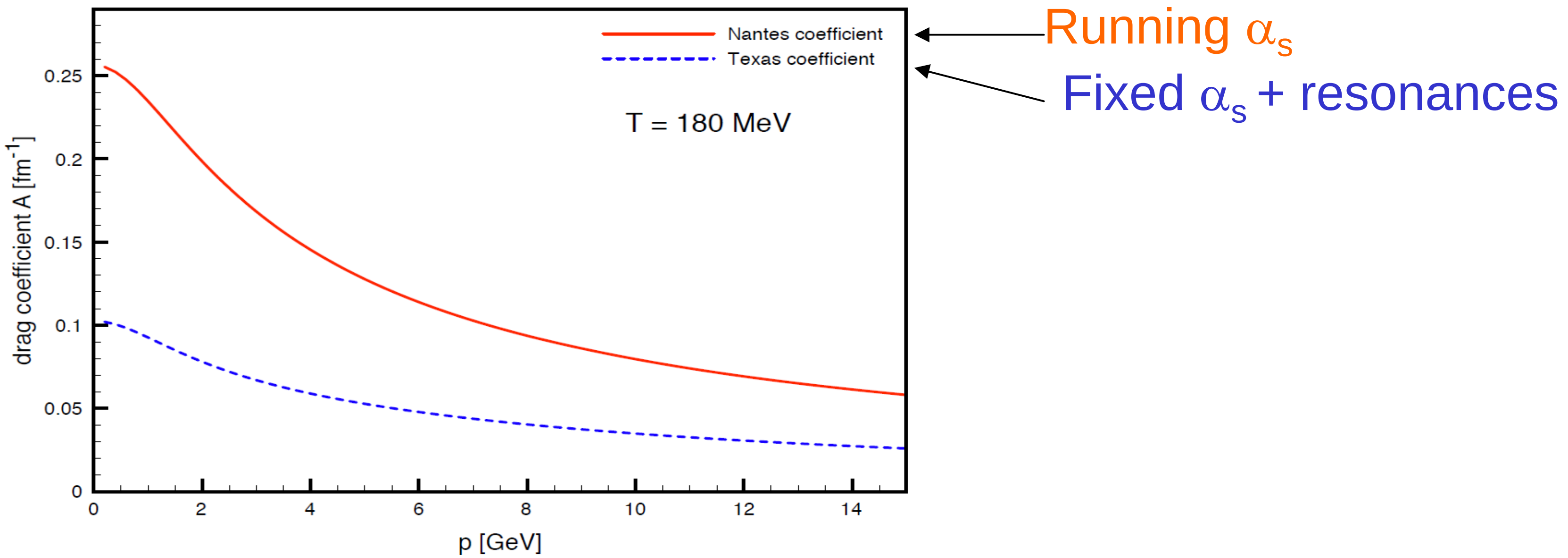
Methodology: exchanging some ingredients of the models into a single framework (the Nantes MC@sHQ)

Need for a collaboration: big thanks to Ralf and Hendrik

Caution: The aim of this study is not to reproduce the data at all price

Basic ingredients

Drag coefficient: $$\frac{d\langle\vec{p}\rangle}{dt} = -A(p,T)\vec{p}$$



Diffusion coefficient *B* tuned to satisfy Einstein relation: asymptotic thermal distribution

Basic ingredients

Transport: both solve FP equation through a Langevin Monte Carlo realization…

Back to the continuous stochastic process: $\dot{\xi} = -\gamma(\xi) + g(\xi)w(t)$

Multiplicative noise — Gaussian auto-correlated

- Spurious drift: $\frac{d\langle\xi\rangle}{dt} = -\gamma(\xi) + \boxed{g(\xi)g'(\xi)}$
- Ambiguity: for a small dt, at which value of ξ should we consider g ? Ito vs Stratonovich

But: a) we do not have a continuous process (smallest time scale: the collision)

b) we can evaluate the average momentum loss directly from the elastic cross section (fundamental blocks are not γ and g but $Ap=-\gamma+g\,g'$ and $B=2g^2$): No ambigity

However: Several realizations are possible

$$dp_i^{(k)} = -A(\vec{p}^{(k)})p_i^{(k)}dt + \left(g(\vec{p}^{(k)})\right)_{ij} w_j\sqrt{dt}$$ Ito pre point (Nantes)

$$dp_i^{(k)} = -\Gamma(\vec{p}^{(k)})p_i^{(k)}dt + \left(g(\vec{p}^{(k)}\boxed{+\,\xi\overrightarrow{dp}^{(k)})}\right)_{ij} w_j\sqrt{dt}$$ Ito post point (Texas AM)

In principle equivalent if one chooses: $\Gamma p_i = A p_i + \frac{\partial g_{ik}}{\partial p_l} g_{lk}$

Basic ingredients

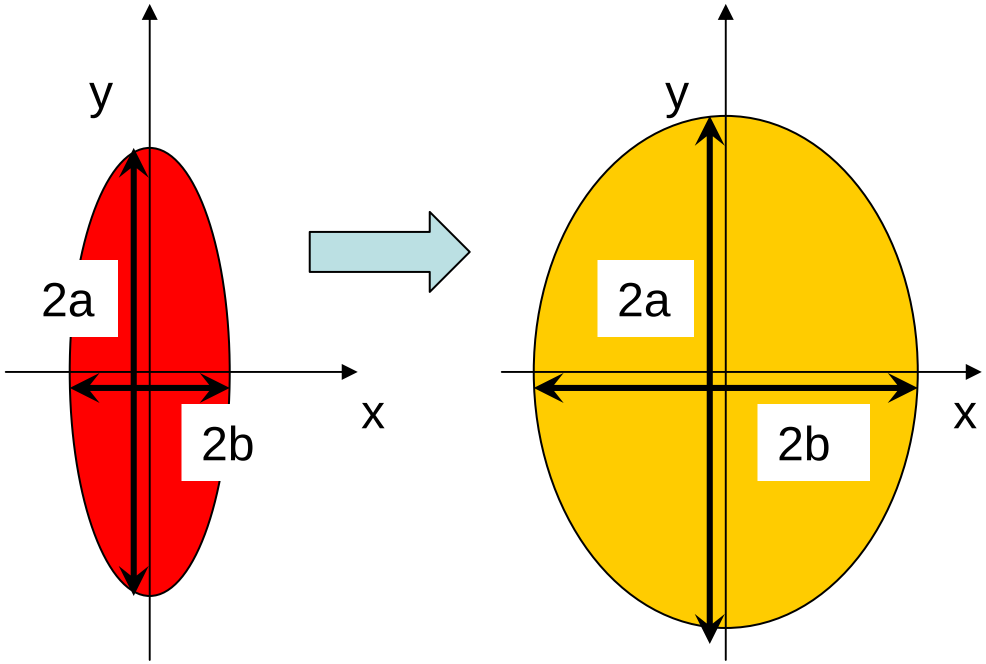
The medium:

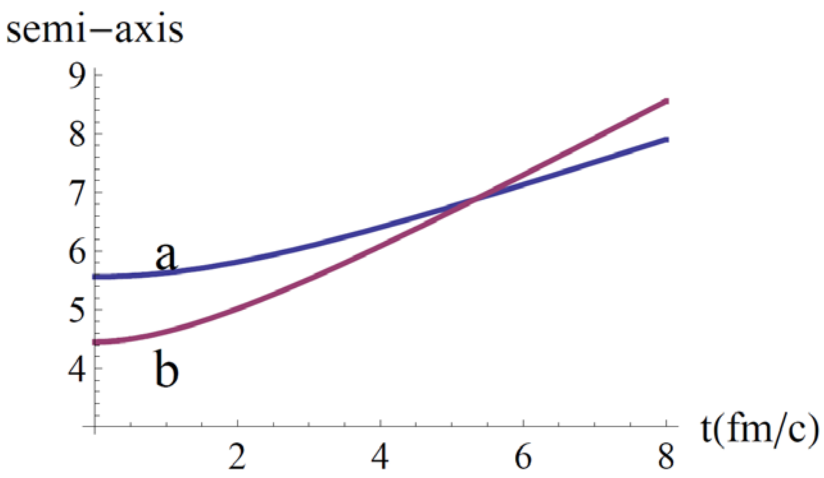
Nantes: Kolb-Heinz 2D+1 ideal hydrodynamics with Bjorken invariance (azhydro V2.0)

Texas AM: Fireball of the Landau-Type: T(t), **s(t)=S/V(t),** ε(t)

Volume: $V(t) = \pi a(t) b(b)(z0 + t)$

Transverse plane:

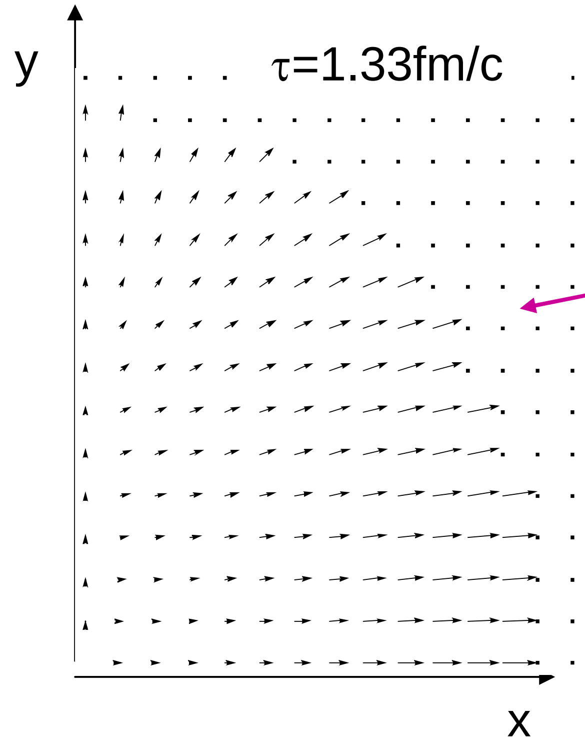




Basic ingredients

The medium:

Texas AM: Everything uniform but the **velocity field**



Calibration

lations). Special care has to be taken in the parameterization of the elliptic flow in noncentral Au-Au collisions: the contours of constant flow velocity are taken as confocal ellipses in the transverse plane with the pertinent transverse flow set consistently in perpendicular direction. The time evolution of the surface velocity of the semi-axes of the elliptic fire cylinder parameterizes the corresponding results of the hydrodynamic calculations in Ref. 135, in particular the time-dependence of the elliptic-flow parameter, v_2, for the light quarks. The parameters are adjusted such that the average surface velocity reaches $v_\perp^{(s)} = 0.5c$ and the anisotropy parameter $v_2 = 5.5\%$ at the end of the mixed phase. Finally, the velocity field is specified by scaling the boundary velocity linearly with distance from the center of the fireball, again in accordance with the hydrodynamic calculation 135.

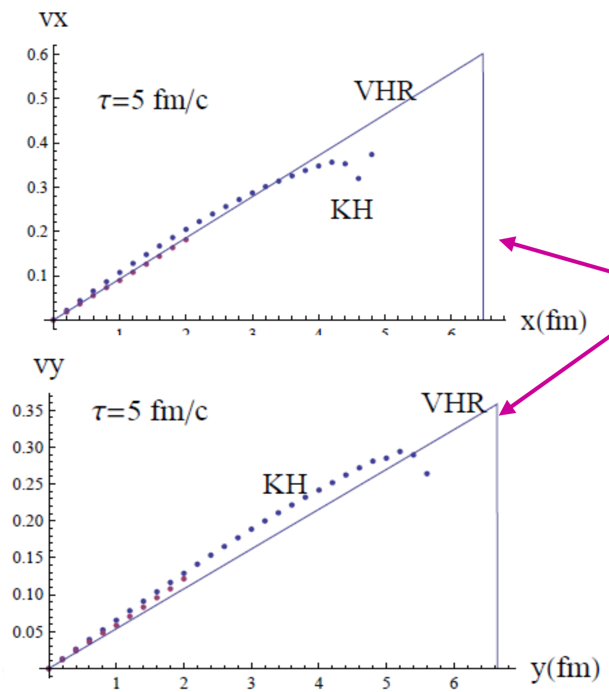
Kolb-Sollfrank-Heinz (2000)

Basic ingredients

The medium:

Texas AM: Everything uniform but the **velocity field**

Calibration



lations). Special care has to be taken in the parameterization of the elliptic flow in noncentral Au-Au collisions: the contours of constant flow velocity are taken as confocal ellipses in the transverse plane with the pertinent transverse flow set consistently in perpendicular direction. The time evolution of the surface velocity of the semi-axes of the elliptic fire cylinder parameterizes the corresponding results of the hydrodynamic calculations in Ref.[135]~~, in particular the time-dependence of the~~ elliptic-flow parameter, v_2, for the light quarks. The parameters are adjusted such that the average surface velocity reaches $v_\perp^{(s)} = 0.5c$ and the anisotropy parameter $v_2 = 5.5\%$ at the end of the mixed phase. Finally, the velocity field is specified by scaling the boundary velocity linearly with distance from the center of the fireball, again in accordance with the hydrodynamic calculation[135].

