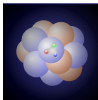


T-Matrix Approach to Heavy-Quark Diffusion and Quarkonia

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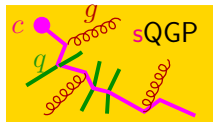
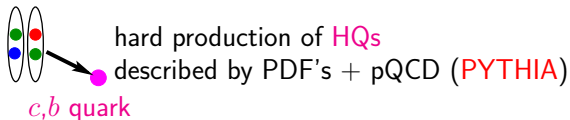


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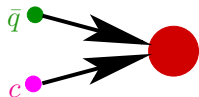


- 1 Heavy-quark interactions in the sQGP
 - Heavy quarks in heavy-ion collisions
 - Heavy-quark diffusion: The Langevin Equation
 - Elastic pQCD heavy-quark scattering
- 2 Microscopic model for non-perturbative HQ interactions
 - Static heavy-quark potentials from lattice QCD
 - T-matrix approach
- 3 Non-photonic electrons at RHIC
- 4 T-matrix approach to Quarkonium-Bound-State Problem
- 5 Summary and Outlook

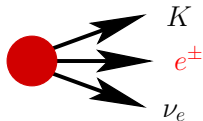
Heavy Quarks in Heavy-Ion collisions



HQ rescattering in QGP: Langevin simulation
drag and diffusion coefficients from
microscopic model for HQ interactions in the sQGP



Hadronization to D, B mesons via
quark coalescence + fragmentation
V. Greco, C. M. Ko, R. Rapp, PLB **595**, 202 (2004)



semileptonic decay $\Rightarrow$
“non-photonic” electron observables
 $R_{AA}^{e^+e^-}(p_T)$, $v_2^{e^+e^-}(p_T)$

Relativistic Langevin process

- **Langevin process**: **friction force** + **Gaussian random force**
- in the (local) rest frame of the heat bath

$$d\vec{x} = \frac{\vec{p}}{E_p} dt,$$

$$d\vec{p} = -\mathbf{A}\vec{p}dt + \sqrt{2dt}[\sqrt{B_0}P_\perp + \sqrt{B_1}P_\parallel]\vec{w}$$

- $\vec{w}$: normal-distributed random variable
- $\mathbf{A}$: friction (drag) coefficient
- $B_{0,1}$: diffusion coefficients
- dependent on **realization of stochastic process**

Local-Equilibrium Limit

- to guarantee correct (local) equilibrium limit:
 - use **Hänggi-Klimontovich calculus**, i.e., use $B_{0/1}(t, \vec{p} + d\vec{p})$
 - Einstein dissipation-fluctuation relation $B_0 = B_1 = E_p T A$.
- to implement flow of the medium
 - use **Lorentz** boost to change into local “heat-bath frame”
 - use **update rule** in heat-bath frame
 - boost back into “lab frame”

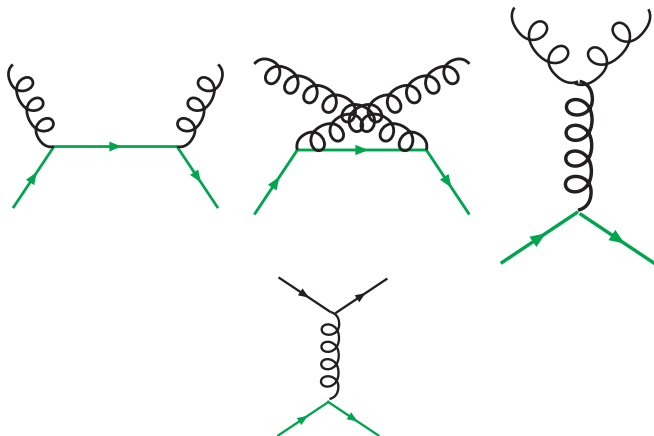
- Realizes **Milekhin-freeze-out distribution** for $t \rightarrow \infty$

$$\frac{dN}{d^3x d^3p} = \frac{g}{(2\pi)^3} \frac{p \cdot u(x)}{E} \exp \left[-\frac{p \cdot u(x)}{T(x)} \right].$$

