

# Inclusive Production of Heavy Flavor and Quarkonium

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# Heavy Flavor

heavy quark:  $Q = b$  or  $c$

charm hadrons:

$$H_c = D^0, D^+, D_s^+, \\ D^{*0}, D^{*+}, D_s^{*+}, \\ \Lambda_c^+, \dots$$

mesons

baryons

bottom hadrons:

$$H_b = B^-, \bar{B}^0, B_s^0, B_c^-, \\ B^{*-}, \bar{B}^{*0}, B_s^{*0}, B_s^{*0}, \\ \Lambda_b, \dots$$

mesons

baryons

# Production of Heavy Flavor

## Creation of the heavy quark

- What are the relevant parton processes?
- Can they be calculated using perturbative QCD?

## Binding with light quarks to form heavy hadron

- Can it be parametrized by a few functions or (better yet) by a few constants?\*
- Can they be measured in experiment or (better yet) calculated using QCD?

\* Albert Einstein:

“Make everything as simple as possible, but not simpler.”

# Heavy Quark Cross Section

$$\sigma[b] = \sum_{H_b} \sigma[H_b]$$

- summed over **heavy** hadrons
- integrated over momentum
- calculable using **perturbative QCD**  
as power series in  $\alpha_s(m_b)$
- calculated to **NNLO**

# *Differential Cross Section* for *Specific Heavy Hadron*

Simple model

consistent with **heavy quark** cross section

$$d\sigma [H_b] = d\sigma [b] \times P_{b \rightarrow H_b}$$

$P_{b \rightarrow H_b}$ : probability for  **$b$**  to hadronize into  **$H_b$**   
assumed independent of process ( $e^+e^-$ ,  $ep$ ,  $pp$ ,  $pp$ )  
independent of kinematics ( $p_T, y$ )

# Differential Cross Section for **b** Quark

Leading Order:  $\alpha_s^2(m_b)$

$$q\bar{q} \rightarrow b\bar{b}$$

$$gg \rightarrow b\bar{b}$$

NLO

Nason, Dawson, and Ellis | 1989

NNLO

$q\bar{q}, gq, g\bar{q}$ : complete

Korner, Merebashvili, & Rogal, Anastasios & Aybat,  
Kniehl et al, Czakon 2008

$gg$ : only LO in  $1/N_c$  or expansion in  $m_b^2/s$

Production of Heavy Flavor

# Heavy Quark Cross Section

Most reliable predictions are for highest possible energies:

Tevatron (and now LHC)

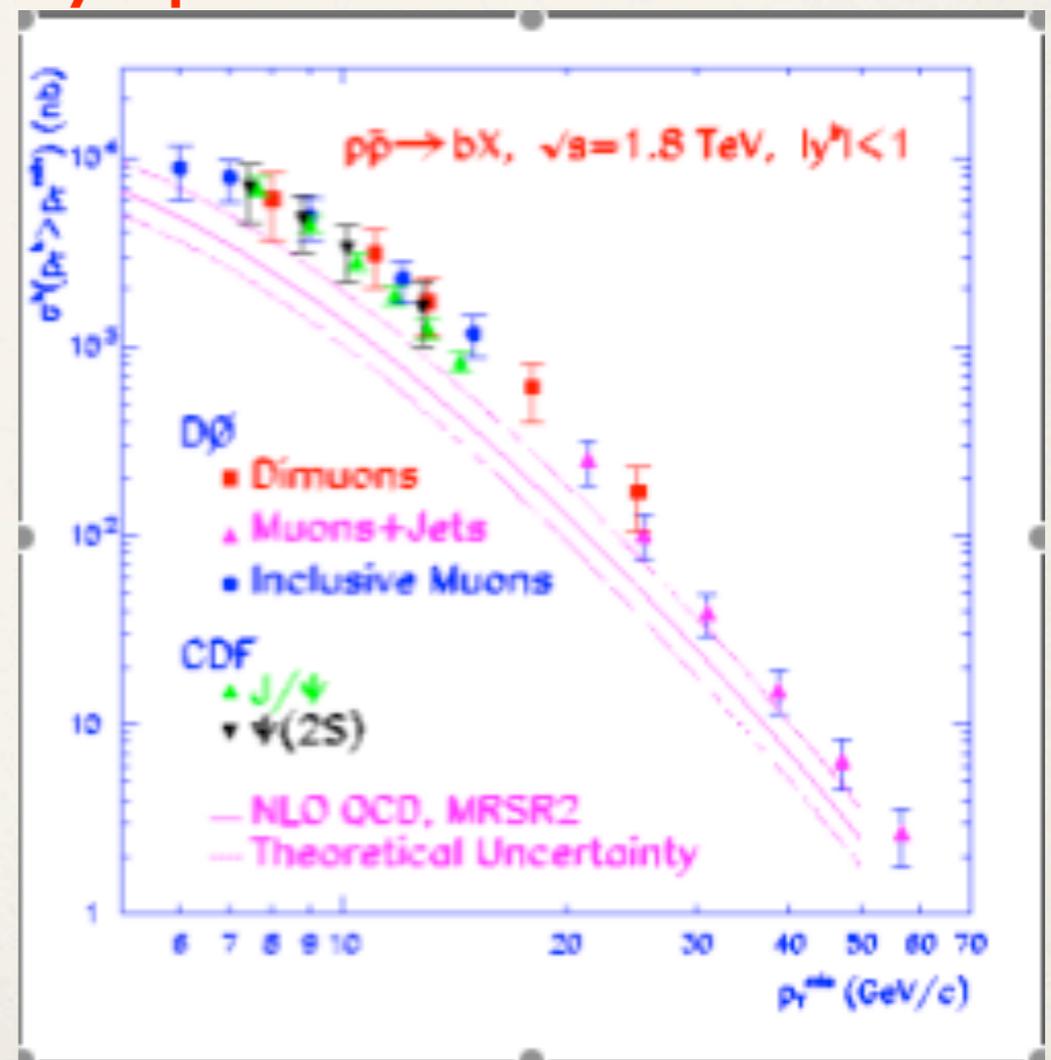
Predictions should be more reliable for bottom than charm

