# Heavy quark diffusion and hadronization in dense matter

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Purdue University, Jan.4,2011

# Outline

- Introduction: heavy quark as a probe for the hot medium
- Langevin simulation of heavy quark diffusion
  ------ Fokker-Planck-Langevin formalism vs equilibrium
  ------ heavy quark thermal relaxation rate
  ------ description of the medium: AZHYDRO
  ------ numerical results for charm/bottom quark
- Hadronization scheme
  ------ hadronization criterion
  ------ Resonance Recombination
  ------ coalescence vs fragmentation
  ------ numerical results for D/B mesons
- Non-photonic decay: electron observables
  ------ B vs D: decay
  ------ numerical results
- Discussion & Summary: other possible contributions

# Introduction: heavy quark probe

## >> Light partons

- fast thermalization: loss of memory
- constitute the bulk of the medium: sQGP vs hydro

## >> Probe the medium

- interacting with and modified by the medium
- jet quenching, J/psi suppression …

## >> Heavy quark

- too heavy to get fully equilibrated with the surrounding medium
- primordial production: hard process
- number conserved through medium evolution

$$\tau_Q = \frac{m_Q}{T}\tau_q \sim 6 * \tau_q > \tau_{QGP}$$

➔Heavy quarks make thus a precious probe for the medium

# Introduction: continue

## >> Energy loss: radiative vs (elastic) collisional

- gluon-bremsstrahlung suppressed: dead cone, Dokshitzer & Kharzeev, 2001
- elastic collisions dominate at low pT: van Hees & Rapp , 2005
- pQCD scheme is insufficient to describe the energy loss and quenching: Moore & Teaney,2005

## >> Uncorrelated momentum kicks

- if thermalized: $p_{th} \sim \sqrt{m_Q T} \gg T \sim q_{tr}$
- Fokker-Planck description of heavy quark diffusion

## >> Experimental observables

- Heavy hadrons' non-photonic electron decay

$$R_{AA}(p_t;b) = \frac{\mathrm{d}N_Q^{AA}(b)/\mathrm{d}p_t}{N_{\mathrm{coll}}(b)\ \mathrm{d}N_Q^{pp}/\mathrm{d}p_t} \qquad v_2(p_t;b) = \frac{\int \mathrm{d}\phi \frac{\mathrm{d}N_Q^{AA}(b)}{\mathrm{d}p_t \mathrm{d}y \mathrm{d}\phi} \cos(2\phi)}{\int \mathrm{d}\phi \frac{\mathrm{d}N_Q^{AA}(b)}{\mathrm{d}p_t \mathrm{d}y \mathrm{d}\phi}}$$

# Langevin simulation: formalism

## >> Fokker-Planck equation

$$\frac{\partial f_Q(t,\vec{p})}{\partial t} = \frac{\partial}{\partial p_i}\{A_i(\vec{p})f_Q(t,\vec{p}) + \frac{\partial}{\partial p_j}[B_{i,j}(\vec{p})f_Q(t,\vec{p})]\}$$

$$A_i(\vec{p}) = \langle\langle (\vec{p}-\vec{p}\,')_i \rangle\rangle = p_i A(p)$$

drag: related to energy loss: $A(p^2) = (-dE/dx)/p$

$$B_{ij}(\vec{p}) = \frac{1}{2}\langle\langle (\vec{p}-\vec{p}\,')_i (\vec{p}-\vec{p}\,')_j \rangle\rangle = D(p)\delta_{ij}$$

diffusion matrix: diagonal

## >> Stochastic realization: Langevin

$$dx_j = \frac{p_j}{E}dt$$

$$dp_j = -\Gamma(p)p_j dt + \sqrt{2dt \cdot D(|\vec{p} + \xi d\vec{p}|)}\rho_j.$$

carried out in the heat bath rest frame

## >> Pre-point discretization scheme: $\xi = 0$

$$\Gamma(p)E(p)T = D(E(p)) - T\frac{\partial D(E(p))}{\partial E},$$

$$with \;\; \Gamma(p) = A(p)$$

# Langevin formalism (continue)

## >> Post-point discretization scheme: $\xi = 1$

$$D(E(p)) = \Gamma(p) E(p) T,$$

$$with \;\; \Gamma(p) = A(p) + 2\frac{\partial D(p)}{\partial p^2} = A(p) + \frac{1}{E(p)}\frac{\partial D(p)}{\partial E}.$$

## >> General equilibrium condition: in terms of A(p)

$$\frac{\partial D(E(p))}{\partial E} - \frac{D(E(p))}{T} + E(p)A(p) = 0$$

**this is a relativistic extension of the Einstein relation:** $D = \gamma m_Q T$

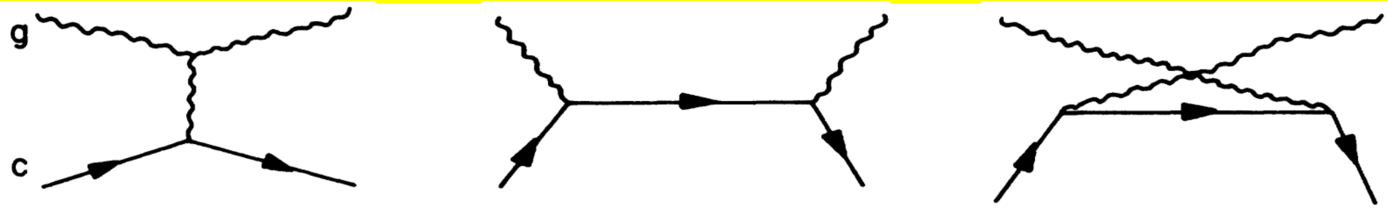
**drag:** $\langle \vec{p}(t) \rangle = \vec{p}_0 \exp(-\gamma t)$ **; diffusion:** $\langle \vec{p}^{\,2}(t) \rangle - \langle \vec{p}(t) \rangle^2 = \frac{3D}{\gamma}[1 - \exp(-2\gamma t)]$

**long time/large drag coefficient Boltzmann equilibrium limit checked:**

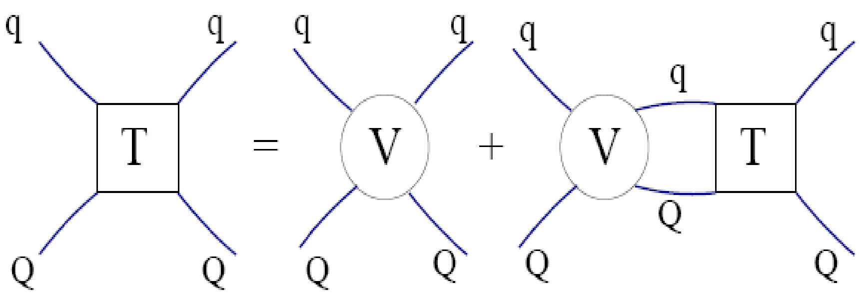
$$f(\vec{p}, \vec{x}) = e^{-E^*(\vec{p}^*)/T} = e^{-p\cdot u(x)/T}$$