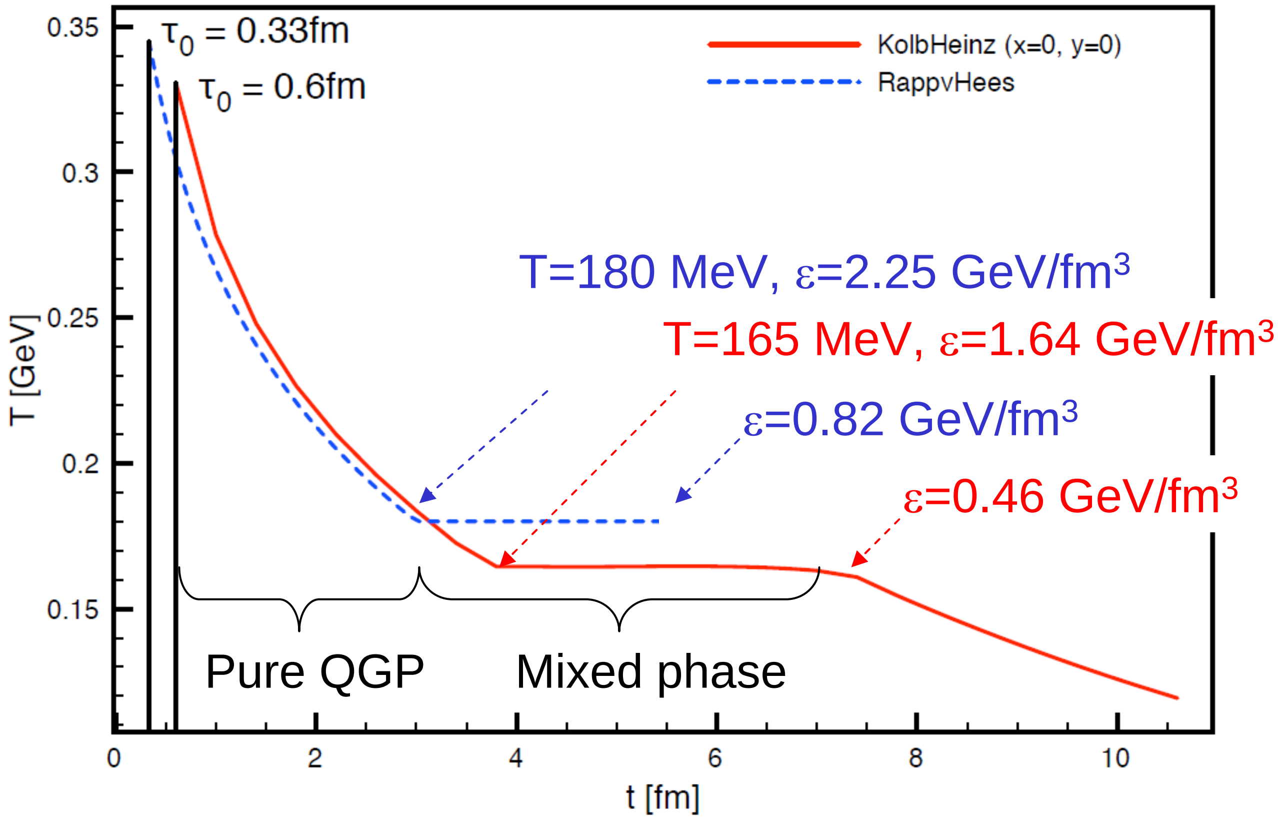
Rather good agreement for the velocity

Good agreement for the bulk flow

Basic ingredients

The medium:

Temperature evolution along time:



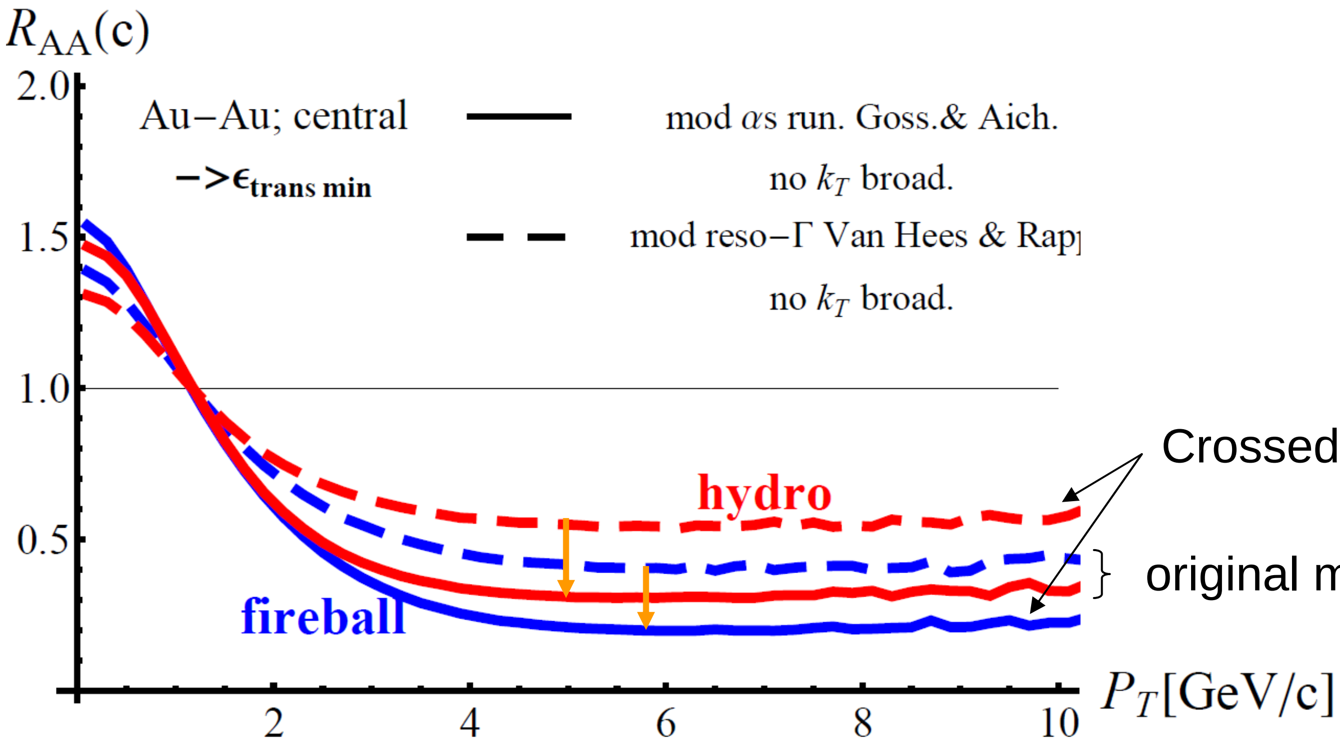
EOS of the same type (QGP described within the MIT bag model, 1st order transition), with rather similar parameters.

Conclusion: Good confidence that the “bulk” fireball is calibrated at the best

Heavy quark evolution

All heavy quarks observables evaluated with the pre-point prescription

Nuclear Modification factor



Crossed ingredients

original models (almost)