For many years, measurements of heavy quark cross section

at the Tevatron were

... consistent with theory for charm

... systematically lower for bottom



Problem may be resolved by taking into account fragmentation

# PQCD Factorization Theorem

for inclusive single-particle production

Collins & Soper 1982

Production of hadron with large transverse momentum  
is dominated by fragmentation

- hard scattering produces parton with larger momentum
- parton hadronizes into a jet that includes the hadron
- probability for hadron to carry fraction  $z$  of jet momentum  
is given by fragmentation function  $D(z)$

# PQCD Factorization Theorem

Collins & Soper 1982

Production of ***b* hadron** with **large transverse momentum** is dominated by **fragmentation** of ***b* quark** or **gluon**

$$\begin{aligned} d\sigma[H_b(P)] &= \int_0^1 dz d\hat{\sigma}[b(P/z)] D_{b \rightarrow H_b}(z) \\ &+ \int_0^1 dz d\hat{\sigma}[g(P/z)] D_{g \rightarrow H_b}(z) \\ &+ \mathcal{O}(m_b^4/p_T^4) + \mathcal{O}(\Lambda_{\text{QCD}}^2/p_T^2) \end{aligned}$$

- **parton cross section  $d\hat{\sigma}$**   
calculate using **PQCD**, power series in  **$\alpha_s(p_T)$**
- **fragmentation functions  $D_{i \rightarrow H}(z)$**   
different for  **$B, B^*, \Lambda_b, \dots$**   
nonperturbative but universal  
determine from  **$e^+e^-$**  annihilation

# $J/\psi$ from $b$ hadron decays

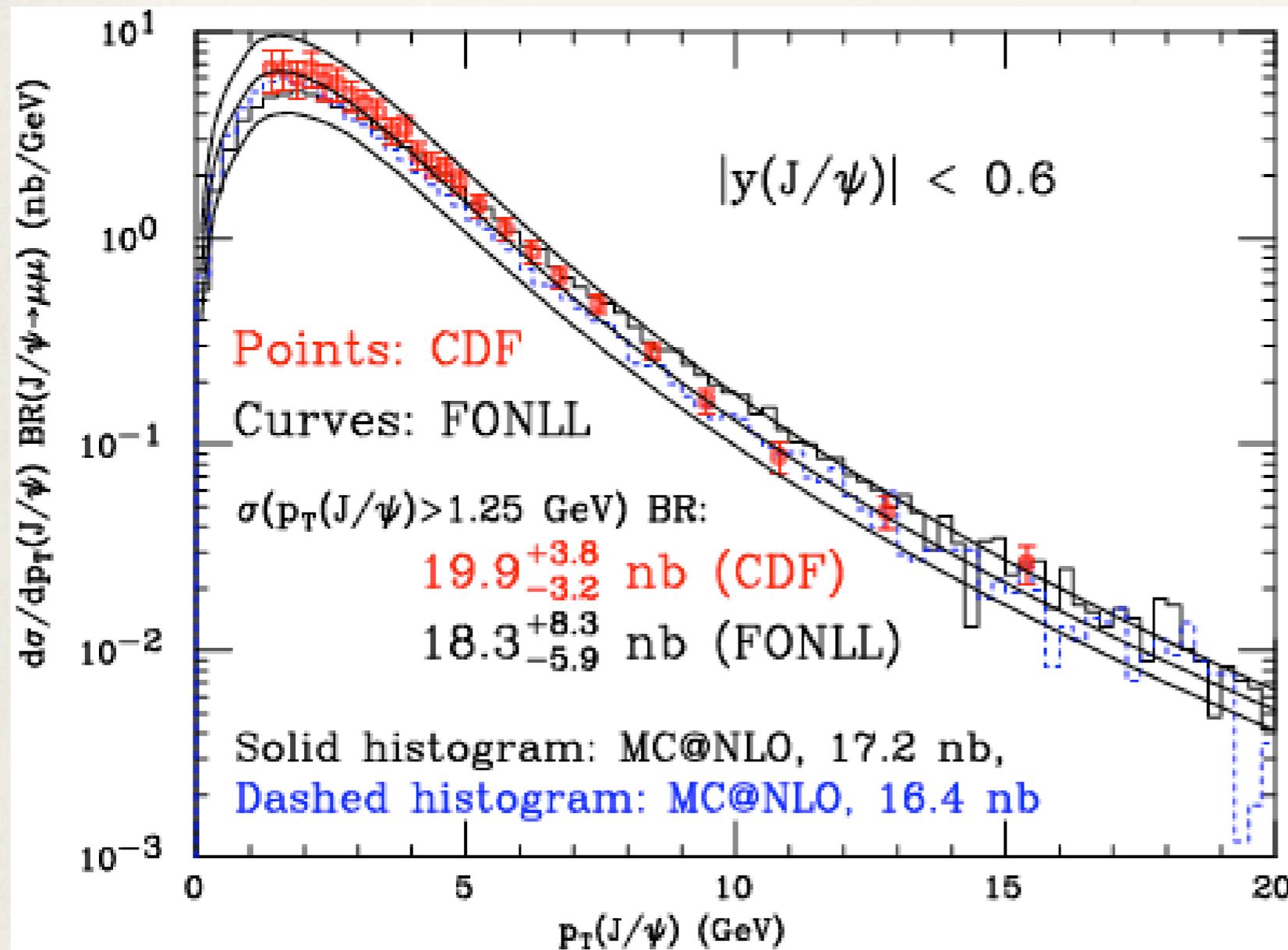
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## State-of-the-art analysis