- $E = p^0 = \sqrt{m^2 + \vec{p}^2}$
- $u(x)$: **four-velocity-flow field of the medium**
- $T(x)$: **temperature field of the medium**
- thermal fireball adjusted such as to lead to correct **light-meson and baryon observables**
- using the same coalescence/fragmentation model for hadronization
- bulk v_2 sensitive to **freeze-out description** (see next talk by P. Gossiaux)

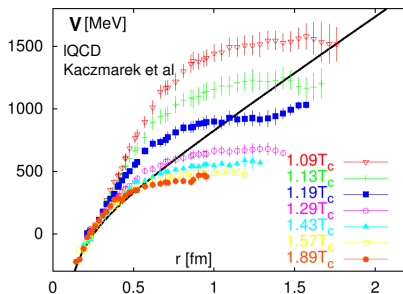
Elastic pQCD processes

- Lowest-order matrix elements [Cambridge 79]



- **Debye-screening mass** for t -channel gluon exch. $\mu_g = gT$, $\alpha_s = 0.4$
- not sufficient to understand RHIC data on “non-photonic” electrons

Microscopic model: Static potentials from lattice QCD



- color-singlet free energy from lattice
- use **internal energy**

$$U_1(r, T) = F_1(r, T) - T \frac{\partial F_1(r, T)}{\partial T},$$

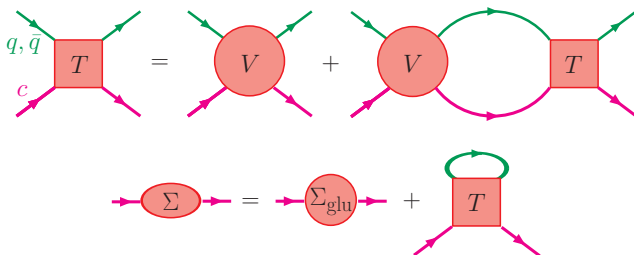
$$V_1(r, T) = U_1(r, T) - U_1(r \rightarrow \infty, T)$$

- Casimir scaling for other color channels [Nakamura et al 05; Döring et al 07]

$$V_3 = \frac{1}{2} V_1, \quad V_6 = -\frac{1}{4} V_1, \quad V_8 = -\frac{1}{8} V_1$$

T-matrix

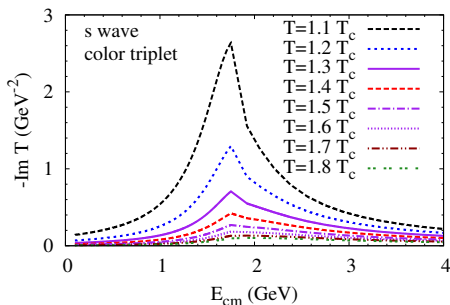
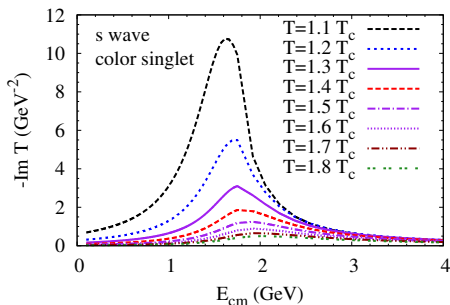
- Brueckner many-body approach for elastic Qq , $Q\bar{q}$ scattering



- reduction scheme: 4D Bethe-Salpeter $\rightarrow$ 3D Lipmann-Schwinger
- S - and P waves
- same scheme for light quarks (self consistent!)
- Relation to invariant **matrix elements**