R_{AA} compatible

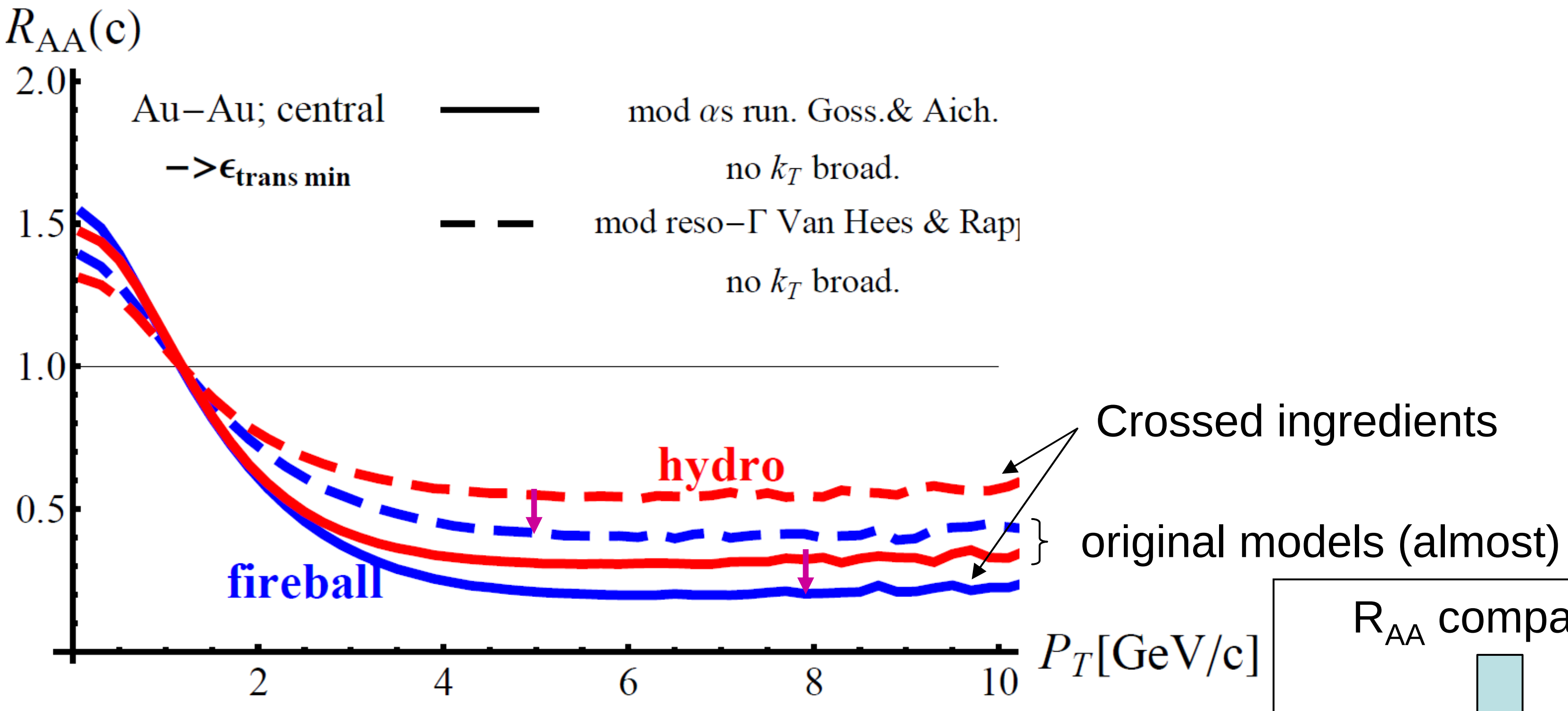
How can we conclude ?

: More coupling with the Nantes microscopic model
(as seen from the drag coefficient)

Heavy quark evolution

All heavy quarks observables evaluated with the pre-point prescription

Nuclear Modification factor



↓ : More quenching with the fireball then with the (original) KH hydro

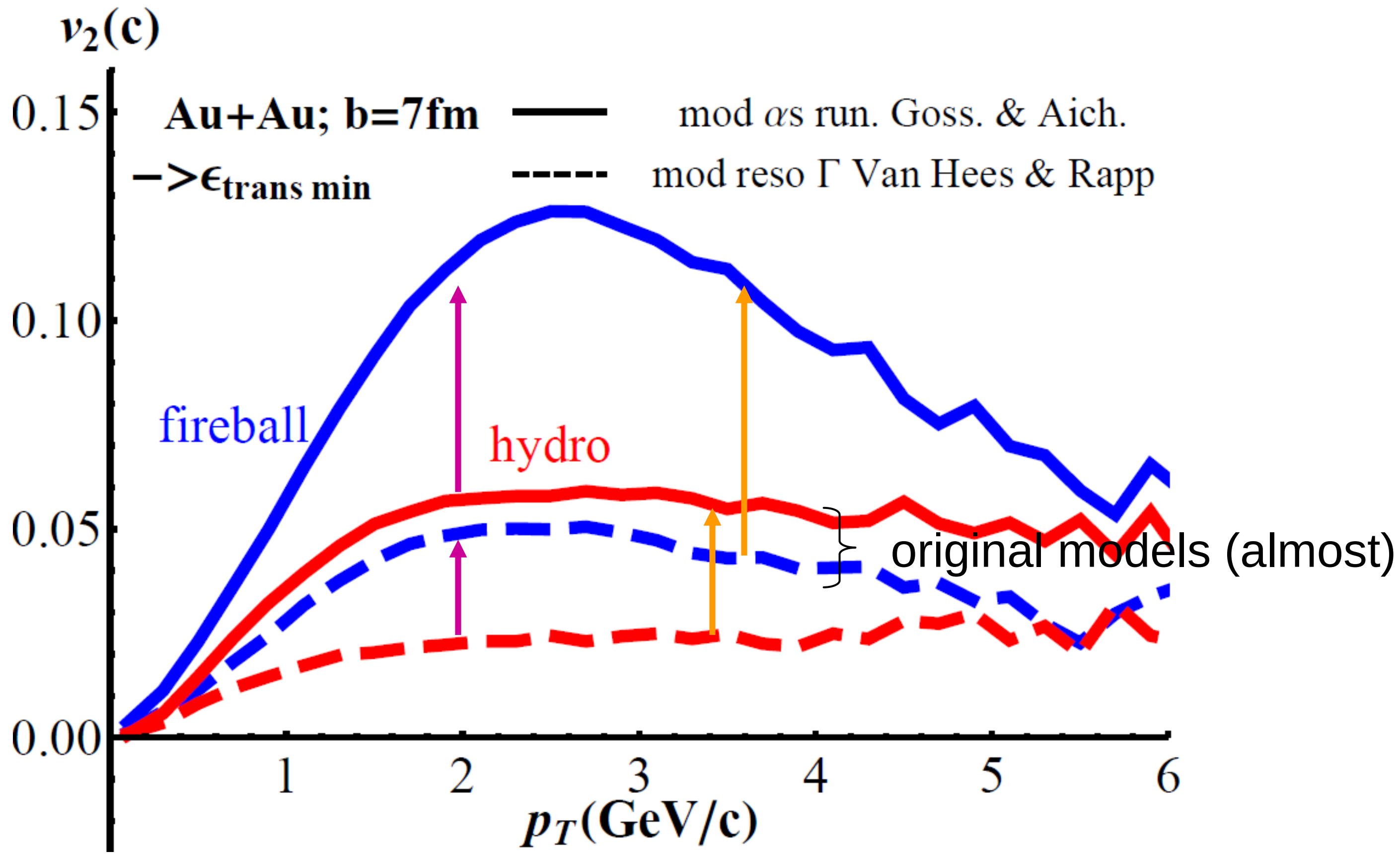
R_{AA} compatible

⇩

How can we conclude ?

