Lattice QCD for thermal heavy-quark physics

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Heavy Quark Production in Heavy-Ion Collisions, Purdue Jan 2011

Outline

- Introduction: bottomonium physics at finite temperature
- Bottomonium in lattice QCD:
  - Effective theories: NRQCD and Fermilab; Relativistic simulations.
- Non-zero temperature simulations in lattice QCD:
  - Anisotropic lattices, $\xi = a_s/a_t$ for $N_f = 2(+1)$.
  - Summary of parameters from previous results for charmonium.
  - Theoretical advantages of NRQCD in nonzero temperature simulations.
- Results:
  - Zero temperature.
  - Υ and χ_b in the plasma
- Conclusions and work in progress.

Quarkonium at nonzero temperature

- $Q\bar{Q}$ states important probes of dynamics of QGP.
- Charmonium suppression observed (SPS,RHIC).
- Melting of bound states reduces quarkonium production but the converse is not necessarily true. Interpretation of suppression patterns difficult for charmonium but may be less so for bottomonium. Rapp et al, arXiv:0807.2470].
- Bottomonium copiously produced at LHC energies.
- Precision study of suppression pattern and its link with the spectrum of bound states possible.
- A theoretical study of bottomonium at nonzero temperature is timely.

Methods for $Q\bar{Q}$ at nonzero temperature (T)

Potential Models:

- at nonzero T choice of potential model unclear since potential in Schroedinger equation not rigorously defined. [Mocsy, arXiv:0811.0337].
- recent progress in HQET at $T \neq 0$, where potential can be defined. [Brambilla arXiv:1010.0805 and refs.]
- in these EFTs weak-coupling arguments require scales to be hierarchically ordered eg $M \gg T > g^2 M > gT \gg g^4 M$ or $M \gg g^2 M > T \gg gT \gg g^4 M$.

Path Integral:

- new approach based on path integral formulation. In-medium effects on heavy quark modelled by Coulomb interaction. [Beraudo et al, arXiv.1005.1245].

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Lattice QCD:

- nonperturbative, systematically improvable approach for QCD.
- anisotropic lattices for simulations at nonzero temperature.
- need methods for heavy quarks (an old story).
- interplay of heavy quarks and thermal field theory.
- unquenched ($N_f = 2$) simulations.
  - In quenched calculations (light) quark loops are neglected.
  - Typically $T_c^{N_f=0} \sim 1.5 T_c^{N_f=2+1}$ whereas $T_c^{N_f=2} \sim T_c^{N_f=2+1}$.
  - No thresholds ($D\bar{D}$ or $B\bar{B}$) in quenched QCD.
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Potential models and Lattice QCD: Good to have different approaches which can be compared.

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Dynamical anisotropic lattices at nonzero T

- A large number of points in the time direction required.
- To reach $T = 2T_c$, $\mathcal{O}(10)$ points needed $\Rightarrow a_\tau \sim 0.025 fm$.
- Far too expensive with an (dynamical) isotropic ($a_s = a_\tau$) lattice $\Rightarrow$ anisotropic lattice, $a_\tau \ll a_s$ a solution.
- Gives an independent handle on temperature ($T = 1/a_\tau N_\tau$).

- Introduces additional parameters to the action.
- Non-trivial but now understood tuning problem [PRD 74 014505 (2006)].
- First results for $c\bar{c}$ spectral functions in unquenched QCD [PRD76 (2006) 094513].

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Heavy quarks in lattice qcd: an old story

In lattice QCD care must be taken that $am_q < 1$ to control errors $\mathcal{O}(am_q)$.
$am_b < 1$ on isotropic lattices, is still not (really) feasible.

- effective field theories (EFT):
  - Static: $m_Q \to \infty$; renormalisable; but $m_b \sim 5Gev$ far from ∞.
  - NRQCD: m_Q non-relativistic; expansion in HQ velocity v; nonrenormalisable; systematically improvable,
  - Fermilab: mass-dependent renormalisation of parameters; smooth interpolation from light to heavy; expensive to improve beyond $\mathcal{O}(a)$
- anisotropic lattices: $a_\tau << a_s$ so that $a_\tau m_b < 1$; requires large ξ; $\mathcal{O}(a_s m_q)$ effects?

For bottomonium at nonzero temperature combine NRQCD and anisotropic lattices.