# Langevin simulation: relaxation rate

**pQCD gluon-Q scattering: B.L.Combridge,1978**




**Heavy-light scattering T-matrix: Riek & Rapp,2010**



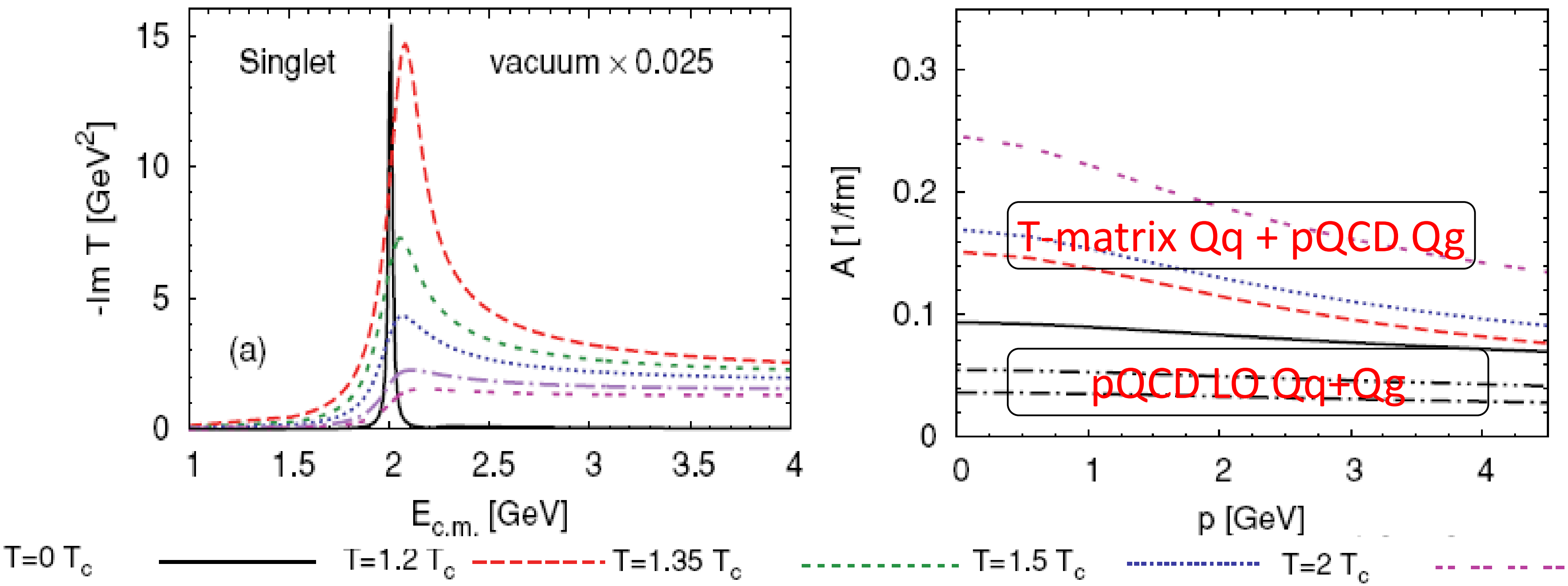

$$T_{l,a}(E;q',q) = \mathcal{V}_{l,a}(q',q) + \frac{2}{\pi}\int_0^\infty dk\,k^2\,\mathcal{V}_{l,a}(q',k) \times G_{12}(E;k)\,T_{l,a}(E;k,q)\{1-n_F[\omega_1(k)]-n_F[\omega_2(k)]\}.$$

static approximation: energy transfer $k_0 \simeq \vec{k}^2/2m_Q \ll |\vec{k}|$

lattice potential: open/hidden heavy flavor vacuum spectroscopy reproduced

heavy quarkonia (bound states) and heavy quark transport (scattering states) incorporated in the same framework and on a common basis (mutual constraints)

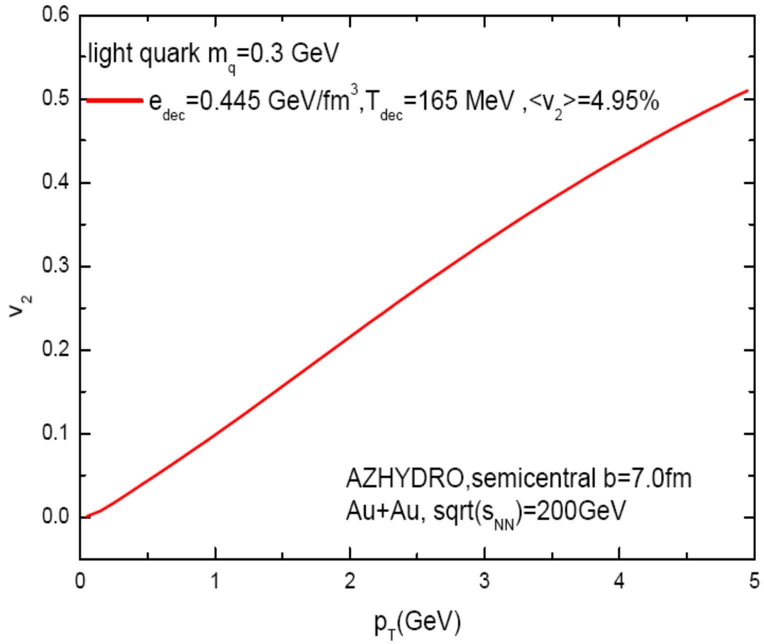
# Relaxation rate: T-matrix (continue)