- FONLL scheme
  - at moderate  $p_T$ , Fixed Order (NLO) calculation
  - at large  $p_T$ , fragmentation approximation
    - with logarithms of  $p_T/m_b$  summed to Next-to-Leading-Log accuracy
  - matching at intermediate  $p_T$
- Nonperturbative inputs
  - gluon and quark parton distributions
  - fragmentation function  $D(z)$  for  $b$  quark to  $H_b$ 
    - fit to data from LEP
  - branching fraction and decay spectrum for  $H_b$  to  $J/\psi$ 
    - fit to data from Belle, Babar

# $J/\psi$ from $b$ hadron decays at the Tevatron

- ✧ Cacciari, Frixione, Mangano, Nason, Ridolfi 2004



- Good agreement with the data
- Scale, mass and PDF uncertainties summed in quadrature

# Heavy Quark Cross Section

Solution to the problem of the  
 $b$  quark cross section at the Tevatron

- should be tested at the LHC
- should also apply to charm (but with larger errors)
- requires nonperturbative functions rather than constants\*

\* Albert Einstein:

“Make everything as simple as possible, but not simpler.”

# Production at **Large Rapidity** or $x_F$

## **Leading particle effect**

enhancement of **charm hadrons**

whose **light flavors** coincide with **constituents** of colliding **hadron**  
in forward direction of colliding **hadron**

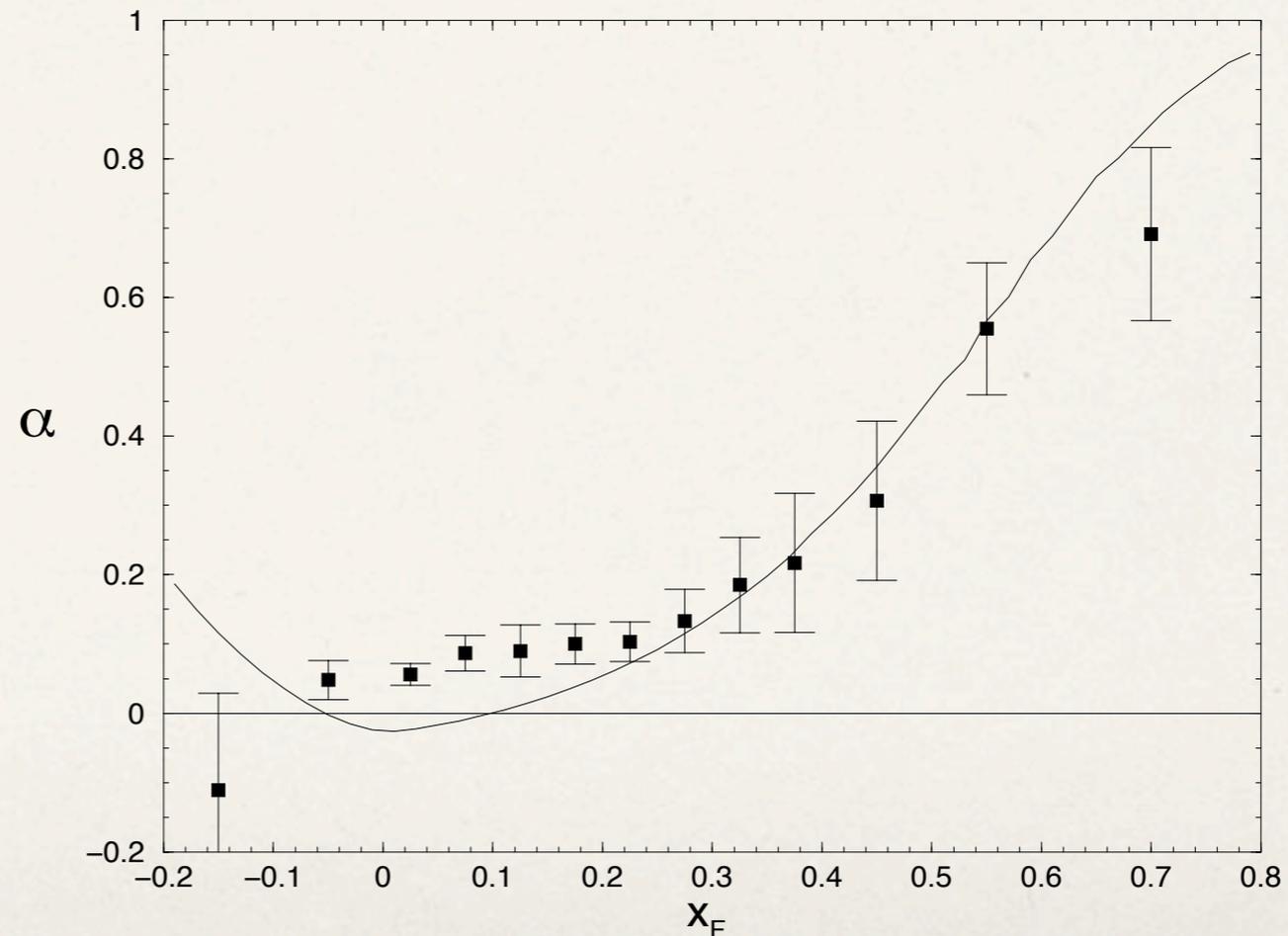
(e.g. enhancement of  $\bar{D}^0 = \bar{c}u$ ,  $D^- = \bar{c}d$   
in forward direction of colliding **proton** =  $uud$ )

leads to dramatic asymmetries between **charm hadrons**  
and their **antiparticles**