NRQCD and finite temperature

- In the effective thermal field theory setup NRQCD is the first theory obtained when integrating out UV dof.
- NRQCD relies only on the scale separation $M \gg T$ and we study temperatures up to $2T_c \simeq 400$ MeV, so its application is fully justified.
- The T dependence in NRQCD correlators is due to the thermal medium, and not also to thermal boundary conditions (unlike the relativistic case).

NRQCD and finite temperature

in more detail:

- At nonzero T, spectroscopy for relativistic quarks hampered by periodicity of the lattice in the τ direction and reflection symmetry of mesonic correlators, so eg
$$G(\tau) = \int_0^\infty \frac{d\omega}{\pi} \frac{\cosh\left[\omega(\tau - 1/2T)\right]}{\sinh\left(\omega/2T\right)}\rho(\omega).$$
Nontrivial spectral weight at small ω (energy) yields a constant τ-independent contribution to the correlator, which must be treated with care. Casting doubt on results for the melting or survival of $c\bar{c}$ at high T [Umeda, PRD75(2007) 094502; Petreczky EPJ.C62 (2009) 85].
- In NRQCD
$$G(\tau) = \int_{-2M}^\infty \frac{d\omega'}{\pi} \exp(-\omega'\tau)\rho(\omega').$$
writing $\omega = 2M + \omega'$ and dropping terms suppressed when $M >> T$.
$\rightarrow$ no thermal boundary problems.

NRQCD and finite temperature

What to expect when quarks aren't bound?

Consider free quarks in continuum NRQCD with energy $E_{\mathbf{p}} = \mathbf{p}^2/2M$. The correlators for the S and P waves are of the form [Burnier et al, JHEP0801 (2008) 043].

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\begin{aligned}
G_S(\tau) &\sim \int \frac{d^3p}{(2\pi)^3} \exp(-2E_{\mathbf{p}}\tau) \sim \tau^{-3/2} \\
G_P(\tau) &\sim \int \frac{d^3p}{(2\pi)^3} \mathbf{p}^2 \exp(-2E_{\mathbf{p}}\tau) \sim \tau^{-5/2},
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power law decay for large euclidean time (τ).

With modifications from interactions, finite lattice spacing, volume effects etc in the realistic case.

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Lattices and temperatures

Summary of the lattice data set for $\xi = a_s/a_t = 6$ and lattices $N_s^3 \times N_\tau$ and NRQCD at $\mathcal{O}(v^4)$.

N_s	N_τ	a_τ^{-1}	T(MeV)	T/T_c	No. of Conf.
12	80	7.35GeV	90	0.42	74
12	32	7.06GeV	221	1.05	500
12	24	7.06GeV	294	1.40	500
12	16	7.06GeV	441	2.09	500

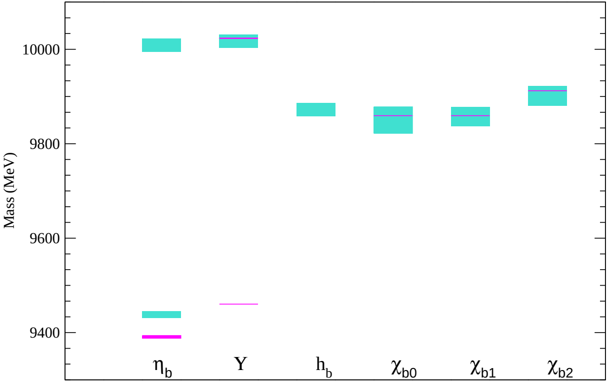
The lattice spacing is set using the $1P - 1S$ spin-averaged splitting in charmonium [ARXIV:1005.1209].

Results: Zero temperature spectroscopy

J^{PC}	state	$a_\tau \Delta E$	Mass (MeV)	Exp. (MeV) (PDG)
0^{-+}	$^1S_0(\eta_b)$	0.118(1)	9438(7)	9390.9(2.8)
0^{-+}	$^1S_0(\eta_b[2S])$	0.197(2)	10009(14)	-
1^{--}	$^3S_1(\Upsilon)$	0.121(1)	9460*	9460.30(26)
1^{--}	$^3S_1(\Upsilon')$	0.198(2)	10017(14)	10023.26(31)
1^{+-}	$^1P_1(h_b)$	0.178(2)	9872(14)	-
0^{++}	$^3P_0(\chi_{b0})$	0.175(4)	9850(28)	9859.44(42)(31)
1^{++}	$^3P_1(\chi_{b1})$	0.176(3)	9858(21)	9892.78(26)(31)
2^{++}	$^3P_2(\chi_{b2})$	0.182(3)	9901(21)	9912.21(26)(31)

Zero temperature bottomonium spectroscopy from NRQCD. *The $1^3S_1(\Upsilon)$ state is used to set the scale.