Heavy quark evolution

Differential Elliptic flow



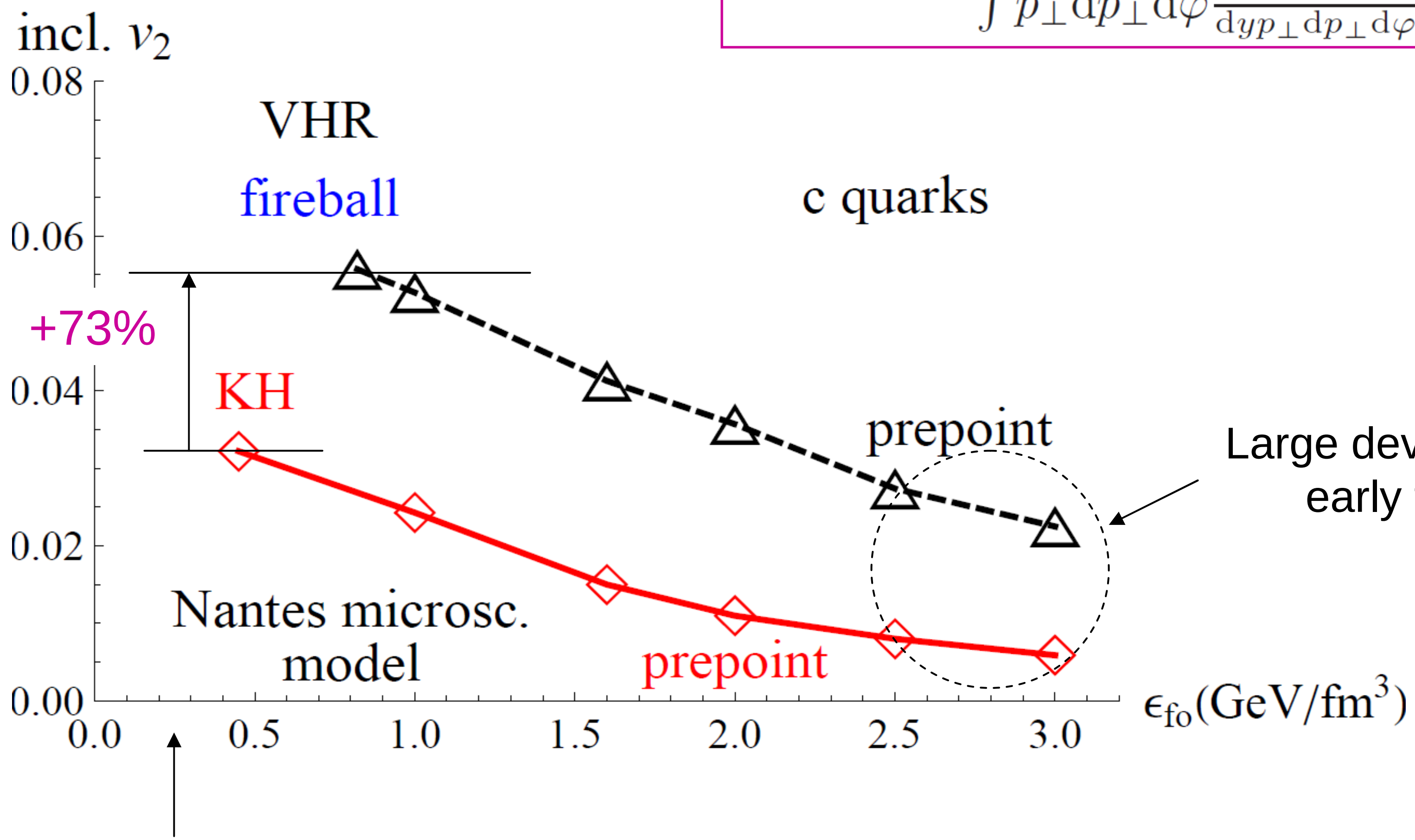
: More coupling with the Nantes microscopic model

: Higher elliptic flow from the fireball

Heavy quark evolution

Inclusive elliptic flow

$$v_2^{\mathrm{incl}} := \frac{\int p_\perp \mathrm{d}p_\perp \mathrm{d}\varphi \cos(2\varphi) \frac{\mathrm{d}^3 N}{\mathrm{d}y p_\perp \mathrm{d}p_\perp \mathrm{d}\varphi}}{\int p_\perp \mathrm{d}p_\perp \mathrm{d}\varphi \frac{\mathrm{d}^3 N}{\mathrm{d}y p_\perp \mathrm{d}p_\perp \mathrm{d}\varphi}}$$



Large deviations at early times

Various gedanken decoupling energy densities for the c quarks

Summary of HQ

Degree of thermalization	Resonances	Running α_s
KH hydro	Low	Intermediate
fireball	Intermediate	High

Is there something special with Heavy Quarks (due to their inertia) as compared to light ones ?

Back on the bulk v2 and on calibration

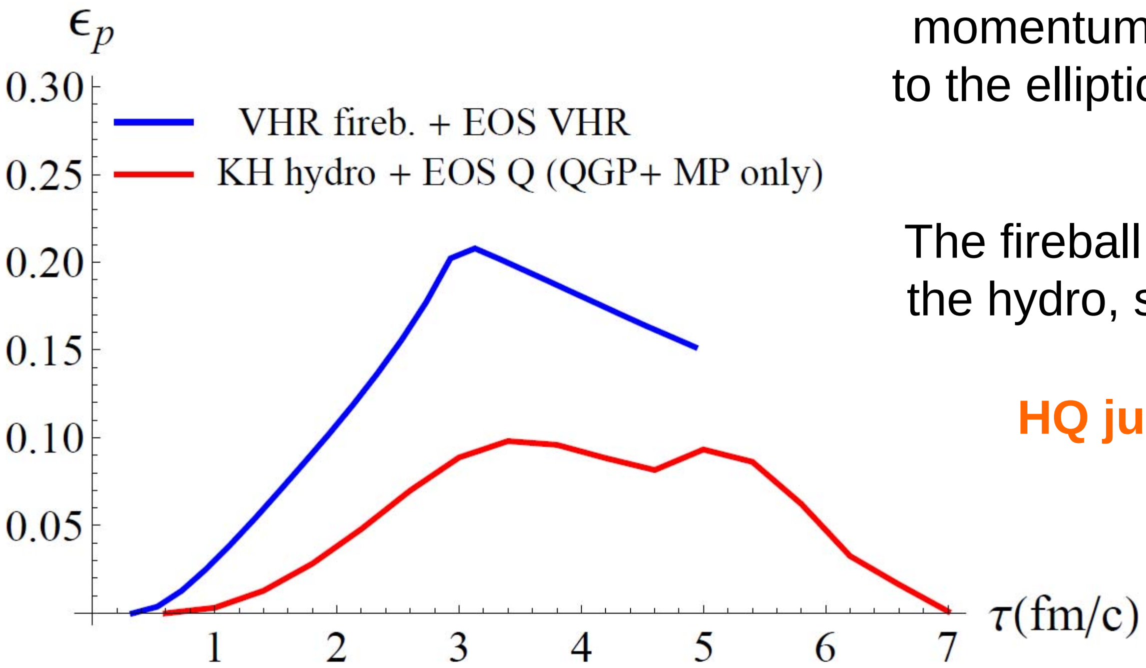
Probes of the bulk v2:

I. So called momentum anisotropy

$$\epsilon_p := \frac{\left.\frac{d\mathcal{T}^{xx}}{d\eta}\right|_{\eta=0} - \left.\frac{d\mathcal{T}^{yy}}{d\eta}\right|_{\eta=0}}{\left.\frac{d\mathcal{T}^{xx}}{d\eta}\right|_{\eta=0} + \left.\frac{d\mathcal{T}^{yy}}{d\eta}\right|_{\eta=0}}$$

with

$$\left.\frac{d\mathcal{T}^{\mu\nu}}{d\eta}\right|_{\eta=0} := t\int d^2x_\perp T^{\mu\nu}(\vec{x}_\perp, z=0)$$