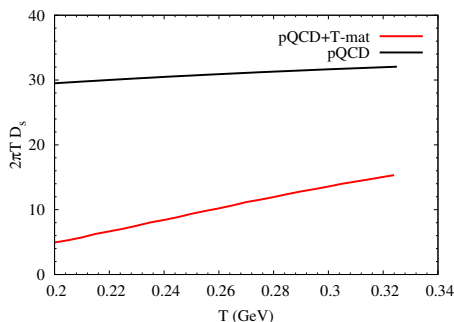
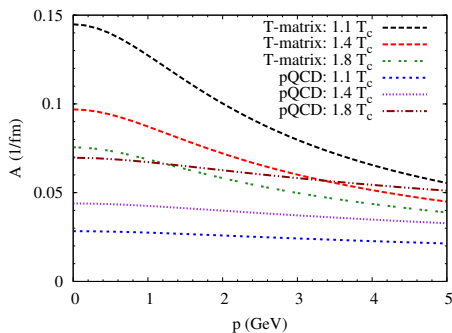
$$\sum |\mathcal{M}(s)|^2 \propto \sum_q d_a (|T_{a,l=0}(s)|^2 + 3|T_{a,l=1}(s)|^2 \cos \theta_{\text{cm}})$$

Microscopic justification for resonances: T-matrix calculation



- use static heavy-quark potentials from IQCD
- resonance formation at lower temperatures $T \simeq T_c$
- melting of resonances at higher T ! $\Rightarrow$ sQGP
- model-independent assessment of elastic Qq , $Q\bar{q}$ scattering
- problems: uncertainties in extracting potential from IQCD
in-medium potential V vs. F ?

Transport coefficients



- from **non-pert.** interactions reach $A_{\text{non-pert}} \simeq 1/(7 \text{ fm}/c) \simeq 4A_{\text{pQCD}}$
- **A decreases with higher temperature**
- higher density (over)compensated by **melting of resonances!**
- spatial diffusion coefficient

$$D_s = \frac{T}{mA}$$

increases with temperature

Time evolution of the fire ball

- Elliptic **fire-ball** parameterization
fitted to hydrodynamical flow pattern [Kolb '00]

$$V(t) = \pi(z_0 + v_z t) a(t) b(t), \quad a, b: \text{semi-axes of ellipse,} \\ v_{a,b} = v_\infty [1 - \exp(-\alpha t)] \mp \Delta v [1 - \exp(-\beta t)]$$

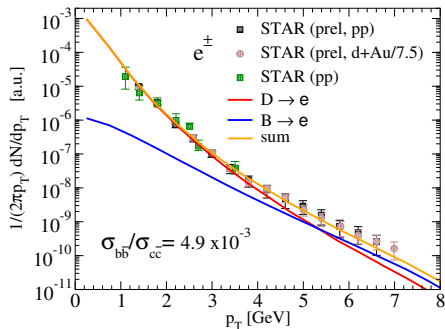
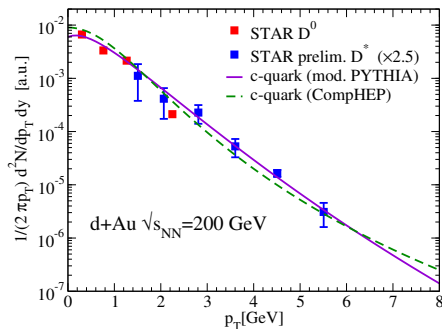
- **Isentropic expansion**: $S = \text{const}$ (fixed from N_{ch})
- **QGP Equation of state**:

$$s = \frac{S}{V(t)} = \frac{4\pi^2}{90} T^3 (16 + 10.5 n_f^*), \quad n_f^* = 2.5$$