## Production at Large $x_F$

charm asymmetry: 
$$\alpha = \frac{\sigma[D^+] - \sigma[D^-]}{\sigma[D^+] + \sigma[D^-]}$$

photoproduction  
data from **E791**



# Charm Asymmetries

Perturbative QCD asymmetries at NLO  
but too small by order of magnitude

Pythia asymmetries too small with standard tunings  
can be tuned to reproduce observed asymmetries

## Recombination

$\bar{c}$  from hard scattering,  $q$  from proton remnant Kartvelishvili, Likhoded  
& Slabopitskii 1978  
 $\bar{c}$  and  $q$  from hard scattering Braaten, Jia & Mehen 2004

## Intrinsic Charm

$\bar{c}$  and  $q$  from proton remnant Brodsky, Hoyer, Peterson & Sakai 1980  
 $q$  from hard scattering,  $\bar{c}$  from proton remnant Cuautle, Herrera  
& Magnea 1998

## Light Quark Fragmentation de Deus & Duraes 2000

$q$  from hard scattering,  $\bar{c}$  from fragmentation of  $q$

# Production at **Large Rapidity** or $x_F$

- dramatic asymmetries

- no rigorous theory

- only **models**

most involve adjustable **functions**

exception: **heavy quark** recombination (Braaten, Jia & Mehen)

involves adjustable **constants** (too simple?)

- to be tested by **LHC-b, Alice?**

# Heavy Quarkonium

## charmonium

$$H_{c\bar{c}} = \eta_c, J/\psi, \eta_c', \psi'$$
$$\chi_{c0}, \chi_{c1}, \chi_{c2}$$

S-waves  
P-waves

## bottomonium

$$H_{b\bar{b}} = \eta_b, \Upsilon, \eta_b', \Upsilon', \eta_b'', \Upsilon''$$
$$\chi_{c0}, \chi_{c1}, \chi_{c2}, \chi_{c0}', \chi_{c1}', \chi_{c2}'$$

S-waves  
P-waves  
D-waves

## $B_c$ mesons

$$H_{\bar{b}c} = B_c, B_c^*, \dots$$

# Production of Heavy Quarkonium

## Creation of heavy quark pair

- What are the relevant parton processes?
- Can they be calculated using perturbative QCD?

## Binding to form heavy quarkonium

- Can it be parametrized by a few functions or better yet by a few constants?\*
- Can they be measured in experiments or better yet calculated using QCD?

\* Albert Einstein:

“Make everything as simple as possible, but not simpler.”

# Production of Heavy Quarkonium

Creation of heavy quark pair?

Binding to form heavy quarkonium?

## Theoretical developments

- Color-Singlet Model 1976-1981
- Color-Evaporation Model 1977
- Fragmentation Mechanism 1993
- NRQCD Factorization 1995
- $Q\bar{Q}$  Fragmentation 2010

# Color-singlet Model

Ellis, Einhorn, Quigg 1976; Carlson and Suaya 1976; Kuhn 1980; Degrand, Toussaint 1980; Kuhn, Nussinov, Ruckl 1980; Wise 1980; Chang 1980; Baier, Ruckl 1981; Berger, Jones 1981

- $c \bar{c}$  is created by **parton collisions**  
with negligible relative momentum
- $c \bar{c}$  can bind into **charmonium** only if it is created  
in same **color/angular momentum** as in **charmonium**
  - $\underline{1}^3S_1$  for  $J/\psi$
  - $\underline{1}^3P_J$  for  $\chi_{cJ}$
- **probability** that  $c \bar{c}$  binds into **charmonium**  
is determined by **wavefunction near origin**
  - $\propto R(0)$  for  $J/\psi, \eta_c$
  - $\propto R'(0)$  for  $\chi_{cJ}, h_c$

one **constant** for each multiplet  
can be determined from **annihilation decays**:

$J/\psi$	$\rightarrow$	$e^+ e^-$
$\chi_{c0}$	$\rightarrow$	$\gamma \gamma$

# Color Evaporation Model

Fritzsch 1977; Halzen 1977

- $c \bar{c}$  pair is created by **parton collisions** with invariant mass below  $D \bar{D}$  threshold (between  $2m_c$  and  $2m_D$ )
- $c \bar{c}$  pair can bind into **charmonium** regardless of its **color/angular momentum** state
- **probability** that  $c \bar{c}$  binds to form **charmonium  $H$**  is adjustable constant  $f_H$  for each multiplet

# Color-singlet Model vs Color Evaporation Model

Theoretical status in early 1990's

- inconsistency of CSM:  
infrared divergences for P-waves  $\chi_{cJ} \rightarrow q \bar{q} g$   
 $b \rightarrow \chi_{cJ} + s + g$
- perturbative corrections  
CSM: LO calculations only  
CEM: NLO, using heavy quark cross section  
Nason, Dawson, Ellis 1988
- Dominant theoretical prejudice  
CSM: can probably be extended  
to a theory based on QCD  
CEM: purely phenomenological model