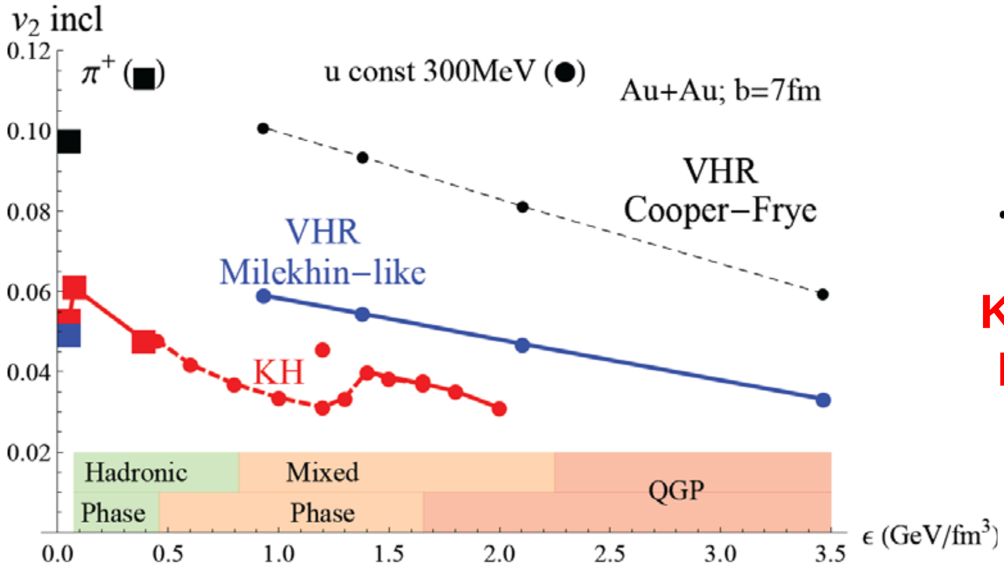
Directly evaluated from the energy-stress momentum; known to be closely related to the elliptic flow (Kolb, Sollfrank & Heinz 2000)

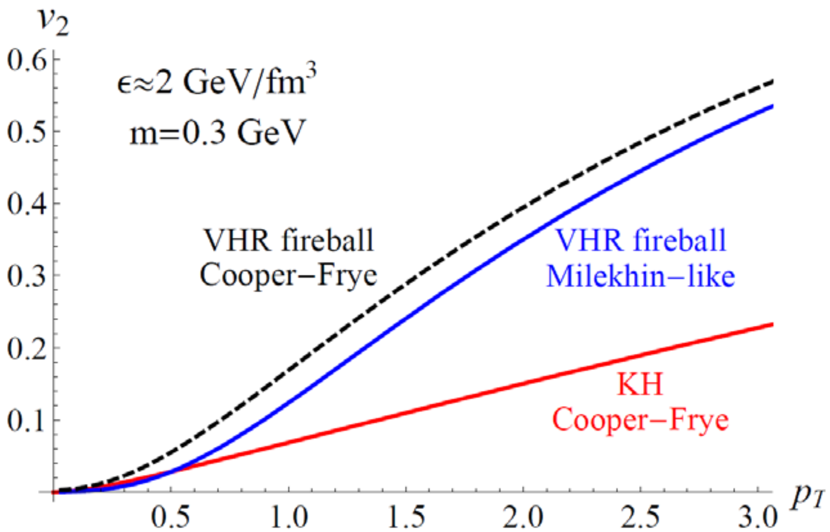
The fireball systematically overshoots the hydro, starting from early times !!!

HQ just inherits this larger anisotropy

Back on the bulk v2 and on calibration

II. Elliptic flow of light quarks and ensuing pions





Particle spectra:

$$\frac{EdN}{\mathrm{d}^3p} = \int \mathrm{d}\sigma_\mu p^\mu f(p,T)$$

$$f = e^{-\frac{p\cdot u}{T}}$$

KH with Cooper-Frye freeze out

Fireball: freeze out at constant lab time t

$$\frac{\mathrm{d}^3N}{\mathrm{d}^3p} = \int_{\mathcal{V}} \frac{\mathrm{d}V}{(2\pi\hbar)^3} f\left(\frac{p\cdot u}{T(t)}\right)$$

$$f = \frac{p\cdot u}{p^0} e^{-\frac{p\cdot u}{T}}$$

Asymptotic distribution in the VHR post-point Langevin

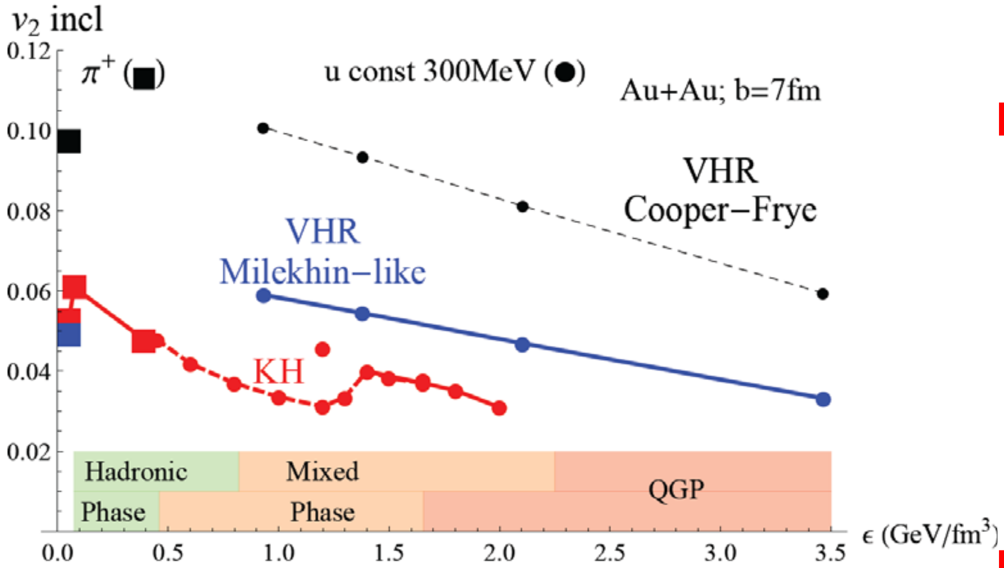
$$f = e^{-\frac{p\cdot u}{T}}$$

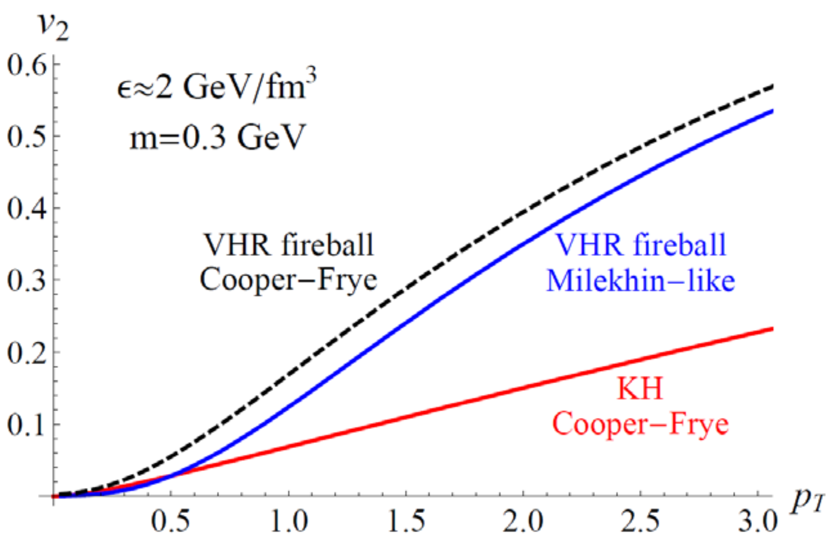
fireball with Cooper-Frye freeze out

fireball with Milekhin-like freeze out

$$d\sigma_\mu = d\bar{V} u_\mu$$

Back on the bulk v2 and on calibration





several levels of subtle and somehow paradoxal conclusions:

KH with *Cooper-Frye* freeze out ⟺ **fireball with *Cooper-Frye* freeze out**

Confirms the excess of intrinsic v2 in the fireball on the absolute level

KH with *Cooper-Frye* freeze out ⟺ **fireball with Milekhin-like freeze out**

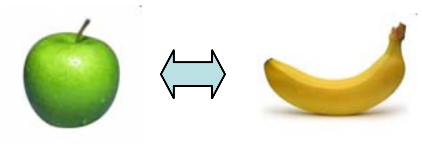
$$f = \frac{p \cdot u}{p^0} e^{-\frac{p \cdot u}{T}}$$

Confirms the proper calibration of the fireball (5.5% v_2 for pions)

Comparing with 2 different freeze out prescriptions:

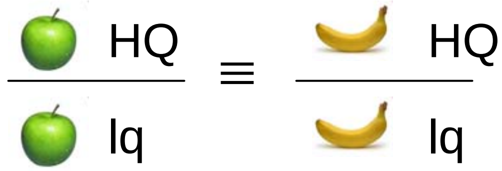
 ⟺  ?

Calibration at a *relative* level



?

Bona fide argument:



"One should calibrate the fireball using the asymptotic distributions (of light quarks) that results from Langevin transport one uses for the heavy quarks"

Hendrik Van Hees

Remember: Several realizations are possible

Ito pre point (Nantes)

$$dp_i^{(k)} = -A(\vec{p}^{(k)})p_i^{(k)}dt + \left(g(\vec{p}^{(k)})\right)_{ij} w_j\sqrt{dt}$$

$$f_{\text{as}} = e^{-\frac{p\cdot u}{T}}$$

Ito post point (Texas AM)

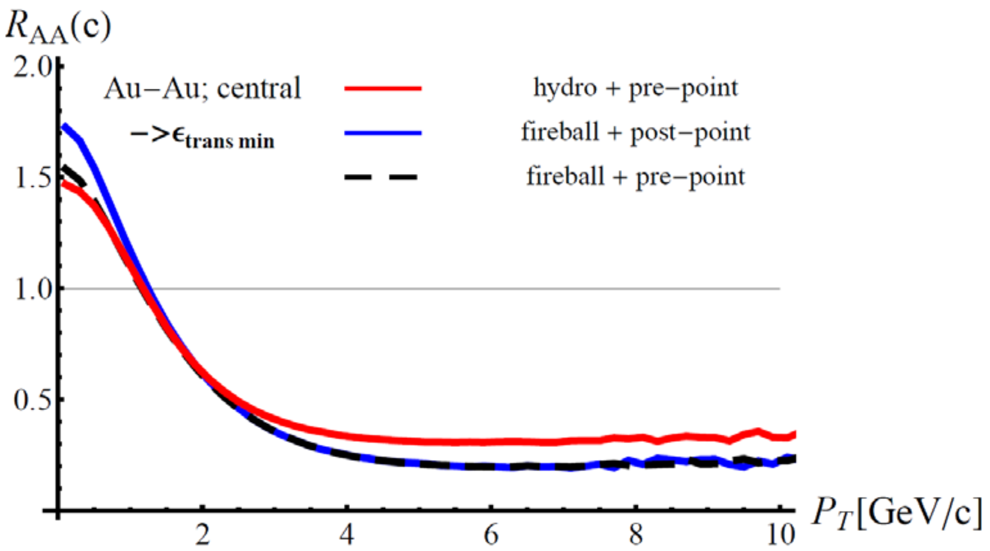
$$dp_i^{(k)} = -\Gamma(\vec{p}^{(k)})p_i^{(k)}dt + \left(g(\vec{p}^{(k)}\boxed{+\xi\overrightarrow{dp}^{(k)})}\right)_{ij} w_j\sqrt{dt}$$

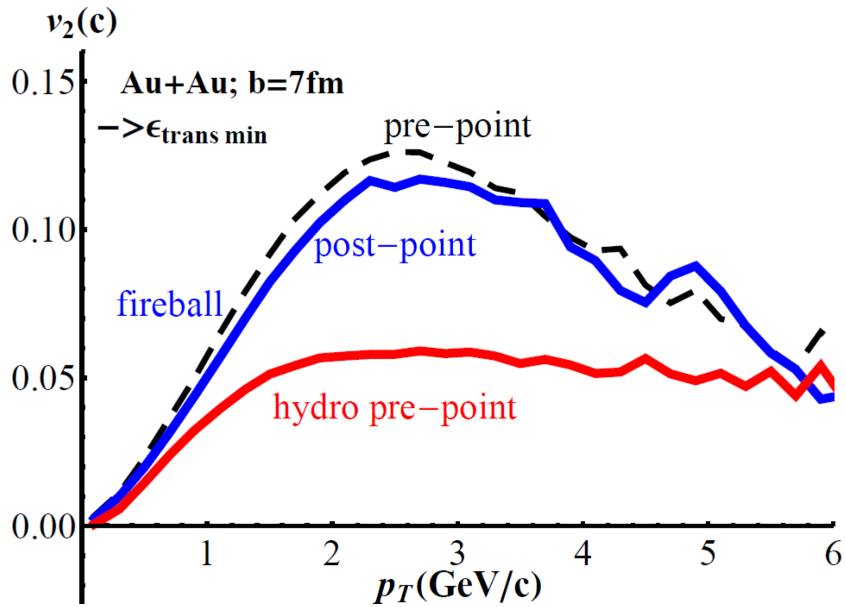
$$f_{\text{as}} = \frac{p\cdot u}{p^0}e^{-\frac{p\cdot u}{T}}$$

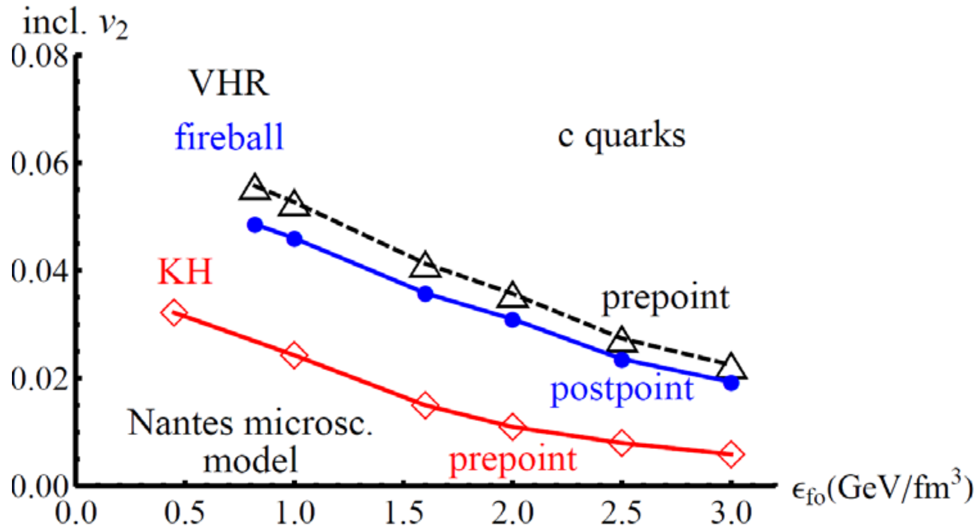
- The medium calibration is intricately linked with the transport model one uses (exchanging the ingredient "medium" alone has no meaning)
- It is not legitimate to perform the HQ simulations in the fireball with the pre-point prescription.