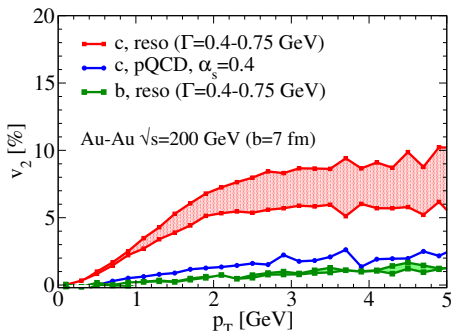
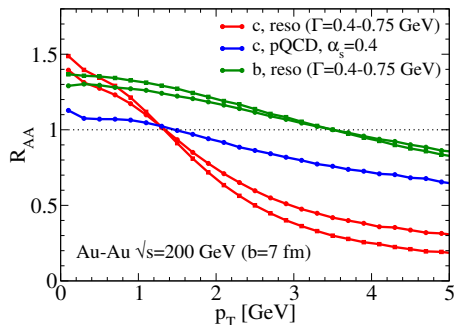
- obtain $T(t) \Rightarrow A(t, p)$, $B_0(t, p)$ and $B_1 = TEA$
- for semicentral collisions ($b = 7$ fm): $T_0 = 340$ MeV,
QGP lifetime $\simeq 5$ fm/ c .
- simulate FP equation as **relativistic Langevin process**

Initial conditions

- need initial p_T -spectra of **charm** and **bottom** quarks
 - (modified) PYTHIA to describe exp. **D** meson spectra, assuming **δ -function fragmentation**
 - exp. **non-photonic single- $e^\pm$ spectra**: Fix bottom/charm ratio



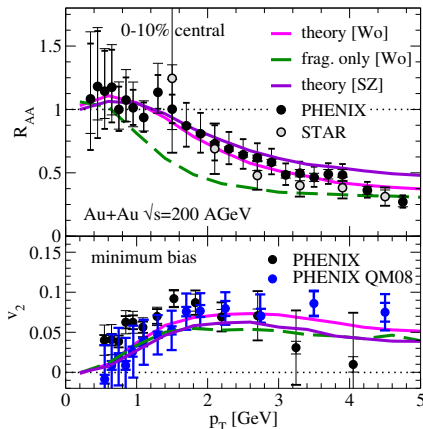
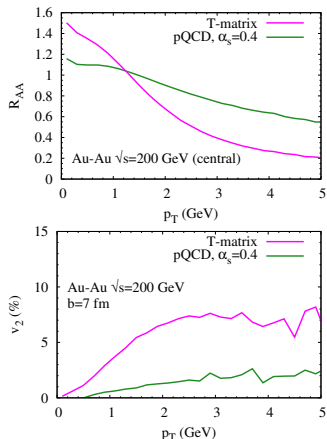
Spectra and elliptic flow for heavy quarks



- $\mu_D = gT$, $\alpha_s = g^2/(4\pi) = 0.4$
- resonances $\Rightarrow c$ -quark thermalization
without upscaling of cross sections
- Fireball parametrization consistent with hydro

Non-photonic electrons at RHIC

- same model for bottom
- quark **coalescence** + **fragmentation** $\rightarrow D/B \rightarrow e + X$

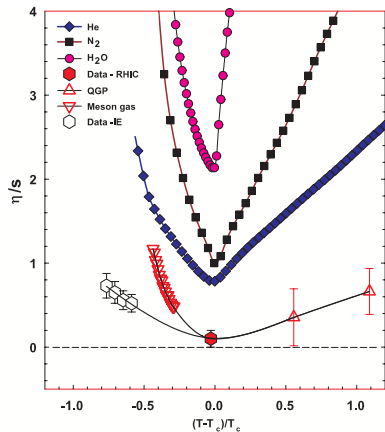
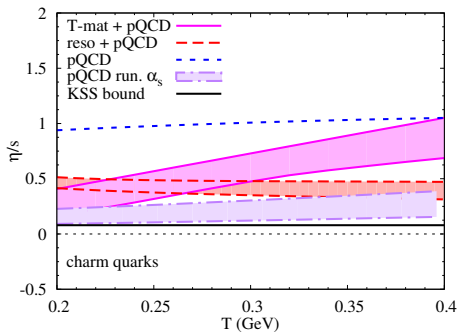