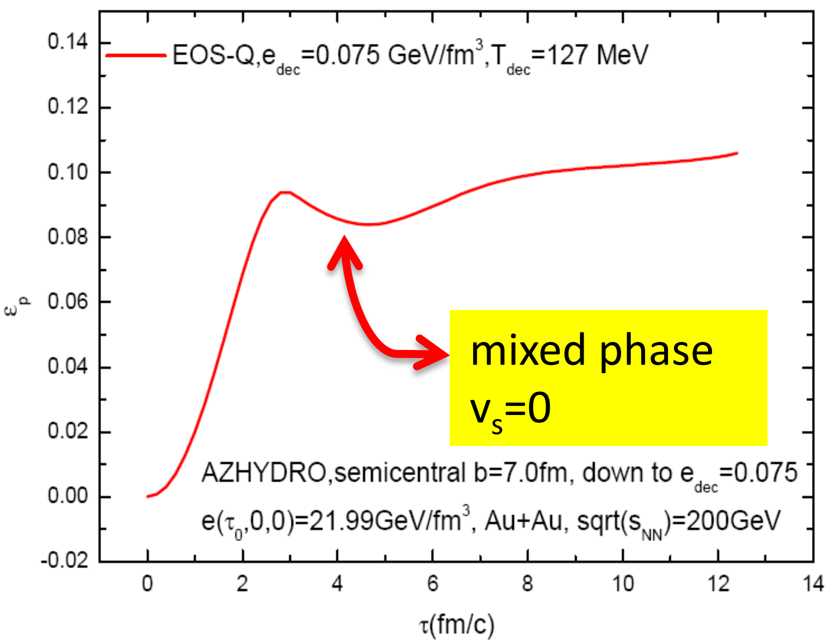



- the color single and antitriplet channels feature broad Feshbach resonance up to ~1.5 $T_C$
- this resonance correlation will be reiterated in our hadronization-coalescence model

- T-matrix relaxation rate: a factor ~4-5 larger than LO pQCD at T=1.2 $T_c$
- T-dependent behavior: screening potential vs light parton density
- p-dependent behavior: less contribution from threshold Feshbach resonance as p increases

# Langevin simulation: QGP medium



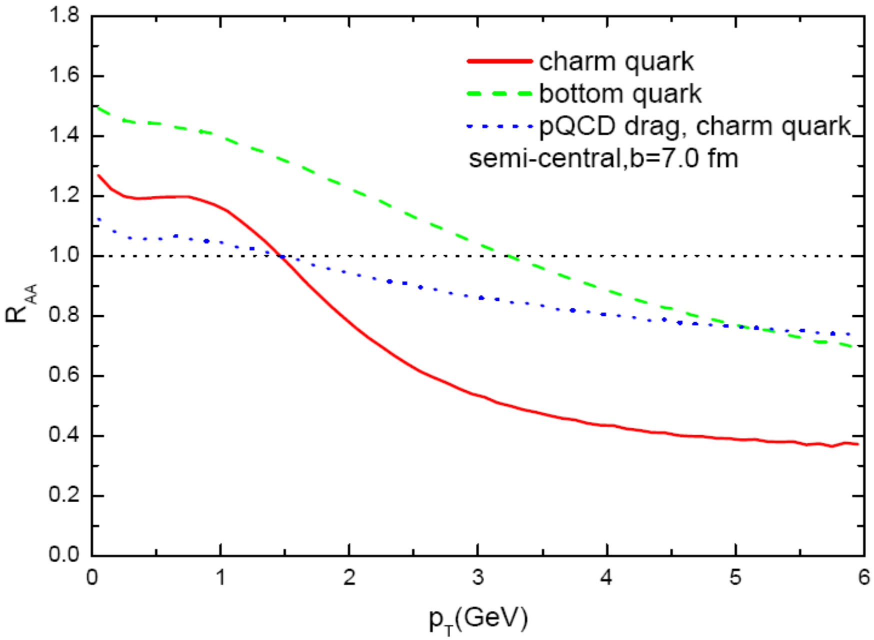



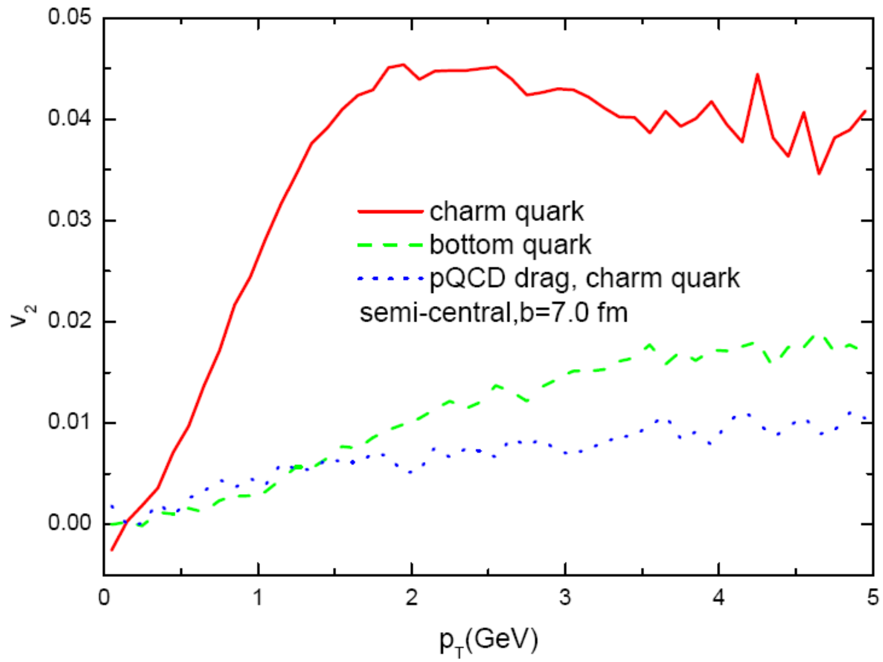

**background QGP medium: AZHYDRO, ideal; initialization time** $\tau_0 = 0.6\, fm/c$

**freeze-out at the end of mixed phase:** $e_{dec}$ **=0.445 GeV/fm³**

**Langevin preserves the boost invariance of hydro: eta →0, y → y - eta**

# Langevin simulation: numerical results




- **initialization: Glauber $n_{BC}(x,y)$ scaling & PYTHIA parametrization for $p_T$ spectrum , van Hees and Rapp, 2005:** $\frac{d^2N_c}{dp_T^2} = C\frac{(p_T + A)^2}{(1 + p_T/B)^\alpha}$
- **quenching: early stage when medium particles' density is high**
- **$v_2$ : develops at later stage when the medium particles' $v_2$ is large**

# Hadronization: Resonance Recombination

>> Hadronization = Resonance formation $c\bar{q} \rightarrow D$

→ *consistent with T-matrix finding of resonance correlations in QGP*

>> Realized by Boltzmann equation Ravagli & Rapp,2007

$$p^\mu \partial_\mu f_M(t,\vec{x},\vec{p}) = -m\Gamma f_M(t,\vec{x},\vec{p}) + p^0 \beta(\vec{x},\vec{p})$$

gain term

$$\beta(\vec{x},\vec{p}) = \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} f_q(\vec{x},\vec{p}_1) f_{\bar{q}}(\vec{x},\vec{p}_2) \times \sigma(s) v_{\rm rel}(\vec{p}_1,\vec{p}_2)\delta^3(\vec{p}-\vec{p}_1-\vec{p}_2)$$