Results: Zero temperature spectroscopy



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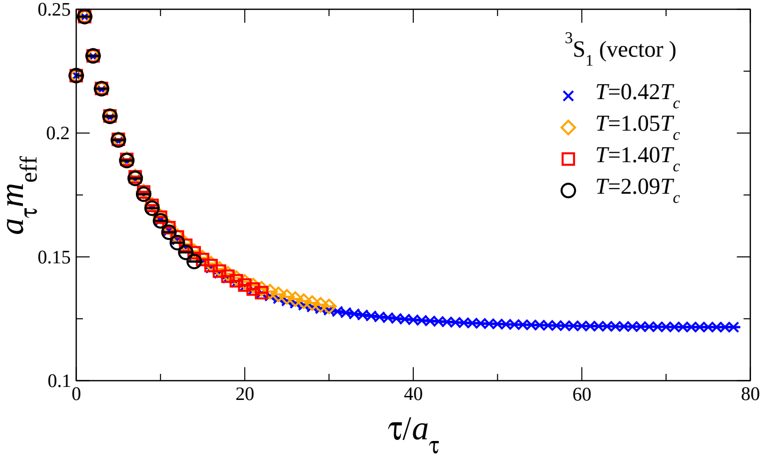
Bottomonium in the plasma

Aim: see transition from exponential decay in the hadronic phase, $G(\tau) \sim \exp(-\Delta E \tau)$, ie bound states, to power law decay, $G(\tau) \sim \tau^{-\gamma}$, ie quasi-free behaviour.

- focus on the S wave 1^{--} (vector) channel and P waves 0^{++} (scalar), 1^{++} (axial-vector), and 2^{++} (tensor) channels.

Results: S wave effective massesd

For bound states $G(\tau) \sim e^{-m\tau}$ at large τ and $m_{\mathrm{eff}}(\tau) = -\log[G(\tau)/G(\tau - a_\tau)]$. Exponential decay $\rightarrow$ τ-independent plateau.



Lowest T has plateau at large τ. Good agreement for all temperatures.

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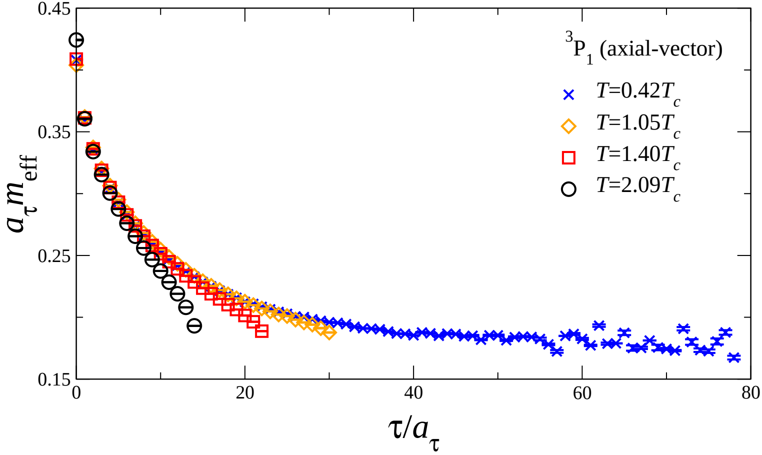
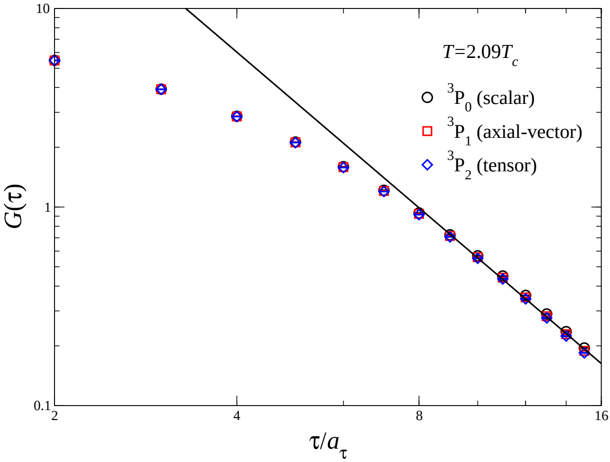


Rules out pure exponential decay at highest Ts.