# Color-singlet Model vs Color Evaporation Model

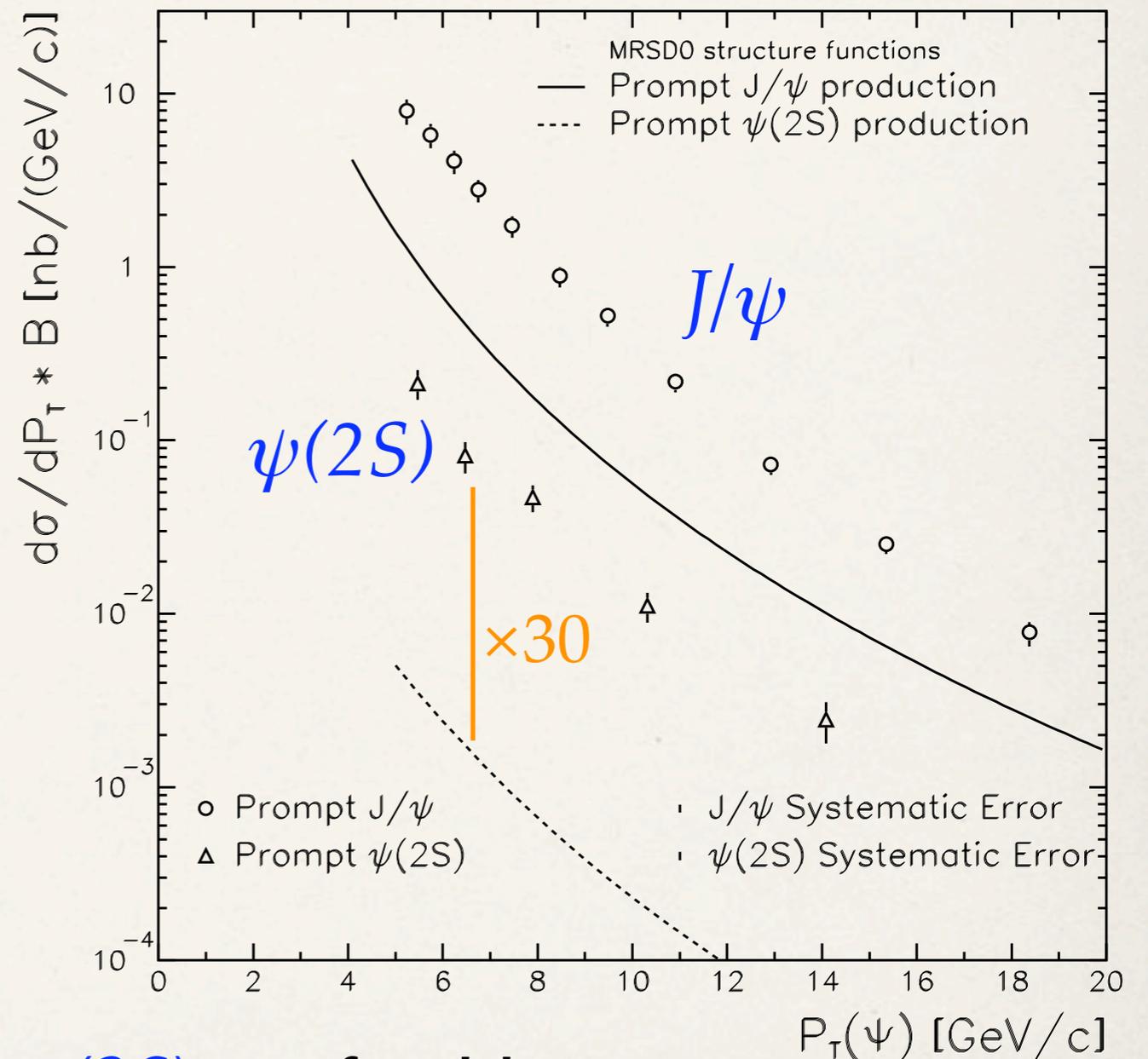
Experimental status in early 1990's

- Fixed target experiments ( $pN, \pi N, \gamma N$ )  
feeddown to  $J/\psi$  from  $\psi(2S), \chi_{cJ}$  decays  
contributions from small  $p_T \Rightarrow$  nonperturbative?  
  
large experimental errors  
roughly compatible with CSM or CEM
- $p \bar{p}$  collisions at the Tevatron  
feeddown to  $J/\psi$  from  $\psi(2S), \chi_{cJ}$  decays  
feeddown from  $B$  decays  
 $p_T > 5 \text{ GeV} \Rightarrow$  perturbative?  
  
production rates much larger than predicted by CSM?

# Demise of Color-singlet Model

CDF collaboration 1997

- use vertex detector to remove  $B$  feeddown



- prompt  $J/\psi$ : complicated by  $\psi(2S)$ ,  $\chi_c$  feeddown
- prompt  $\psi(2S)$ : 30 times larger than CSM prediction  
(in retrospect, compatible with CEM)

# Fragmentation mechanism

PQCD Factorization Theorem Collins & Soper 1983

at sufficiently large transverse momentum  $p_T \gg m_Q$   
quarkonium production must be dominated by fragmentation  
of heavy quark and gluon

$$d\sigma[H_{c\bar{c}}(P)] = \sum_{i=c,\bar{c},g} \int_0^1 dz d\hat{\sigma}[i(P/z)] D_{i \rightarrow H_{c\bar{c}}}(z) + \mathcal{O}(m_c^2/p_T^2)$$

in Color-Singlet Model,  
fragmentation functions  $D(z)$  for quarkonium can be calculated  
using perturbative QCD Braaten and Yuan 1993  
but not accurate approximation at accessible values of  $p_T$

# NonRelativistic QCD

Caswell and Lepage 1986

- effective field theory for  $Q\bar{Q}$  sector of QCD  
at energies  $\ll m_Q$  from  $Q\bar{Q}$  threshold
- in quarkonium, small velocity  $v$  is generated dynamically  
by balance between potential energy and kinetic energy  
charmonium:  $v^2 \approx 1/3$   
bottomonium:  $v^2 \approx 1/10$
- nonperturbative effects  
can be organized according to their scaling with  $v$
- Lattice NRQCD Lepage et al. 1992  
calculate properties of quarkonium nonperturbatively

