Collaboration

High order Monte Carlo DGLV heavy quark energy loss with dynamic interactions in expanding diffuse A+A systems

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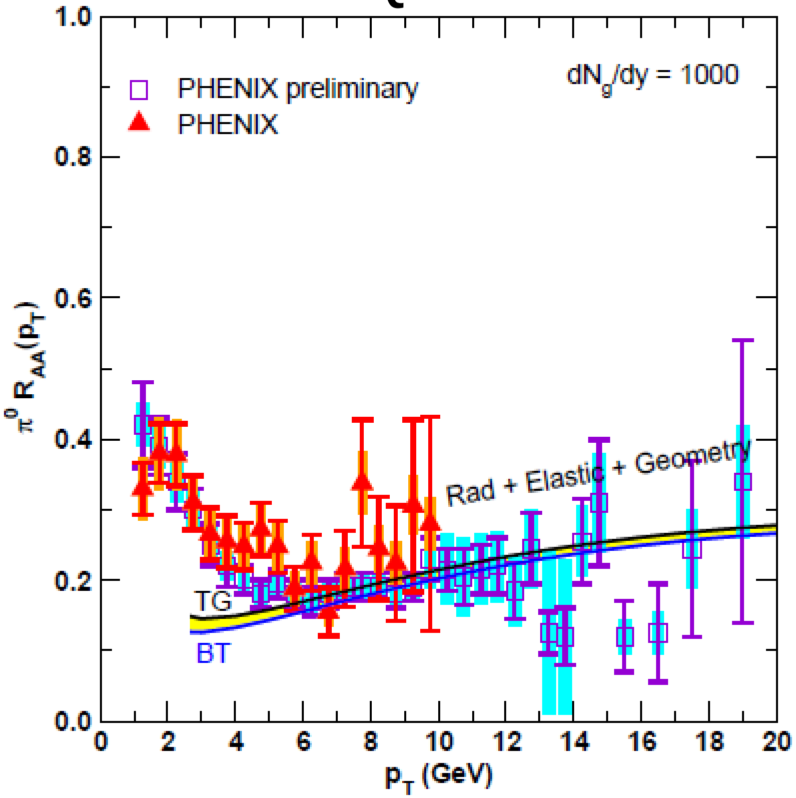
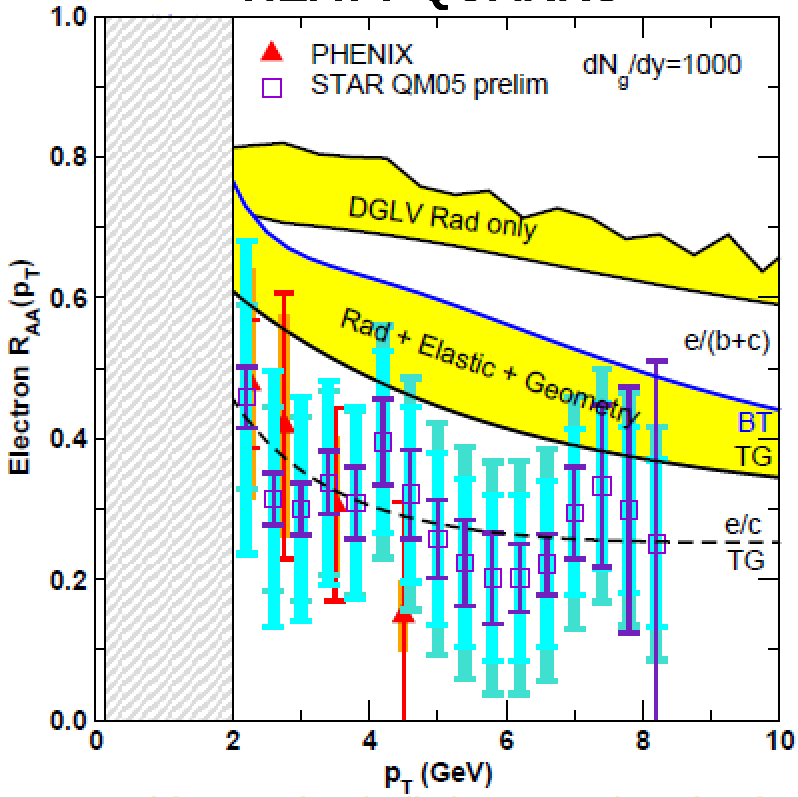
Light and Heavy Quarks R_{AA}

LIGHT QUARKS



HEAVY QUARKS



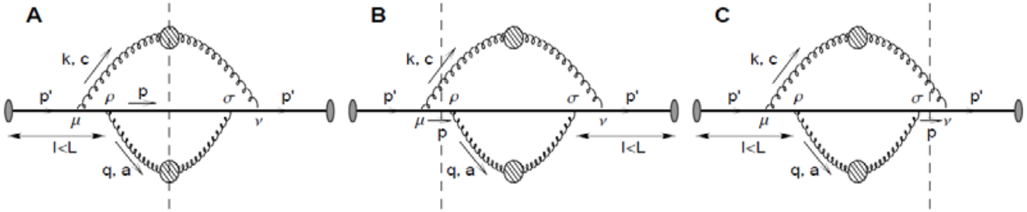
Wicks, Horowitz, Djordjevic, Gyulassy / NPA (2007)

DGLV is not sufficient to explain electron data observed at RHIC

– **need to increase Radiative Energy losses for charm and bottom quarks**

Opacity expansion: DGLV, MD

- **DGLV** (M. Djordjevic and M. Gyulassy, Nucl. Phys. A 733, 265, 2004)
  - Energy loss is obtained as a series in powers of opacity L/λ
  - Assumes static scattering centers, modeled by Yukawa potential
- **MD (Magdalena Djordjevic)** (Djordjevic, Heinz / Phys.Rev.Lett.101:022302,2008)
  - Dynamical model: includes recoils of scattering centers
  - New effective potential: $\frac{1}{(q^2+\mu^2)^2} \rightarrow \frac{1}{q^2(q^2+\mu^2)}$
  - No magnetic screening at order gT
  - Diagrams evaluated in Thermal Field Theory, only first order in opacity has been computed