Breit-Wigner

$$\sigma(s) = g_\sigma \frac{4\pi}{k^2} \frac{(\Gamma m)^2}{(s-m^2)+(\Gamma m)^2}$$

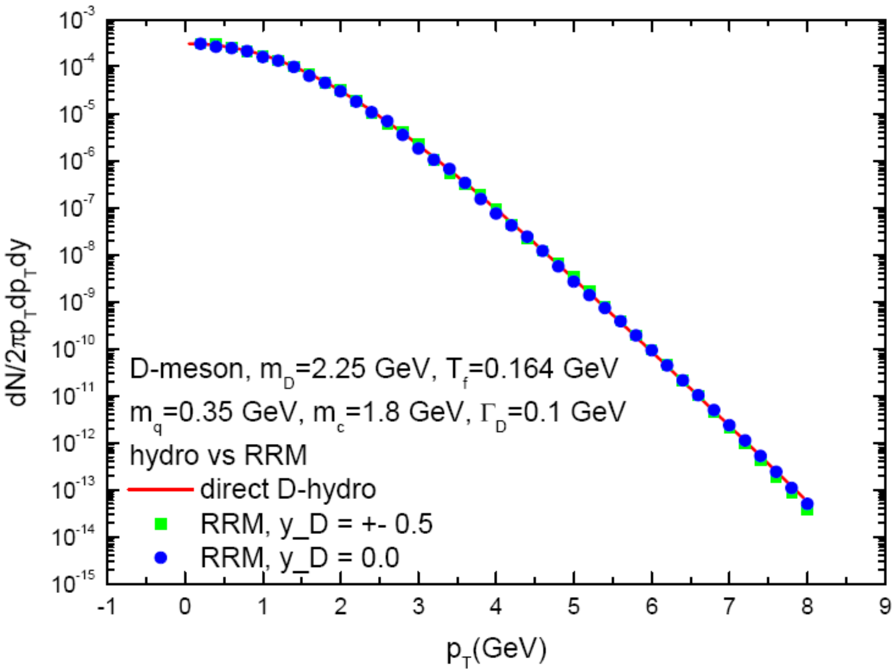
>> Equilibrium limit

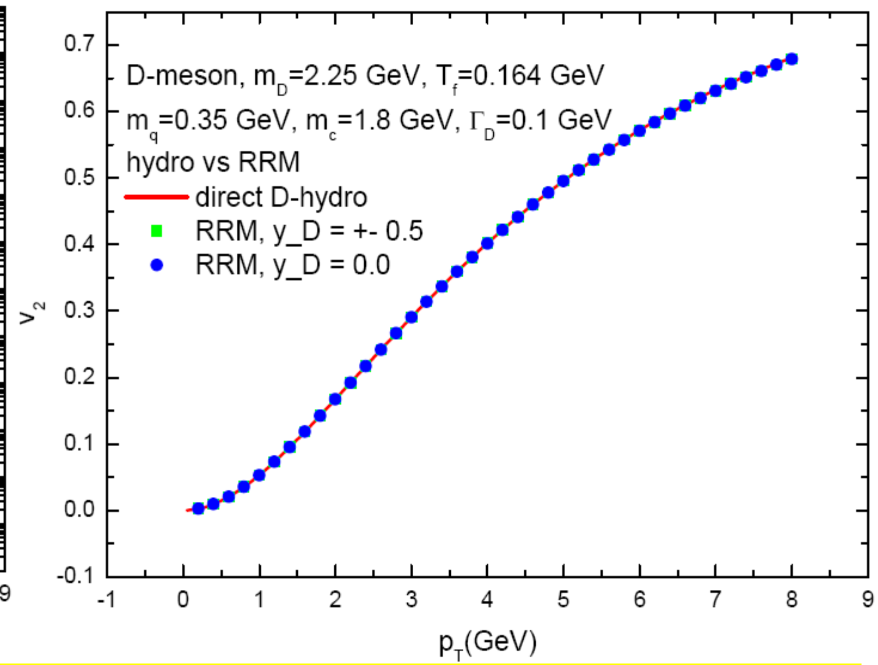
$$f_M^{\rm eq}(\vec{p}) = \frac{E_M(\vec{p})}{m\Gamma}\int d^3x \beta(\vec{x},\vec{p})$$