Heavy quarks with pre-point vs post-point

All heavy quarks observables evaluated with the Nantes FP coefficients





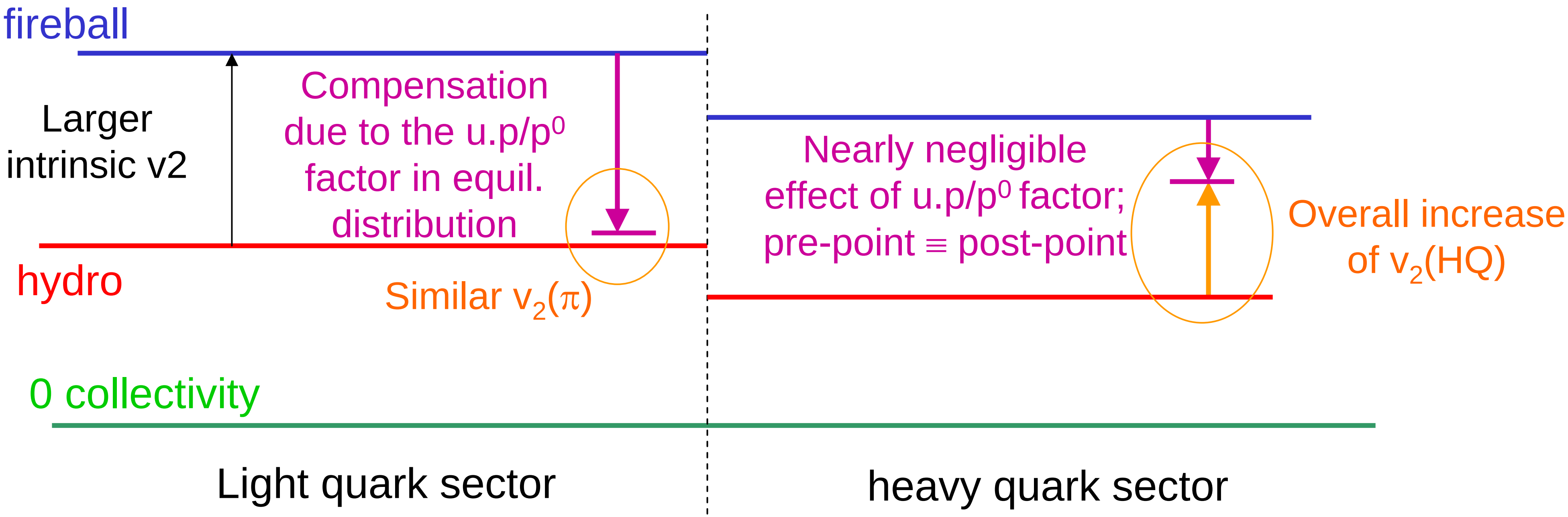


The post-point vs pre-point prescription has little influence on the heavy quark observables (consequences of u.p/p^0 are not mass independent).

In particular, the post-point prescription does not bring the “fireball results” in the range of the “pre-point KH hydro”

Summary and conclusions (1)

With respect to the Kolb Heinz hydro + Cooper-Frye reference:



This all together explain why one is able to describe the heavy flavor data with smaller FP coefficients within the fireball than in the KH hydro, although they both reproduce the pions elliptic flow

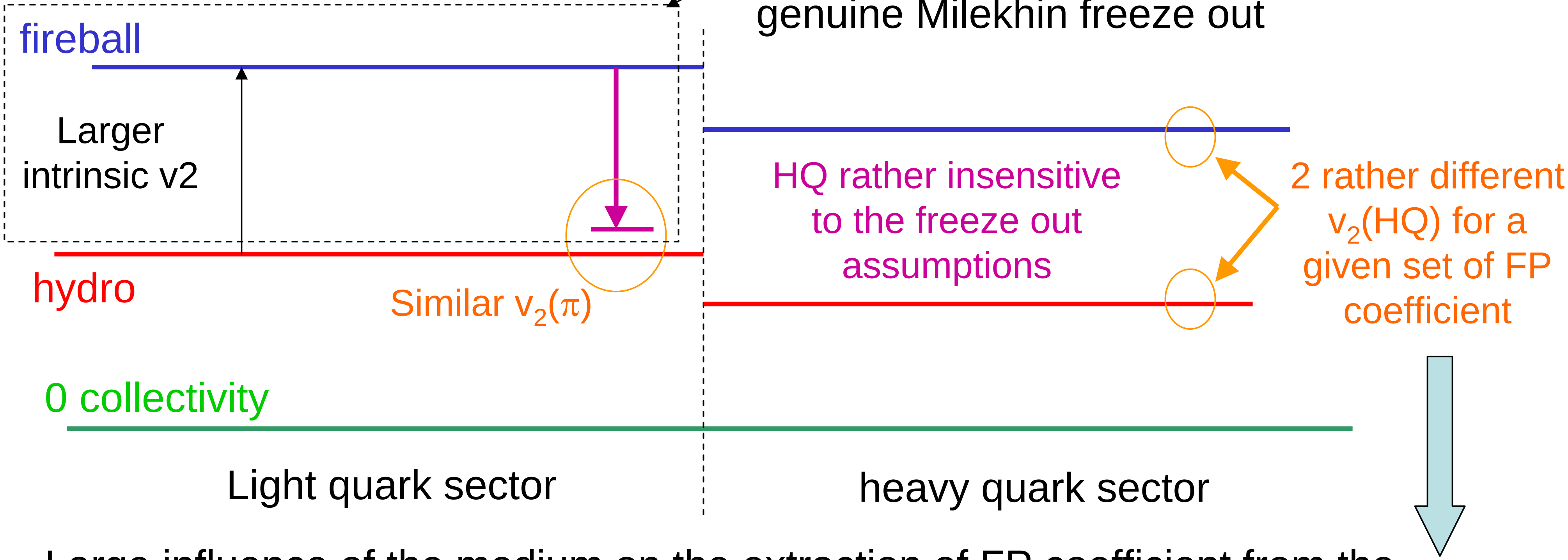
“Bona fide” is sometimes too optimistic

Summary and conclusions (2)

Third global level of interpretation:

After all, the KH hydro with a kinetic freeze out a given rather all energy density is a model among others…

Good candidate for medium with a genuine Milekhin freeze out



Large influence of the medium on the extraction of FP coefficient from the experimental data

First study, not exhaustive and to be pursued