- **Multigluon emission included via Poisson ansatz**

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Dynamical model in details

$$x\frac{dN}{dx}=\frac{C_R\alpha_s}{\pi}\frac{L}{\lambda_{dyn}}\int dz\frac{d\boldsymbol{k}}{\pi}\frac{d\boldsymbol{q}}{\pi}\frac{\mu^2}{q^2(q^2+\mu^2)}$$

$$\frac{2(\boldsymbol{k}+\boldsymbol{q})}{(\boldsymbol{k}+\boldsymbol{q})^2+\chi}\left(\frac{(\boldsymbol{k}+\boldsymbol{q})}{(\boldsymbol{k}+\boldsymbol{q})^2+\chi}-\frac{\boldsymbol{k}}{k^2+\chi}\right)(1-\mathrm{Cos}\,\omega z)\rho(z)$$

- $\boldsymbol{\omega}=\frac{(\boldsymbol{k}+\boldsymbol{q})^2+\chi}{xE}$
- $\boldsymbol{\rho(z)=\theta(L-z)/L}$ **(normalized uniform scattering center distribution)**
- $\chi\equiv M^2x^2+m_g{}^2(1-x)$ **with** $m_g{}^2=\frac{\mu^2}{2}$
- $\frac{1}{\lambda_{static}}=\sigma_{qg}\rho_q+\sigma_{gg}\rho_g=\frac{1}{\lambda_{dynamical}}\left(\frac{Zeta[3]}{\pi^2}\left(6+\frac{3}{2}n_f\right)\right)=\frac{1}{\lambda_{dynamical}}c_{nf}(n_f)$

NOTE: Pure gluonic medium in our computations ($n_f=0$)

Fluctuations

- **Include effects of fluctuations of the number of emitted gluons via Poisson ansatz**

In the approximation that the fluctuations of the gluon number are uncorrelated, the spectrum of the total radiative energy loss fraction, $\epsilon = \sum_i \omega_i/E$, can be expressed via a Poisson expansion $P(\epsilon, E) = \sum_{n=0}^{\infty} P_n(\epsilon, E)$ with $P_1(\epsilon, E) = e^{-\langle N^g \rangle} \rho(\epsilon, E)$ and

$$P_{n+1}(\epsilon, E) = \frac{1}{n+1} \int_{x_0}^{1-x_0} dx_n \; \rho(x_n, E) P_n(\epsilon - x_n, E)$$

$$= \frac{e^{-\langle N^g(E) \rangle}}{(n+1)!} \int dx_1 \cdots dx_n \; \rho(x_1, E) \cdots \rho(x_n, E) \rho(\epsilon - x_1 - \cdots - x_n, E) \; .$$

The form of this spectrum guarantees that the mean value is

$$\int_0^{\infty} d\epsilon \; P(\epsilon, E)\epsilon = \frac{\Delta E}{E} \; .$$

Gyulassy, Levai, Vitev / Phys.Lett.B538:282-288,2002

$$P(\varepsilon) = P_0 \delta(\varepsilon) + P(\varepsilon)\big|_0^1 + P_{stop} \delta(1 - \varepsilon)$$