# NRQCD Factorization

Bodwin, Braaten & Lepage 1995

Conjectured factorization formula

for inclusive production of charmonium  $H$   
motivated by perturbative QCD factorization theorems

$$d\sigma[H] = \sum_n d\hat{\sigma}[c\bar{c}(n)] \langle \mathcal{O}_n^H \rangle$$

- sum over color/angular momentum channels  
1 or 8  $^1S_0, ^3S_1, ^1P_1, ^3P_0, ^3P_1, ^3P_2, \dots$
- parton cross sections for creating  $c\bar{c}$   
expand in powers of  $\alpha_s(m_c)$
- NRQCD matrix elements for formation of  $H$   
scale as definite powers of  $v$

# NRQCD factorization

- **Color-Singlet Model** reproduced  
by color-singlet terms of leading order in  $v$
- solves problem of infrared divergences for P-waves
- **Color Evaporation Model** reproduced by S-wave terms

Simplest viable truncation of expansion in  $v$

- for **S-waves**, truncate after order  $v^7$

$$J/\psi : \quad \langle \underline{1} \ ^3S_1 \rangle \sim v^3$$
$$\langle \underline{8} \ ^3P_J \rangle, \langle \underline{8} \ ^1S_0 \rangle, \langle \underline{8} \ ^3S_1 \rangle \sim v^7$$

$\Rightarrow$  4 universal constants for  $J/\psi$ ,  $\eta_c$

(I determined by  $J/\psi \rightarrow e^+e^-$ )

- for **P-waves**, truncate after order  $v^5$

$$\chi_{cJ} : \quad \langle \underline{1} \ ^3P_J \rangle, \langle \underline{8} \ ^3S_1 \rangle \sim v^5$$

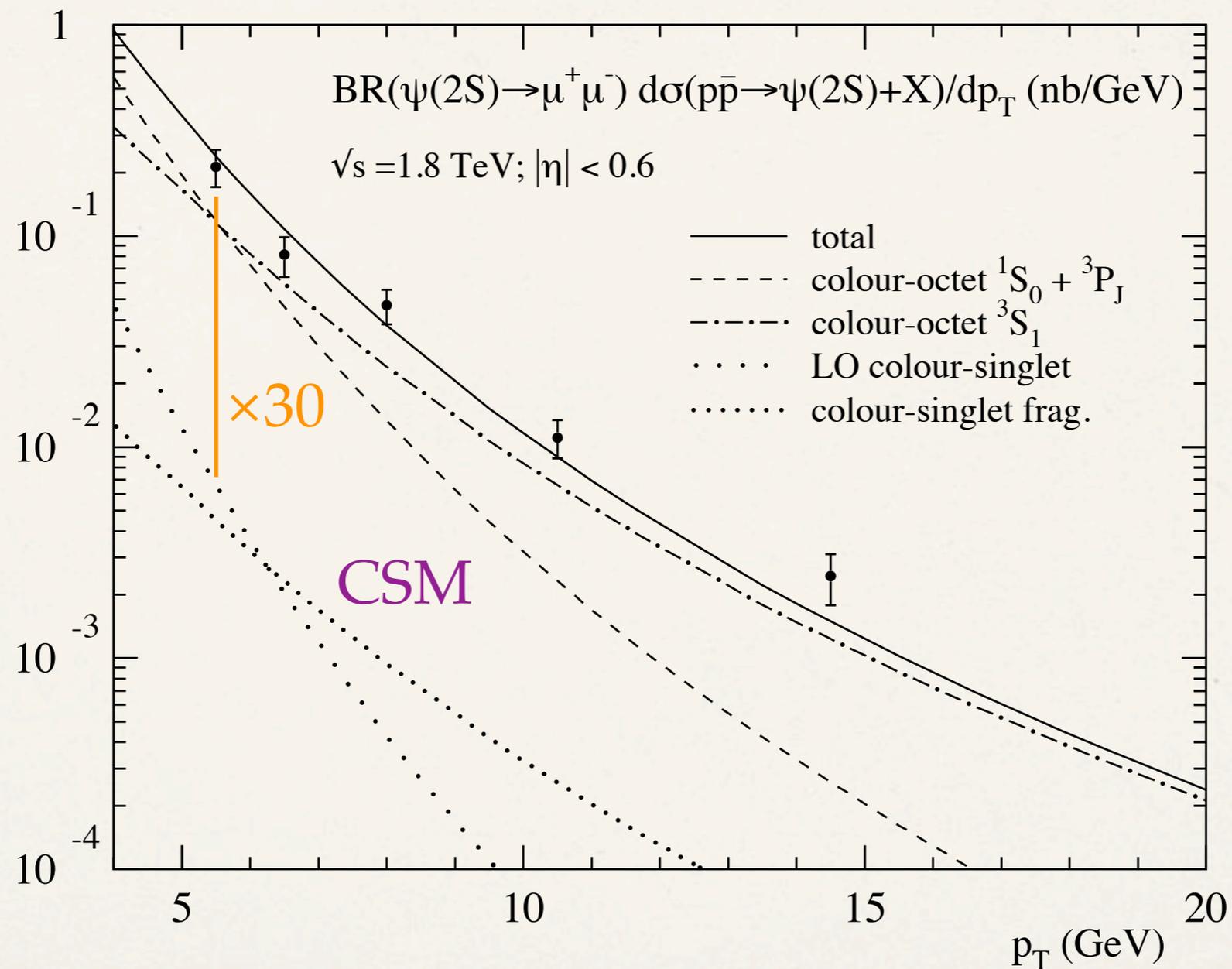
$\Rightarrow$  2 universal constants for  $\chi_{c0}$ ,  $\chi_{c1}$ ,  $\chi_{c2}$ ,  $h_c$

(I determined by  $\chi_{c0} \rightarrow \Upsilon\Upsilon$ )