- **coalescence** crucial for description of data
- increases **both**, R_{AA} and $v_2 \Leftrightarrow$ “momentum kick” from light quarks!
- “resonance formation” **towards T_c** $\Rightarrow$ **coalescence natural** [Ravagli, Rapp 07]

Transport properties of the sQGP

- spatial diffusion coefficient: **Fokker-Planck** $\Rightarrow D_s = \frac{T}{m_A} = \frac{T^2}{D}$
- measure for coupling strength in plasma: η/s

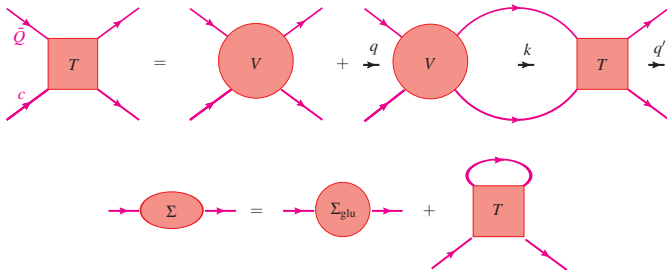
$$\frac{\eta}{s} \simeq \frac{1}{2} T D_s \quad (\text{AdS/CFT}), \quad \frac{\eta}{s} \simeq \frac{1}{5} T D_s \quad (\text{wQGP})$$



[Lacey, Taranenko (2006)]

T-matrix approach for quarkonium-bound-state problem

- **T-matrix Brückner approach** for heavy quarkonia as for HQ diffusion
- consistency between HQ diffusion and $\bar{Q}Q$ suppression!

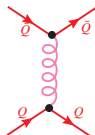


- 4D **Bethe-Salpeter equation** $\rightarrow$ 3D **Lippmann-Schwinger equation**
- relativistic interaction $\rightarrow$ **static heavy-quark potential** (IQCD)

$$T_{\alpha}(E; q', q) = V_{\alpha}(q', q) + \frac{2}{\pi} \int_0^{\infty} dk \, k^2 V_{\alpha}(q', k) G_{Q\bar{Q}}(E; k) T_{\alpha}(E; k, q) \\ \times \{1 - n_F[\omega_1(\vec{k})] - n_F[\omega_2(k)]\}$$

- q, q', k relative 3-momentum of initial, final, intermediate $\bar{Q}Q$ state
[F. Riek, R. Rapp, PRC **82**, 035201 (2010)]

The potential



- non-perturbative static **gluon** propagator

$$D_{00}(\vec{k}) = 1/(\vec{k}^2 + \mu_D^2) + m_G^2/(\vec{k}^2 + \tilde{m}_D^2)^2$$

- **finite-T** HQ **color-singlet-free energy** from Polyakov loops

$$\begin{aligned} \exp[-F_1(r, T)/T] &= \left\langle \text{Tr}[\Omega(x)\Omega^\dagger(y)]/N_c \right\rangle \\ &= \exp \left[\frac{g^2}{2N_c T^2} \left\langle A_{0,\alpha}(x)A_{0,\alpha}(y) - A_{0,\alpha}^2(x) \right\rangle \right] + \mathcal{O}(g^6) \end{aligned}$$

- identify $\langle A_{0,\alpha}(x)A_{0,\alpha}(y) \rangle = D_{00}(x - y)$
- **color-singlet free energy**

$$F_1(r, T) = -\frac{4}{3}\alpha_s \left\{ \frac{\exp(-m_D r)}{r} + \frac{m_G^2}{2\tilde{m}_D} [\exp(-\tilde{m}_D r) - 1] + m_D \right\}$$