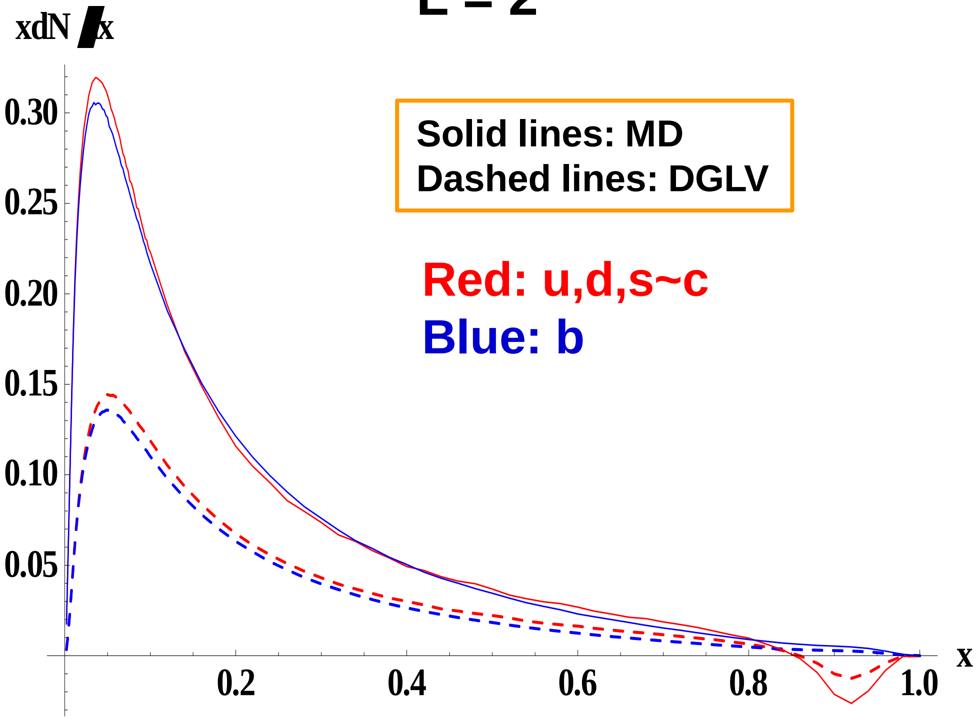
Contributes to R_{AA}

Contributes to $\frac{\Delta E}{E}$

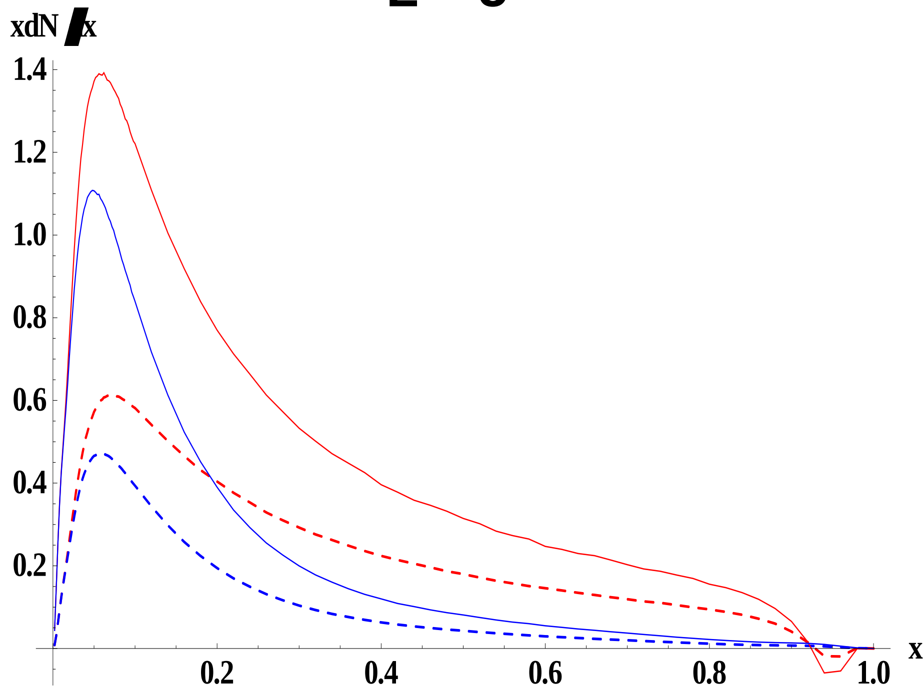
DGLV vs MD – xdN/dx

BRICK problem, En = 20GeV, T~250MeV

L = 2



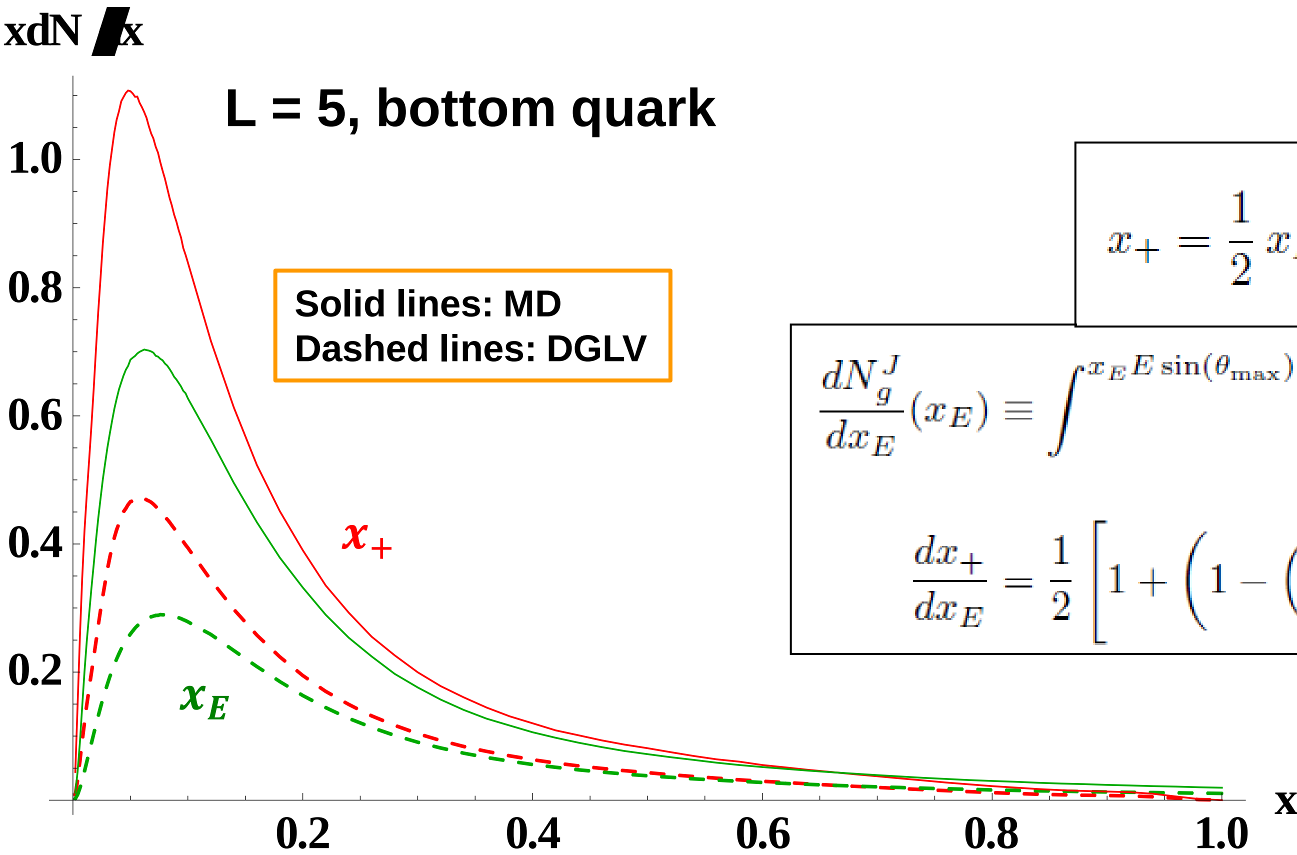
L = 5



k_T sensitivity

- **Collinear approximation:** $x_E = x_+ \left(1 + O\left(\frac{k_T}{x_+ E^+}\right)^2\right)$
  - **DGLV formula has the same functional form for x_E or x_+**
  - **Different kinematic limits:** $k_T^{max} = x_E E$