>> Energy conservation + detailed balance

→ equilibrium mapping

# Equilibrium Quark ➔ Equilibrium Meson

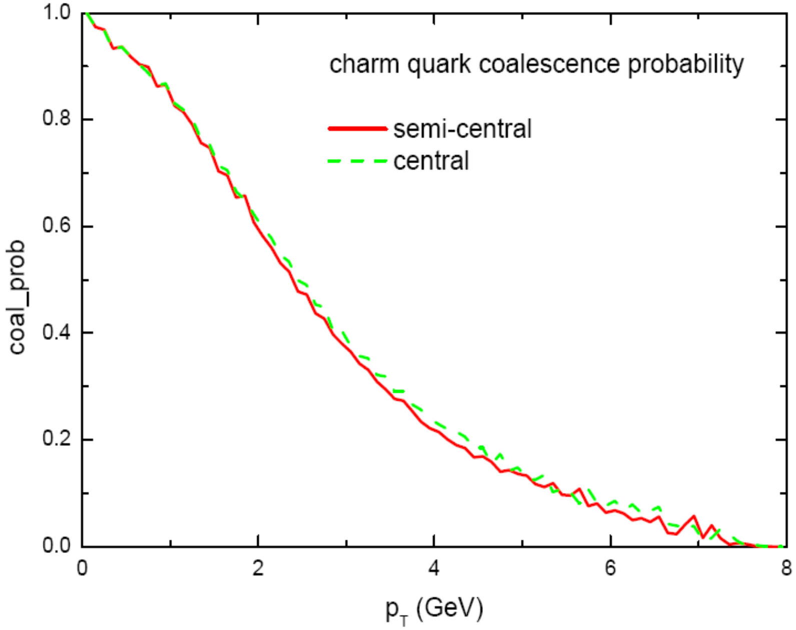


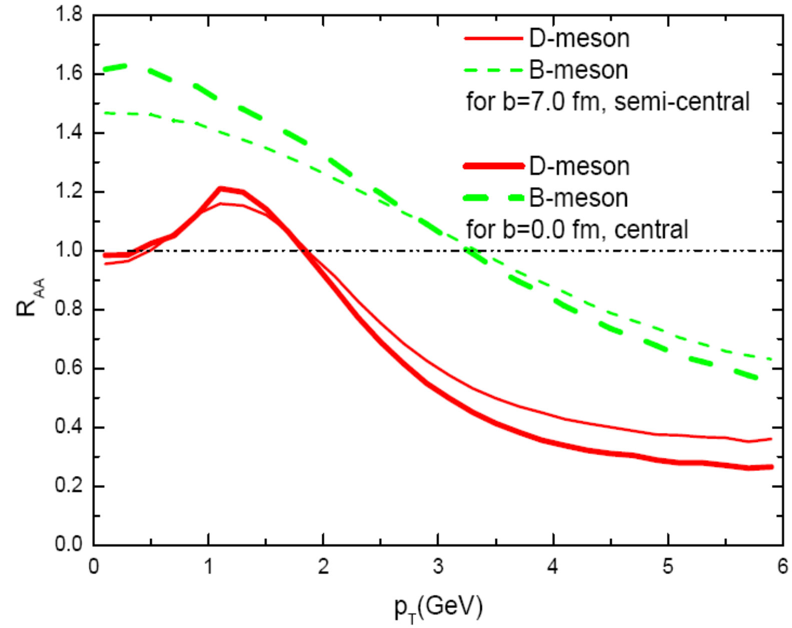




- **Kolb-Heinz hydro: space-momentum correlation in hydro included**
- **excellent equilibrium mapping achieved; boost invariance preserved**
- **c.f. M. He, R. J. Fries & R. Rapp, 2010**
- **large drag coeffi. Langevin-RRM (coalescence D-meson) equilibrium checked**

**➔RRM (compared to the conventional coalescence models) quite facilitates the description of the transition from low $p_T$ (equilibrium) to intermediate $p_T$ (kinetic) region in heavy-light quark recombination**

# Hadronization: coal. vs frag.

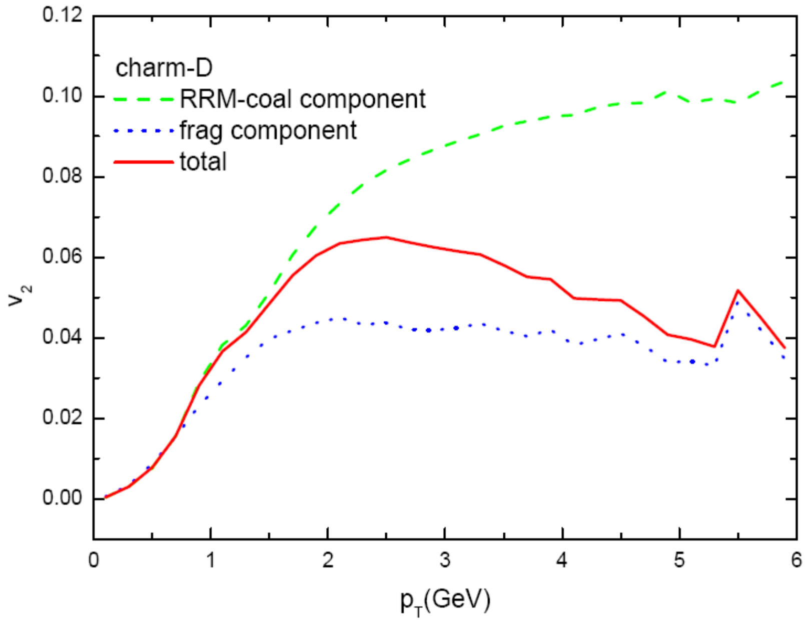


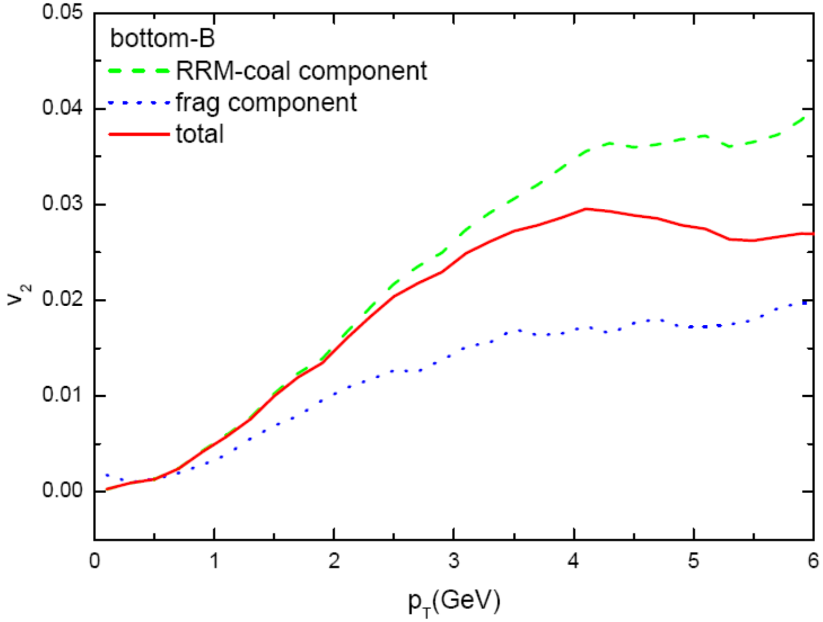




**charm quark coal. probability: high $p_T$ c-quark finds light quark of low phase density ; normalized to 100% at zero momentum**