P waves: propagators on a log-log scale, at $T = 2.09T_c$



Line $G(\tau) = c\tau^{-d}$; gives $c = 223.2(5)$, $d = 2.605(1)$, $[\tau/a_\tau = 10 - 15]$.
(recall continuum noninteracting value 5/2.)

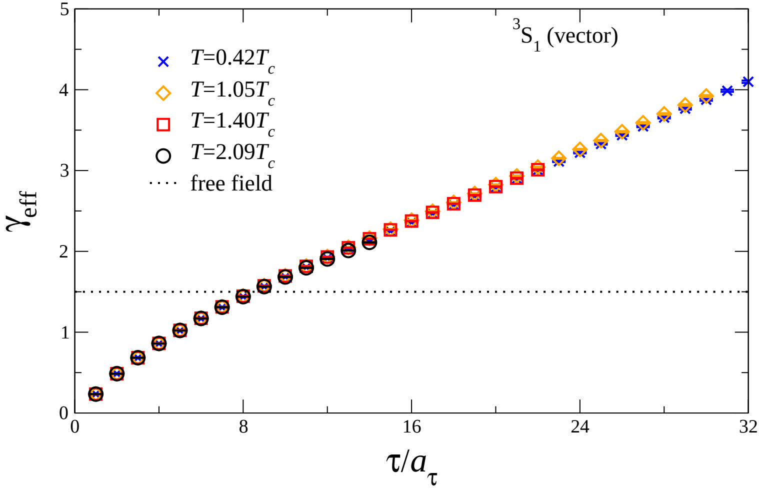
Effective power plots

Visualise approach to quasi-free behaviour using

$$\gamma_{\text{eff}}(\tau) = -\tau \frac{G'(\tau)}{G(\tau)} = -\tau \frac{G(\tau + a_\tau) - G(\tau - a_\tau)}{2a_\tau G(\tau)},$$

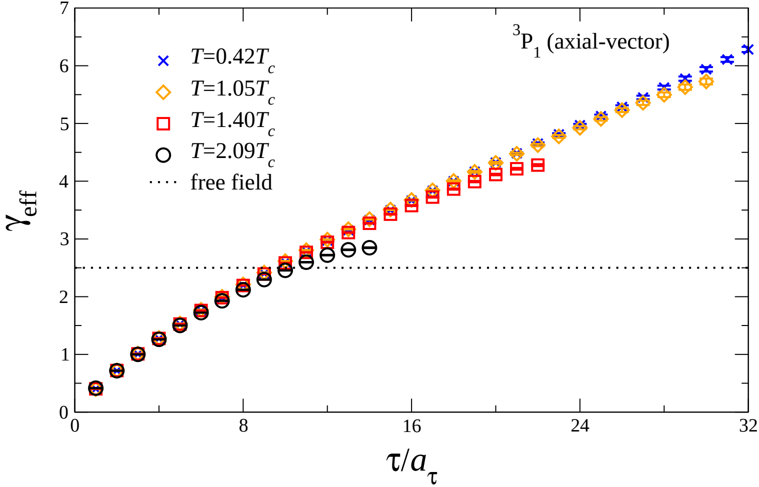
- For power decay, $G(\tau) \sim \tau^{-\gamma}$, a constant result.
- for exponential decay, $G(\tau) \sim \exp(-\Delta E \tau)$, a linearly rising result, $\gamma_{\text{eff}}(\tau) = \Delta E \tau$.

Effective power: S waves



- essentially no temperature dependence.
- dotted line - effective exponent in the continuum noninteracting limit.

Effective power: P waves



- tendency to flatten out: power decay at large euclidean time.
- effective exponent tends towards noninteracting result at highest temperature we consider.

Conclusions, outlook and further work

- Studied S and P wave bottomonium at high T with anisotropic lattice simulations, $N_f = 2$ and NRQCD.
- NRQCD is well justified in the range of temperatures explored, and has many technical advantages.
- S wave vector correlator: no T-dependence up to $2.09T_c$, P wave correlators sensitive to the thermal medium immediately above T_c.
- Power-law decay of P-wave propagators visible at $T = 1.4T_c$, while at $T \simeq 2T_c$, we found consistency with nearly-free dynamics.
- Indications that Υ is not sensitive to QGP up to $T \sim 2T_c$ which P waves may melt at much lower T.
- γ_{eff}, is temperature dependent, approaches the noninteracting result at the highest temperature considered.
- Future work includes: extracting spectral functions using MEM and a comparision with relativistic bottomonium results.