    $k_T^{max} = 2EMin[x_+, 1 - x_+]$



$$x_+ = \frac{1}{2} x_E \left(1 + \sqrt{1 - \left(\frac{k_T}{x_E E}\right)^2}\right)$$

$$\frac{dN_g^J}{dx_E}(x_E) \equiv \int^{x_E E \sin(\theta_{\max})} dk_T \frac{dx_+}{dx_E} \frac{dN_g}{dx_+ dk_T}(x_+(x_E)),$$

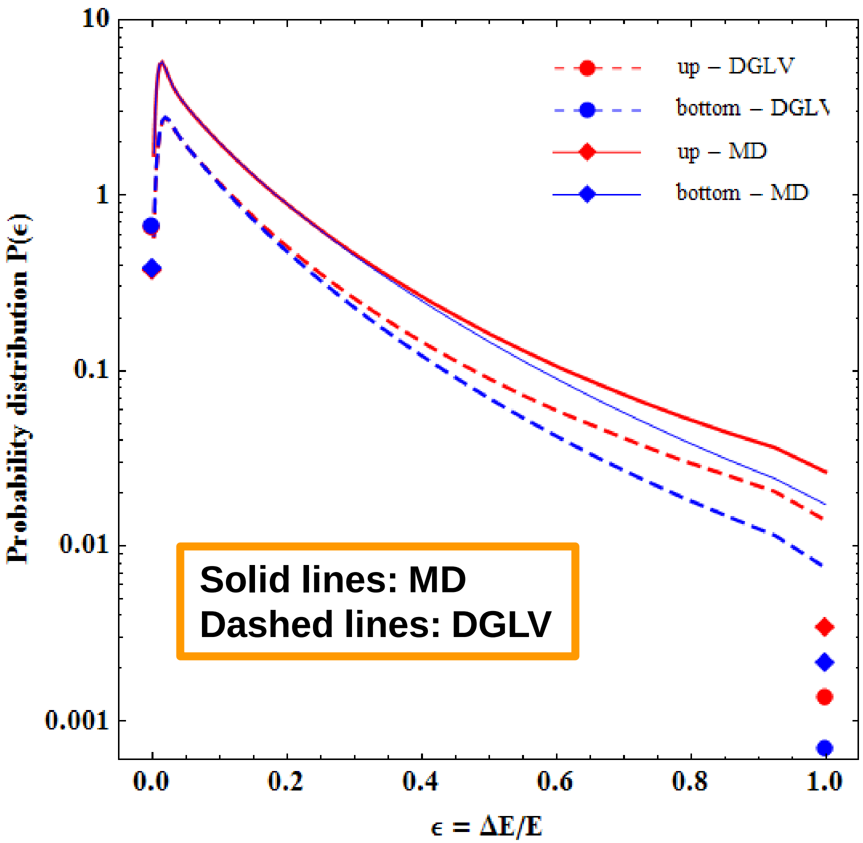
$$\frac{dx_+}{dx_E} = \frac{1}{2}\left[1 + \left(1 - \left(\frac{k_T}{x_E E}\right)^2\right)^{-1}\right].$$