- **in vacuo** $m_D, \tilde{m}_D \rightarrow 0$

$$F_1(r) = -\frac{4}{3} \frac{\alpha_s}{r} + \sigma r, \quad \sigma = \frac{2\alpha_s m_G^2}{3}$$

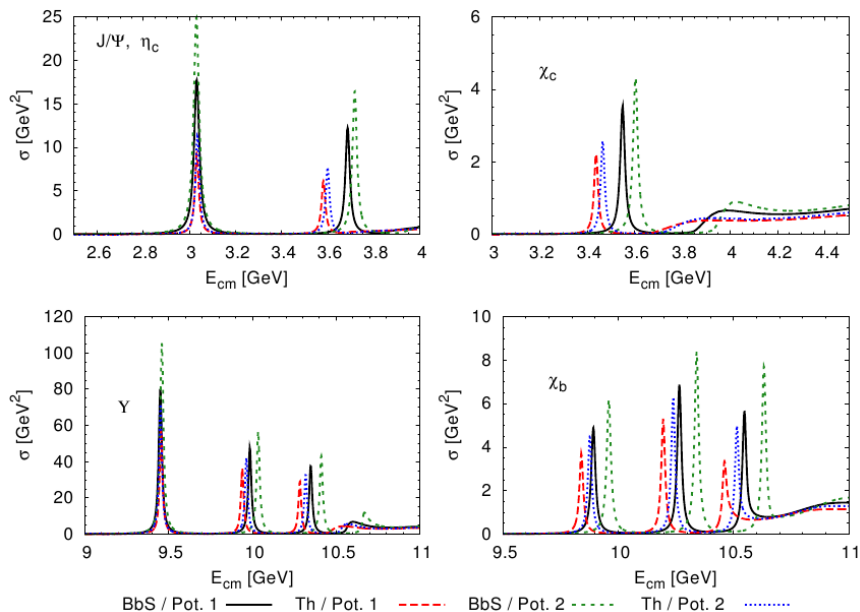
[F. Riek, R. Rapp, arXiv:1005.0769 [hep-ph]]

Heavy quarkonia

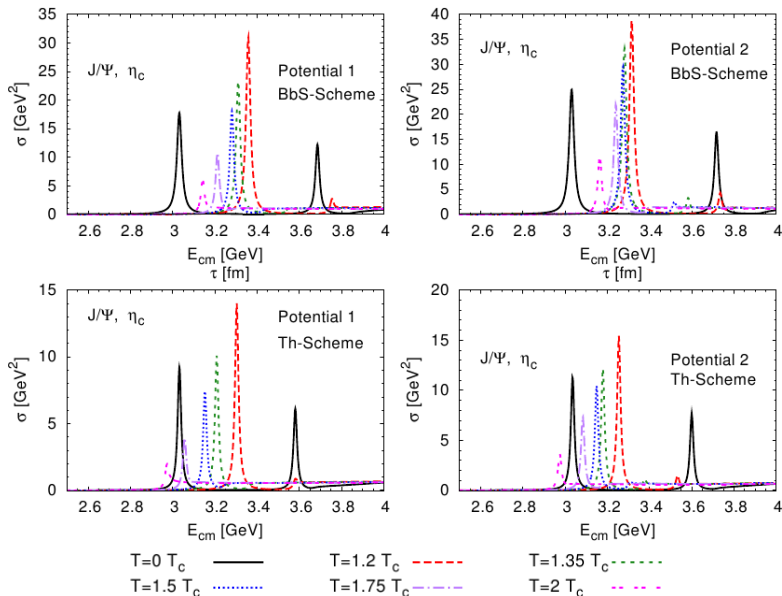
- fit parameters, $\alpha_s(T)$, $m_D(T)$, $\tilde{m}_D(T)$, $\tilde{m}_G(T)$ to IQCD
- calculate **internal energy** $U(r, T) = F(r, T) - T \frac{\partial}{\partial T} F(r, T)$
- solve **Lippmann-Schwinger equation** $\Rightarrow$ adjust m_Q to get s -wave charmonia/bottomonia masses in vacuum
- in the following
 - **potential 1**: $N_f = 2 + 1$ [O. Kaczmarek]
 - **potential 2**: $N_f = 3$ [P. Petreczky]
 - **BbS**: Blencowe-Blecher-Sugar reduction scheme
 - **Th**: Thompson reduction scheme
- vacuum-mass splittings
 - uncertainty for charmonia 50-100 MeV
 - uncertainty for bottomonia 30-70 MeV
 - overall uncertainty $\simeq 10\%$
- melting temperatures with U and F
 - s -wave ($\eta_c, J/\psi$): $2-2.5T_c, \gtrsim 1.3T_c$,
 Υ : $> 2T_c, \gtrsim 1.7T_c, 1T_c, \gtrsim 2T_c, 1T_c, 1T_c$
 - p -wave (χ_c): $\gtrsim 1.2T_c, \gtrsim 1T_c, \chi_b$: $\gtrsim 1.7T_c, 1.2T_c$, all $\gtrsim 1T_c$