**the remaining c-quarks get hadronized through independent fragmentation:** $D(z) = \delta(z-1)$

# Hadronization: coal. vs frag.



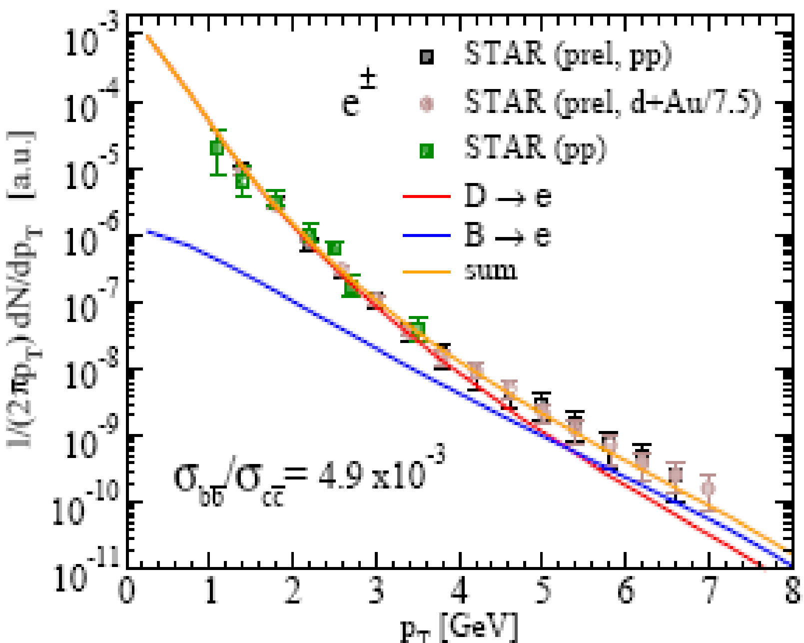




coal. vs frag. : relatively normalized with the calculated coal. probability

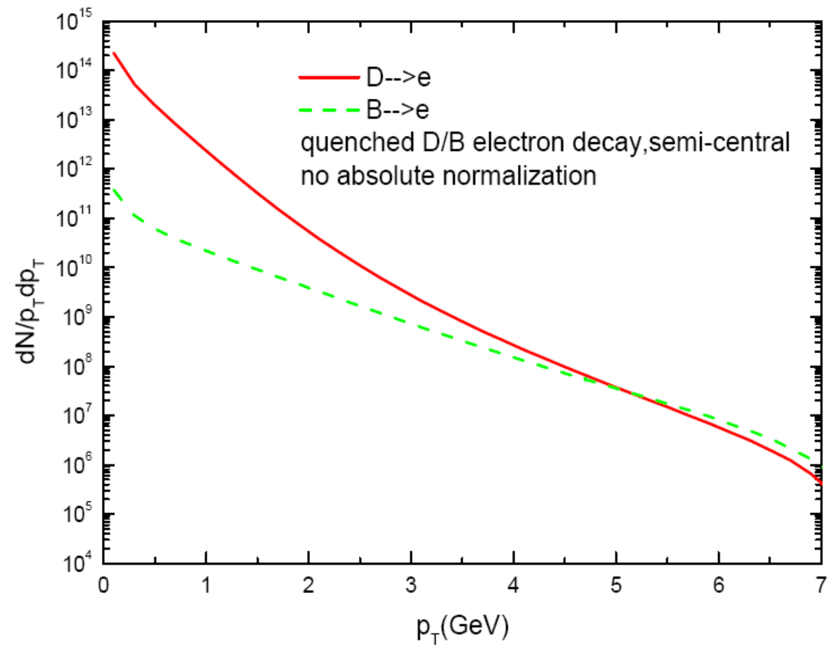
$$\frac{\mathrm{d}N_D^{\mathrm{tot}}}{\mathrm{d}y\,\mathrm{d}^2p_T} = \frac{\mathrm{d}N_D^{\mathrm{coal}}}{\mathrm{d}y\,\mathrm{d}^2p_T} + \frac{\mathrm{d}N_c^{\mathrm{frag}}}{\mathrm{d}y\,\mathrm{d}^2p_T}$$

fragmentation: preserves the charm quark $v_2$ from charm → D
coalescence: adds $p_T$ and $v_2$ from the light quarks to charm quarks

# Non-photonic decay: B vs D

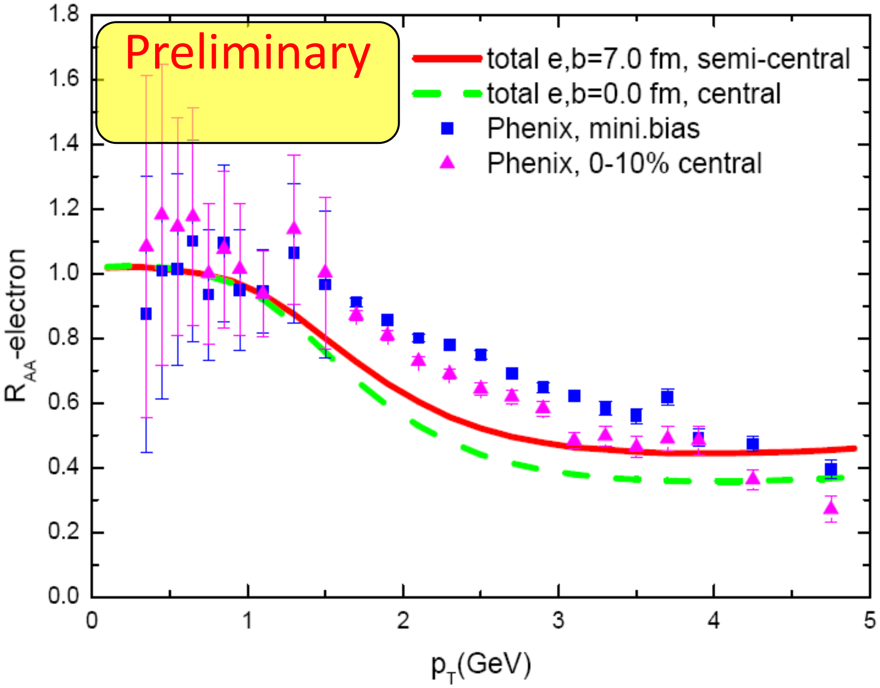


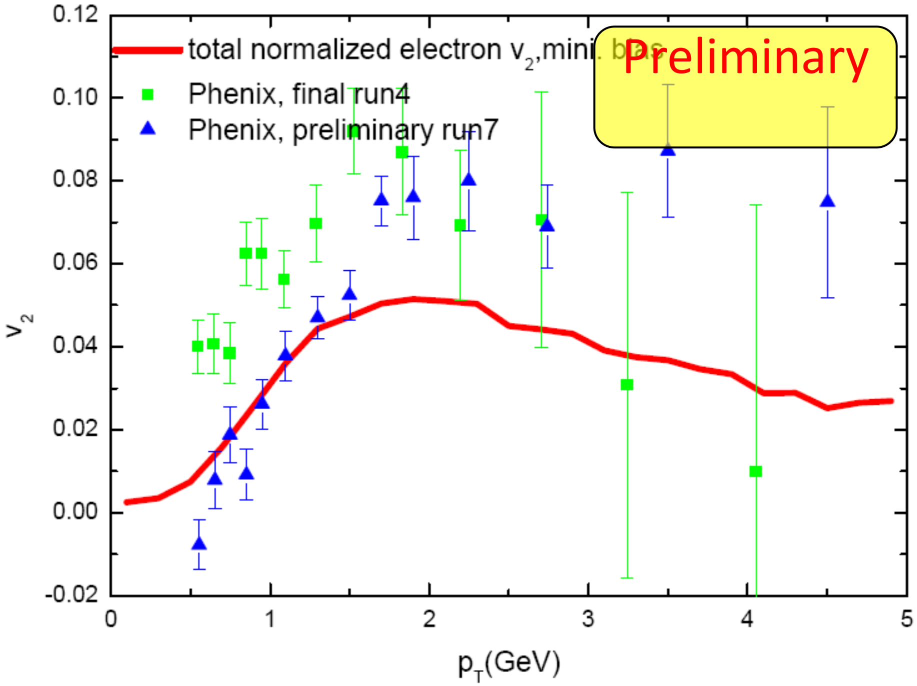

initial spectra: B vs D electron crossing at $p_T$~5.4 GeV, c.f. Rapp, Greco, van Hees, 2006; the cross section ratio consistent with pQCD prediction: Cacciari, Nason and Vogt, 2005




quenched spectra: B vs D electron crossing at $p_T$ ~ 5 GeV
B & D: relatively normalized