BRICK – Probability distributions

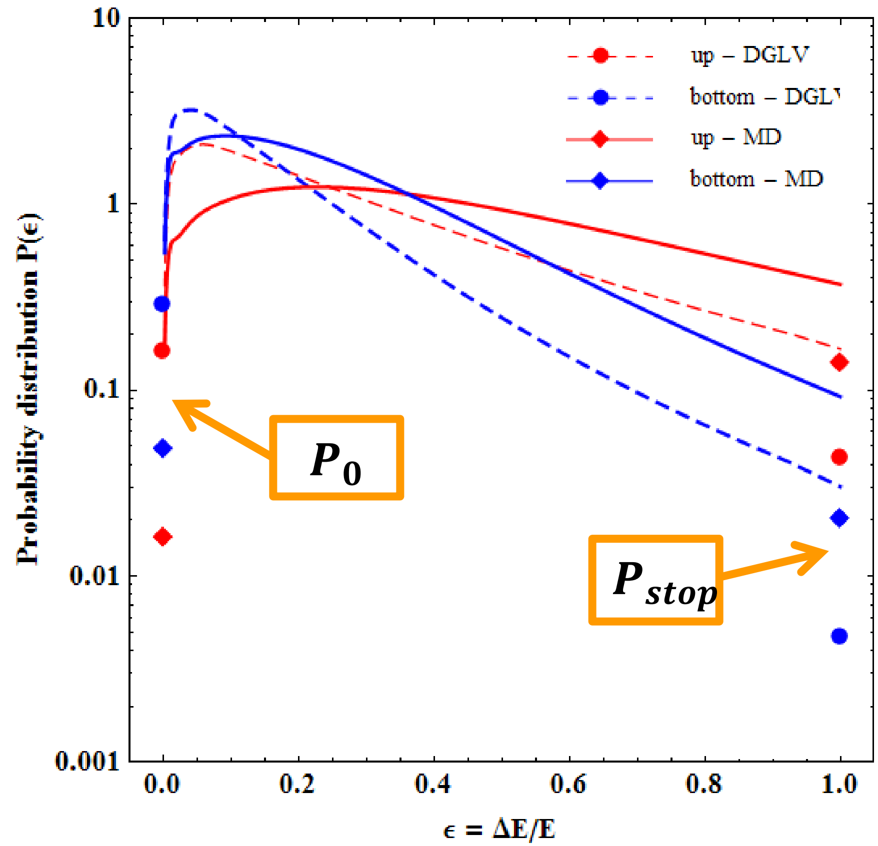
En = 20GeV, T~250MeV

– For short path lengths, P(ε) for u and b quarks are similar

L = 2

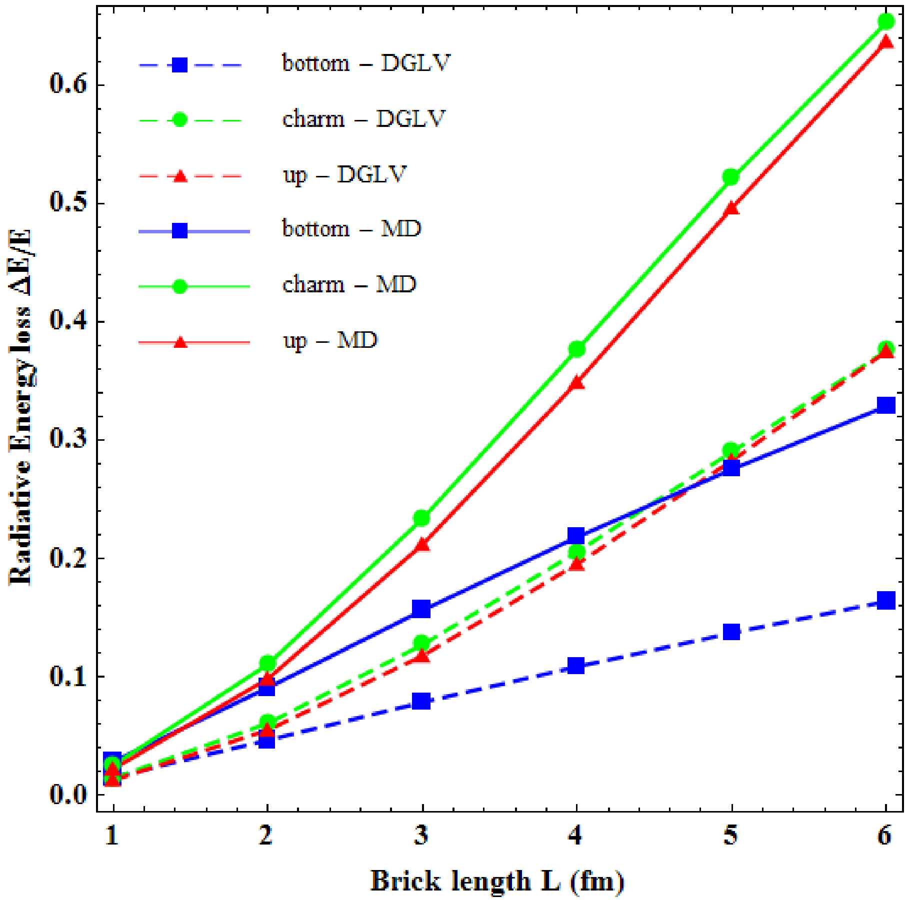


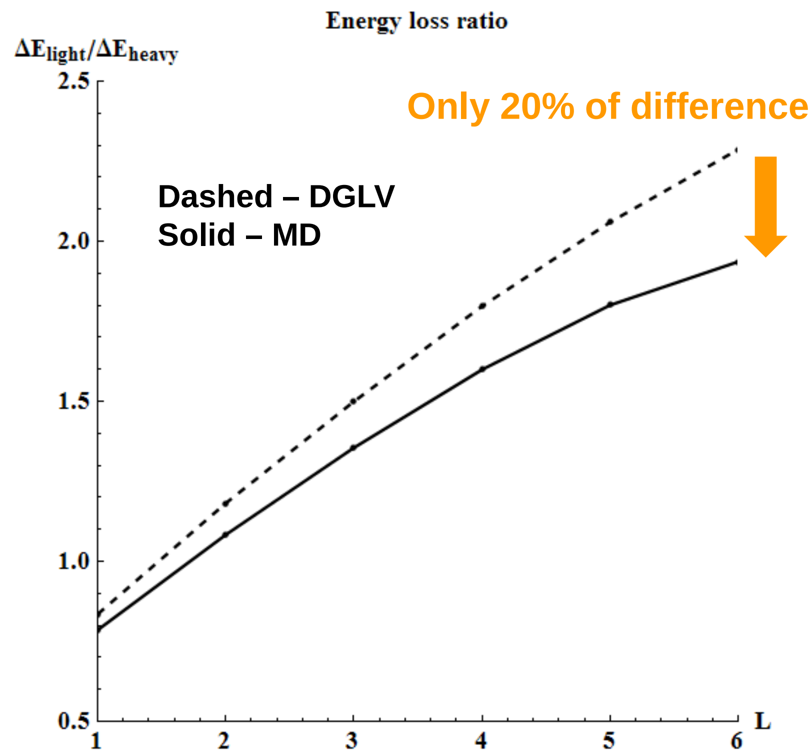
L = 5



BRICK – Energy loss

En = 20GeV, T~250MeV





- The energy loss for both up and bottom quarks is enhanced by a factor ~2
- $\Delta E_{up} \simeq \Delta E_{bottom}$ for $L \lesssim 2fm$
- Charm quark behavior is similar to light quarks