[F. Riek, R. Rapp, arXiv:1005.0769 [hep-ph]]

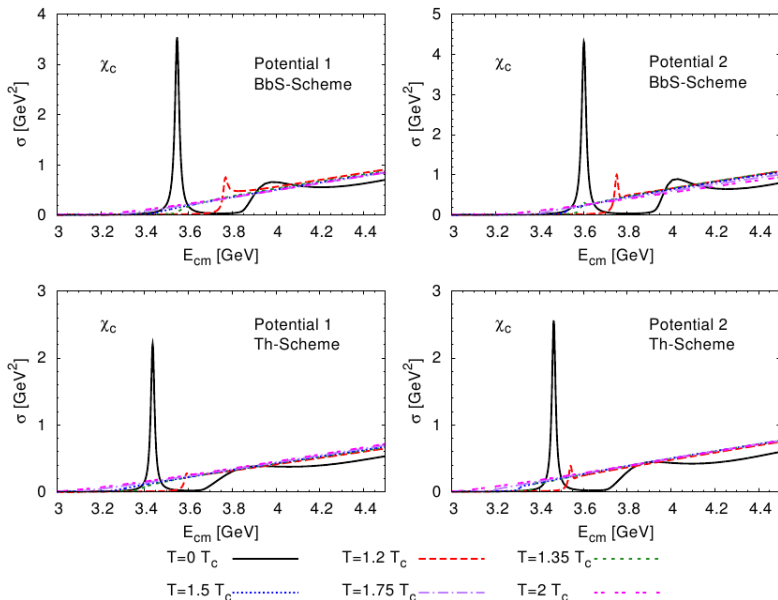
Quarkonium-spectral functions in the vacuum



In-medium charmonium-spectral functions (s states)



In-medium charmonium-spectral functions (p states)



Summary and Outlook

• Summary

- Heavy quarks in the sQGP
- non-perturbative interactions
 - mechanism for strong coupling: resonance formation at $T \gtrsim T_c$
 - IQCD potentials parameter free
 - res. melt at higher temperatures $\Leftrightarrow$ consistency betw. R_{AA} and v_2 !
 - same model also used for quarkonia in medium
- also provides “natural” mechanism for quark coalescence
- resonance-recombination model [L. Ravagli, HvH, R. Rapp, Phys. Rev. C **79**, 064902 (2009)]
- problems
 - potential approach at finite T : F , V or combination?

• Outlook

- use more realistic bulk-medium description ($\rightarrow$ following talks by P. Gossiaux and M. He)
- include inelastic heavy-quark processes (gluo-radiative processes)
- take into account D/B-meson rescattering in the hadronic phase
- other heavy-quark observables like charmonium suppression/regeneration ($\rightarrow$ talk by X. Zhao on Thursday)