# NRQCD Factorization

NRQCD factorization can accommodate the Tevatron data

CDF data on prompt  $\psi(2S)$



fit  $\langle \underline{8} \ ^3S_1 \rangle$  and  $\langle \underline{8} \ ^1S_0 \rangle$  (or  $\langle \underline{8} \ ^3P_J \rangle$ )

Kramer 2001

# Parton cross sections

$$d\sigma[H] = \sum_n d\hat{\sigma}[c\bar{c}(n)] \langle \mathcal{O}_n^H \rangle$$

accurate predictions require at least **NLO** in  $\alpha_s$

for **charmonium**,  $\alpha_s(m_c) \approx 0.25$

for **bottomonium**,  $\alpha_s(m_b) \approx 0.18$

- **photoproduction**

Kramer, Zunft, Steegborn, Zerwas 1995; Kramer 1996  
 Artoisenet, Campbell, Maltoni, Tramontano 2009  
 Chang, Li, Wang 2009; Li, Chao 2009  
 Butenschoen, Kniehl 2009

- **$\gamma\gamma$  collisions**

Klasen, Kniehl, Mihaila, Steinhauser 2005

- $e^+ e^- \rightarrow$  **double charmonium**

Zhang, Gao, Chao 2005; Zhang, Ma, Chao 2008  
 Gong, Wang 2008

- $e^+ e^- \rightarrow$  **charmonium + X**

Zhang, Chao 2006; Ma, Zhang, Chao 2008  
 Gong, Wang 2008, 2009  
 Zhang, Ma, Wang, Chao 2009

- **hadron collisions**

Petrelli, Cacciari, Greco, Maltoni, Mangano 1988  
 Campbell, Maltoni, Tramontano 2008; Artoisenet, Lansberg, Maltoni, 2008  
 Li, Wang 2008; Gong, Wang 2008; Gong, Li, Wang 2009  
 Ma, Wang, Chao 2010; Butenschoen, Kniehl 2010

# NRQCD Factorization

## NLO perturbative QCD corrections

- increase cross sections by large  $K$  factors  
in some channels but not in others  
(large  $K$  factors in Color-Singlet Model channels  
decreases importance of color-octet terms)
- have removed most dramatic discrepancies  
between NRQCD factorization and experiment  
for spin-averaged cross sections  
(an important exception is polarization)
- fragmentation contribution appears at  
LO in color-octet  $^3S_1$  channel  
NLO in most channels  
NNLO in color-singlet  $^3S_1$  channel  
⇒ NLO in  $\alpha_s$  is not NLO accuracy at large  $p_T$

# Polarization

## NRQCD factorization

predicts the polarization of quarkonium  
with no additional parameters

dramatic qualitative prediction for hadron collisions:

direct  $J/\psi$ ,  $\Upsilon$  transversely polarised at large  $p_T$

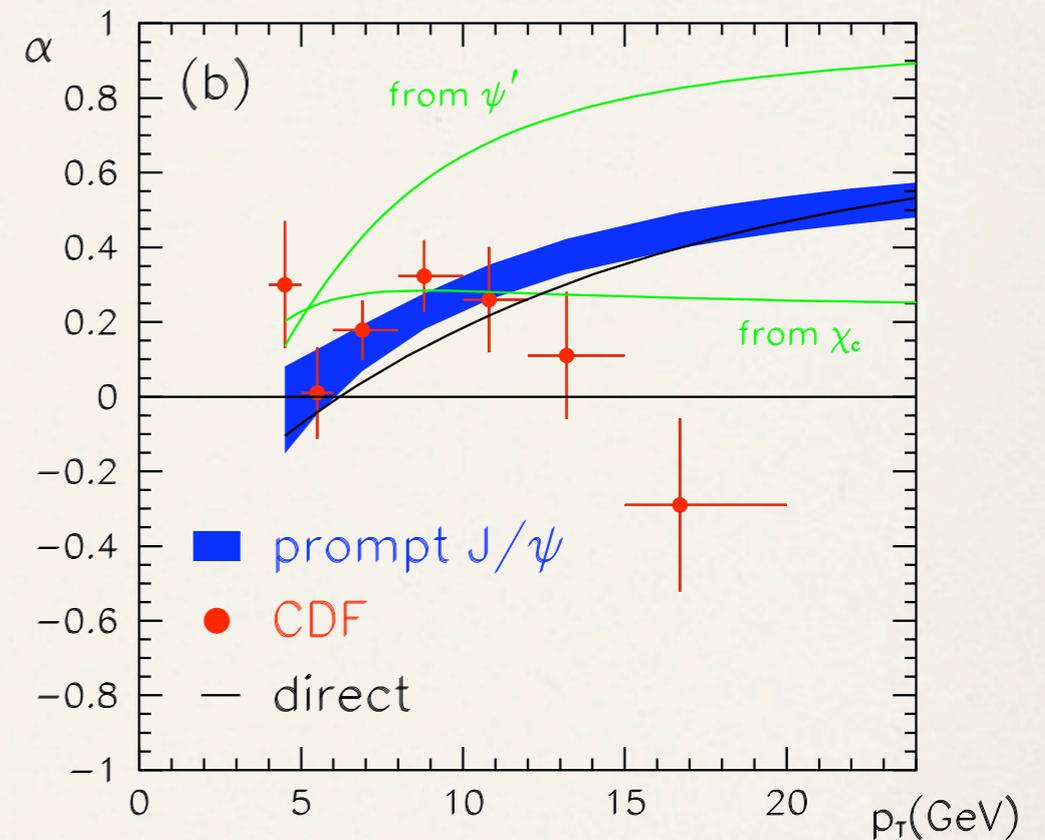
Cho, Wise 1995

- at sufficiently large  $p_T$ , charmonium production is dominated by gluon fragmentation
- at LO in  $\alpha_s$ , gluon fragments into color-octet  $c\bar{c}$  pair that inherits transverse polarization of gluon
- at LO in  $v$ , hadronization into  $^3S_1$  charmonium preserves transverse polarization of  $c\bar{c}$  pair