# Non-photonic decay electrons



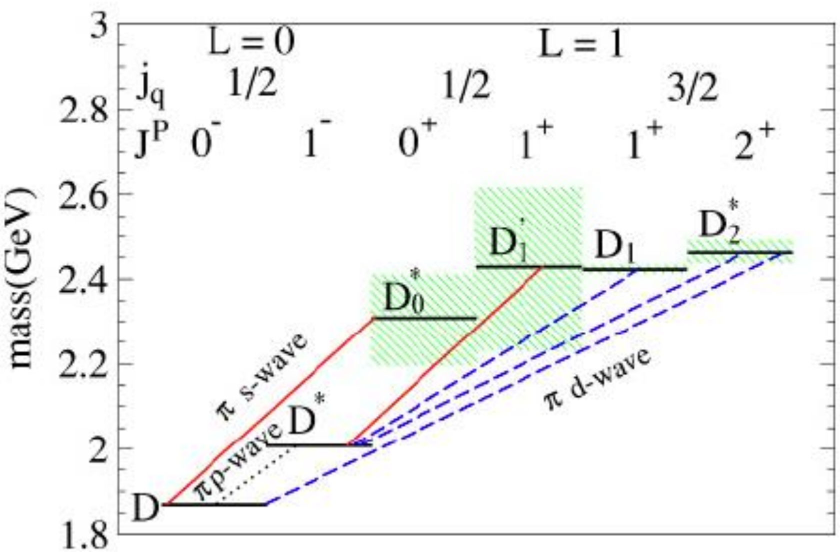

- **Preliminary**: **Monte-Carlo simulation of the non-photonic decay with a constant decay matrix element (only phase space constraint), to be improved**
- **decay channels included:**

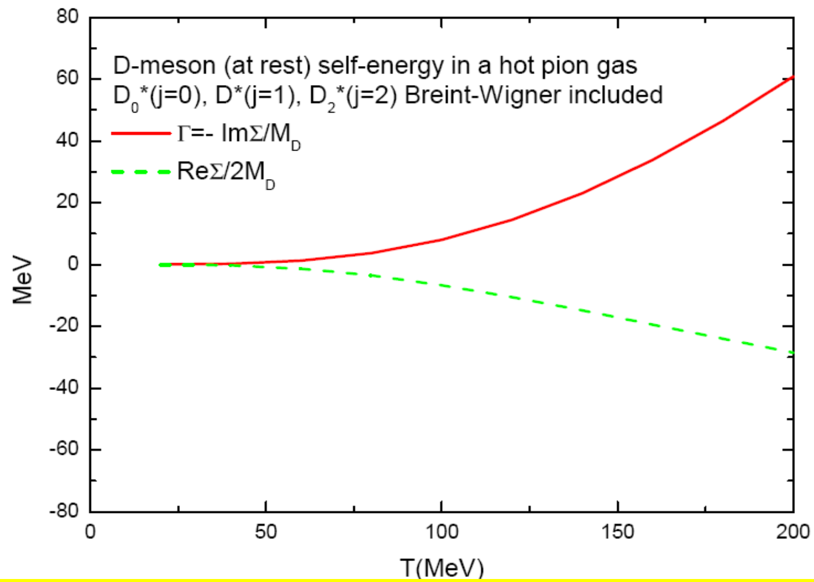
$D \to eK\bar{\nu}_e$ **(8.6%);** $D \to eK^*\bar{\nu}_e$ **(3.66%);** $B \to eX\bar{\nu}_e$ **(10.36%)**

- Au+Au $\sqrt{s} = 200GeV$

# Discussion: other possible contributions to charm relaxation

- Non-perturbative gluon-charm scattering: g-Q T-matrix?!
- Hadronic phase D interactions: pion,K,rho,N…