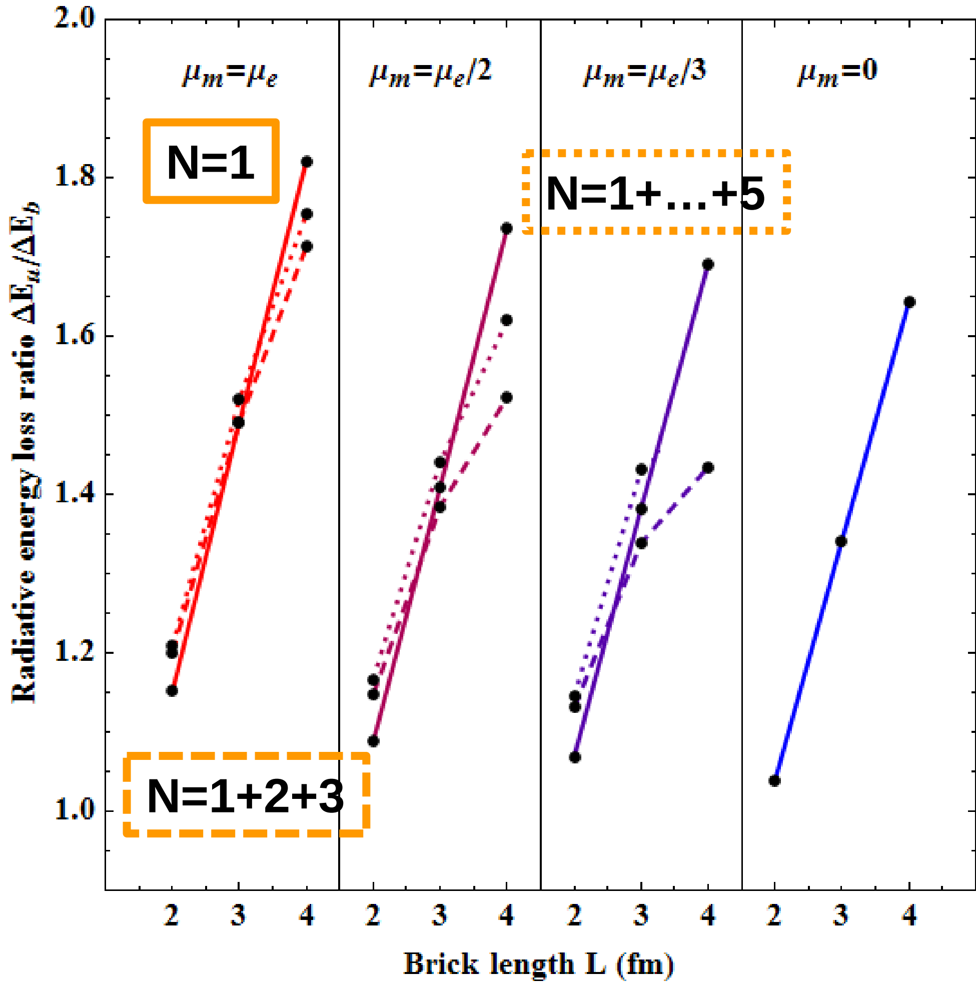
Beyond first order in opacity

Interpolate between DGLV and MD with a new effective potential

$$\frac{1}{(q^2+\mu^2)^2} \xleftarrow{DGLV} \frac{1}{(q^2+\mu_m^2)(q^2+\mu_e^2)} \xrightarrow{MD} \frac{1}{q^2(q^2+\mu^2)}$$

It is possible to study the limit $\mu_m \rightarrow 0$ for values of $\mu_m \gtrsim \mu_e/3$

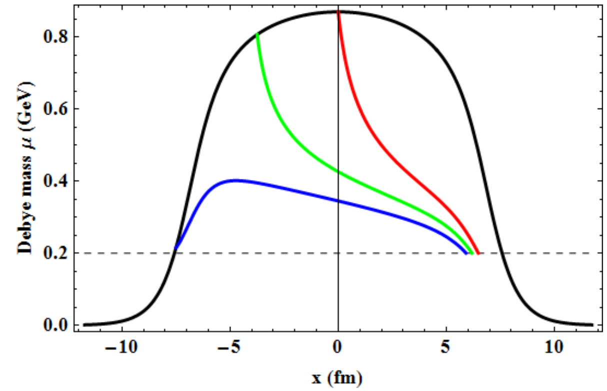
- The mean free path $\frac{1}{\lambda} = \int d\boldsymbol{q} \frac{d\sigma}{d\boldsymbol{q}} \rho$ is divergent for μ_m=0



$\left(\frac{\Delta E_u}{\Delta E_b}\right)$ ratio improves for N>1 and $\mu_m \rightarrow 0$, but likely not enough.

1D Bjorken Expanding geometry

- **Consider a diffuse Woods-Saxon nuclear density profile (for Au+Au central collision)**



  - **The bulk sQGP density profile is computed from the participant transverse density ρ_p:**

Initial rapidity density (~1000 RHIC)

$$\rho_{QGP}(x,y,\tau) = \frac{1}{\tau+\tau_0} * \frac{dN}{dy} * \rho_p(x,y)$$

Formation time (0.5 fm/c)

  - **The jet production profile is given by the binary collision density T_{AA}:**

$$\rho_{Jet}(x,y) = \sigma_{in}\, T_{AA}(x,y)$$

Inelastic cross section

Implementation of the geometry

- From ideal Boson/Fermion gas statistics, we can obtain the temperature profile of the QGP:

$$\mu[x,y,\tau] = \sqrt{4\pi\alpha} * T[x,y,\tau] = \sqrt{4\pi\alpha} * \left(\frac{\pi^2}{Zeta[3]}\frac{1}{16}\rho_g[x,y,\tau]\right)^{1/3}$$

$$\frac{1}{\lambda_{dyn}[x,y,\tau]} = \frac{1}{c_{nf}(0)}\sigma_{gg}\rho_g[x,y,\tau] = \frac{1}{c_{nf}(0)}\frac{\frac{9}{2}\pi\alpha^2}{\mu[x,y,\tau]^2}\rho_g[x,y,\tau]$$

$T[0,0,0] \simeq 450 MeV\ (RHIC)$

$$\frac{\Delta E_{\rm dyn}}{E} = \frac{C_R\alpha_s}{\pi}\frac{L}{\lambda_{\rm dyn}}\int dx\,\frac{d^2k}{\pi}\frac{d^2q}{\pi}\frac{\mu^2}{\boldsymbol{q}^2(\boldsymbol{q}^2+\mu^2)}\left(1-\frac{\sin\left(\frac{(\boldsymbol{k}+\boldsymbol{q})^2+\chi}{xE^+}L\right)}{\frac{(\boldsymbol{k}+\boldsymbol{q})^2+\chi}{xE^+}L}\right)\frac{2(\boldsymbol{k}+\boldsymbol{q})}{(\boldsymbol{k}+\boldsymbol{q})^2+\chi}\left(\frac{(\boldsymbol{k}+\boldsymbol{q})}{(\boldsymbol{k}+\boldsymbol{q})^2+\chi}-\frac{\boldsymbol{k}}{\boldsymbol{k}^2+\chi}\right)$$