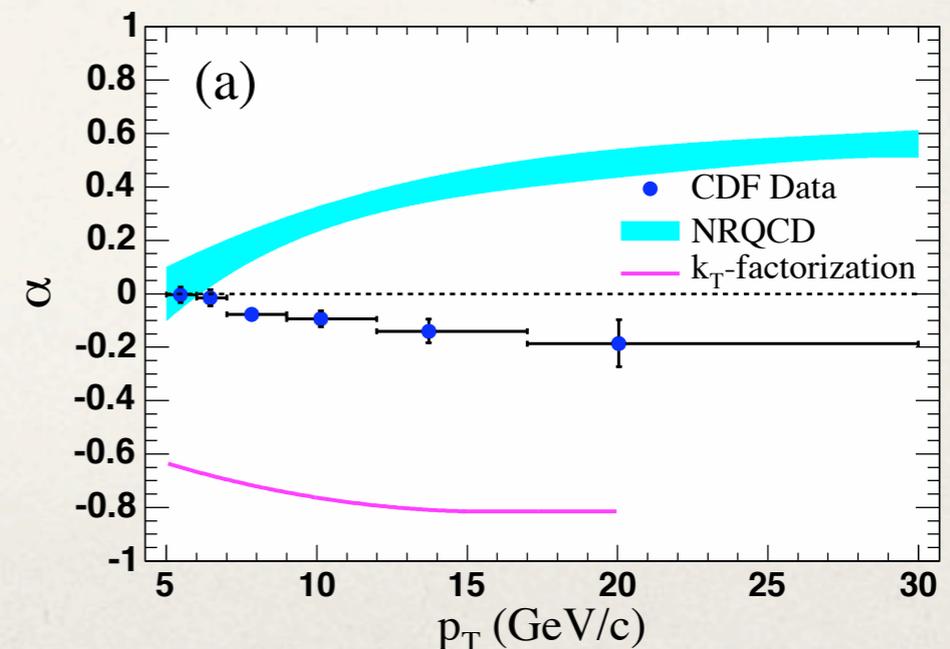
# Polarization of $J/\psi$

- NRQCD predictions at **LO** fail dramatically in comparison with **CDF data**
- **NLO** calculations for **polarization** are incomplete but do not seem to improve agreement
- incompatibility between **CDF data** from **Run I** and **Run II**

Run I:



Run II:



# $Q\bar{Q}$ Fragmentation

Dramatic new development in 2010!

Proof of factorization for **quarkonium** production at **large  $p_T$**   
Kang, Qiu, Sterman (in progress)

- at leading order in  $m_Q/p_T$ ,  
fragmentation of single partons ( $Q, \bar{Q}, g$ )  
Collins & Soper 1983
- at order  $m_Q^2/p_T^2$ ,  
 $Q\bar{Q}$  fragmentation into **quarkonium**
- ● at order  $m_Q^4/p_T^4$ , factorization breaks down

$$\begin{aligned} d\sigma[H_{c\bar{c}}(P)] &= \sum_{i=c,\bar{c},g} \int_0^1 dz d\hat{\sigma}[i(P/z)] D_{i \rightarrow H_{c\bar{c}}}(z) \\ &+ \int_0^1 dz d\hat{\sigma}[c\bar{c}(P/z)] D_{c\bar{c} \rightarrow H_{c\bar{c}}}(z) \\ &+ \mathcal{O}(m_c^4/p_T^4) \end{aligned}$$

# $Q\bar{Q}$ Fragmentation

Rigorous factorization formula for **quarkonium** production  
separates **hardest** momentum scales  $\sim p_T$   
from **softer** momentum scales  $\lesssim m_Q$   
Kang, Qiu, Sterman (in progress)

$\sim p_T$ : **parton cross sections** for producing  $Q$ ,  $\bar{Q}$ ,  $g$ , and  $Q\bar{Q}$   
calculable using **PQCD** as power series in  $\alpha_s(m_Q)$

$\lesssim m_Q$ : **fragmentation functions** for  $Q$ ,  $\bar{Q}$ ,  $g$ , and  $Q\bar{Q}$   
into **quarkonium**  
nonperturbative (but involves **hard** scale  $m_Q$ )

needed: **NLO** calculations of **parton cross sections** for  $Q\bar{Q}$   
determination of **fragmentation functions**

# Quarkonium Production

In rigorous factorization formula for quarkonium production, fragmentation functions for  $Q$ ,  $\bar{Q}$ ,  $g$ , and  $Q\bar{Q}$  into quarkonium involve momentum scales  $\lesssim m_Q$

NRQCD factorization can plausibly be used to separate hard scale  $m_Q$  from softer nonperturbative scales reduce fragmentation functions to a few constants for each multiplet

needed: NLO calculations of fragmentation functions

Albert Einstein:

“Make everything as simple as possible, but not simpler.”

# Summary

## Heavy hadron production

Rigorous factorization formula at **large  $p_T$**   
involves **nonperturbative fragmentation functions**  
**parton cross sections** should soon be available at **NNLO**

## Quarkonium production

Rigorous factorization formula at **large  $p_T$**   
involves **nonperturbative fragmentation functions**  
**NRQCD factorization** could reduce them to **constants**  
**parton cross sections** at **NLO** available for moderate  **$p_T$**   
**NLO** calculations of **fragmentation functions** also needed

Albert Einstein:

“Make everything as simple as possible, but not simpler.”