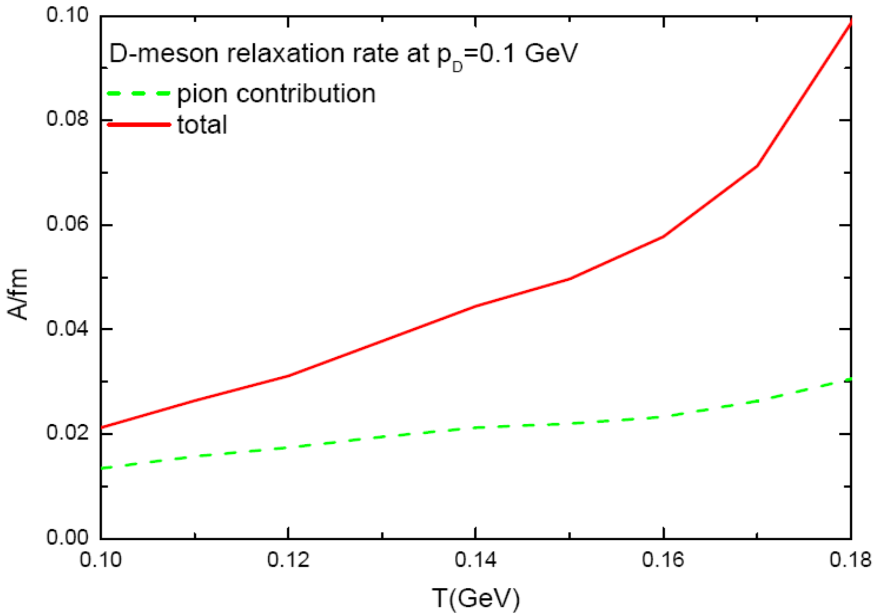



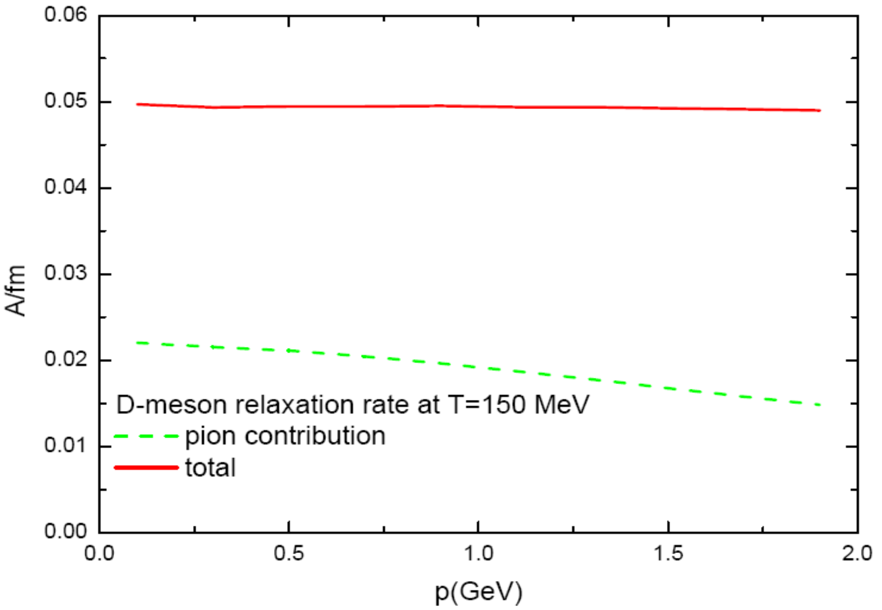

✓ D&$D_0$*, D*&$D_1$' : chiral partners, large pion s-wave decay width ~300 MeV, see Fuchs, et al., 2006

✓ D + pion → D + pion: Breit-Wigner

$$A_{1/2} = \sum_{j=0,1,2} \frac{8\pi\sqrt{s}}{k} \frac{(2j+1)}{(2j_1+1)(2j_2+1)} \frac{-\sqrt{s}\Gamma_j^{D\pi}}{s - M_j^2 + i\sqrt{s}\Gamma_j^{\rm tot}}$$

# Hadronic phase contribution to D thermal relaxation




✓D + pion →$D_0^*$,$D^*$,$D_2^*$→ D + pion: Breit-Wigner; for D + K, rho, K*, N, Delta, use an isotropic cross section of 10 mb (baryon 15 mb) to make a conservative estimate

✓ non-equilibrium chemical potentials in hadronic phase included, e.g.

$$\mu_\rho = 2\mu_\pi \qquad \mu_\Delta = \mu_N + \mu_\pi$$

✓at T~$T_{c,}$D relaxation rate comparable to that of charm quark: quark-hadron duality???

# Summary & Conclusion

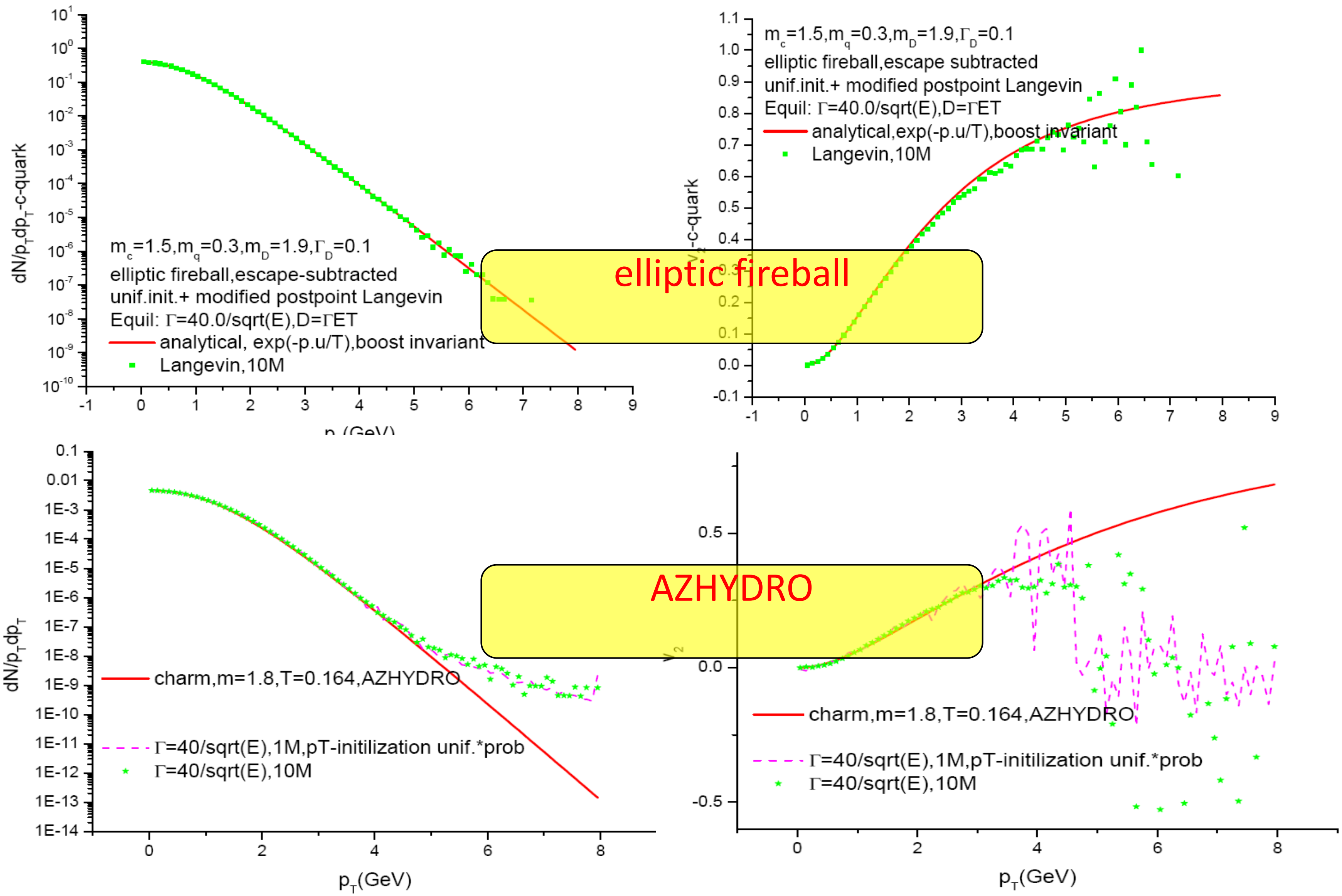
- Hydro + Langevin + RRM(coalescence) conducted; equilibrium limit (important) checked
- Non-photonic decay calculation: to be improved
- The role of resonance correlation is emphasized:

  (a). resonance contribution (Q-q T-matrix calculation) to heavy quark thermal relaxation

  (b). c-q Resonance Recombination to describe the coalescence hadronization

***Thanks for attention!***

# Appendix 1: Langevin equilibrium



# Appendix 2: RRM equilibrium