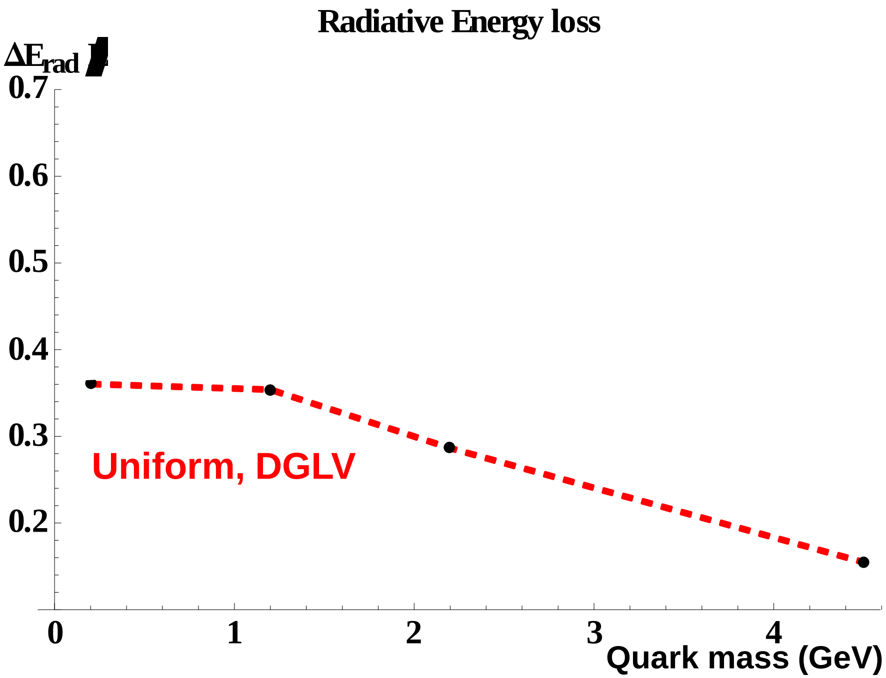
Cutoff at $\mu = \Lambda_{QCD}$

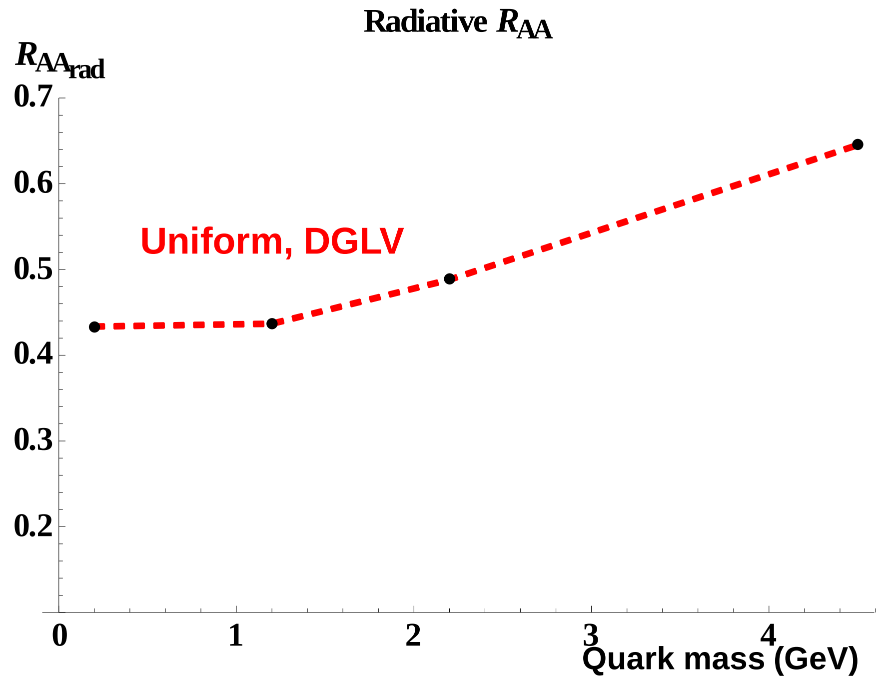
$$x\frac{dN}{dx}[x_0,\phi] = \frac{C_R\alpha_s}{\pi}\int\frac{d^2k}{\pi}\frac{d^2q}{\pi}d\tau\frac{9/2\pi\alpha^2}{q^2(q^2+\mu^2[\rho_{QGP}^{1/3}])} * f * (1-\cos\omega\tau) * \rho_{QGP}[x_0+\tau\hat{n}(\phi),\tau]$$

Energy loss and R_{AA}

First compute the fluctuations: $x\frac{dN}{dx} \rightarrow P(\varepsilon)$

- $\frac{\Delta E}{E} = \int \frac{d\phi}{2\pi} d^2x_0 * \rho_{Jet}(x_0,\phi) * \int_0^1 d\varepsilon * \varepsilon * P(\varepsilon; x_0, \phi)$
- $R_{AA} = \int \frac{d\phi}{2\pi} d^2x_0 * \rho_{Jet}(x_0,\phi) * \int_0^1 d\varepsilon * (1-\varepsilon)^n * P(\varepsilon; x_0, \phi)$





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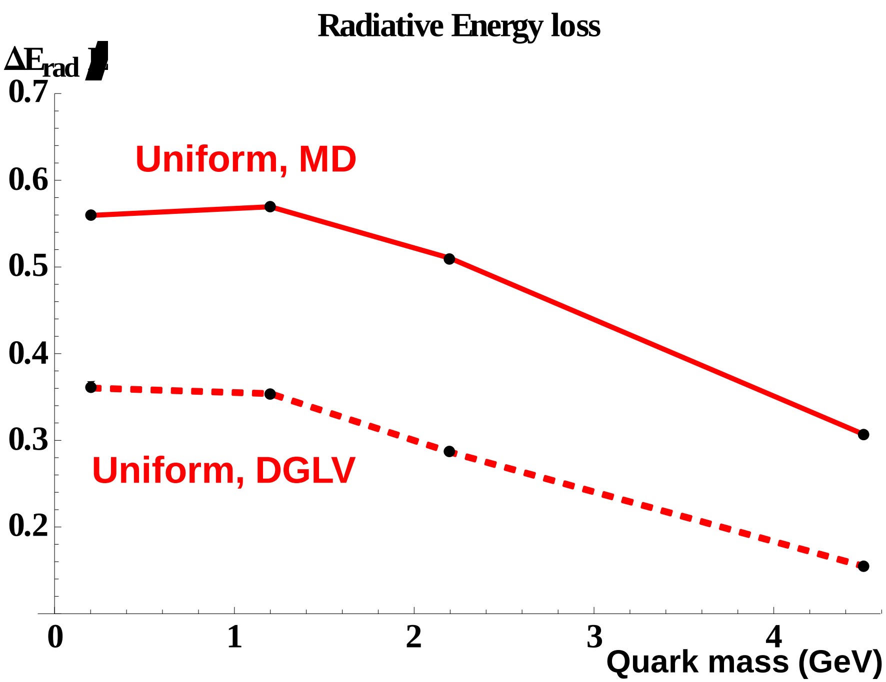

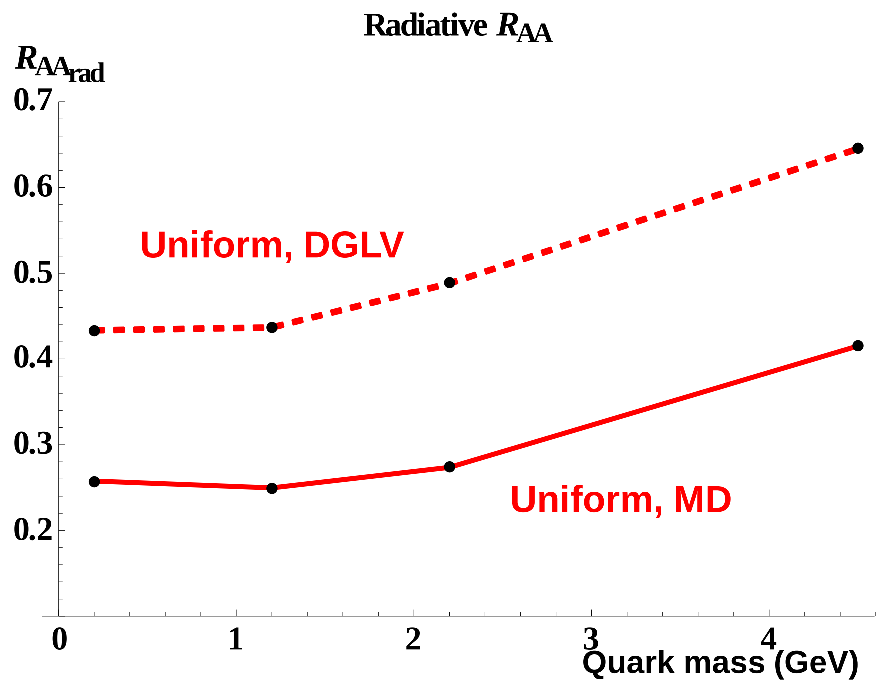

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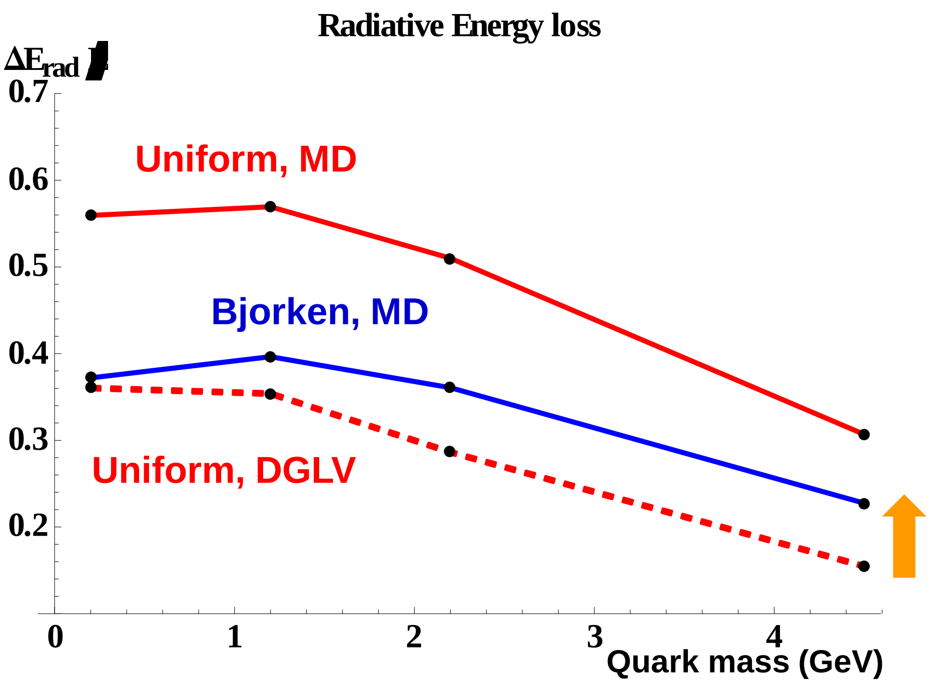
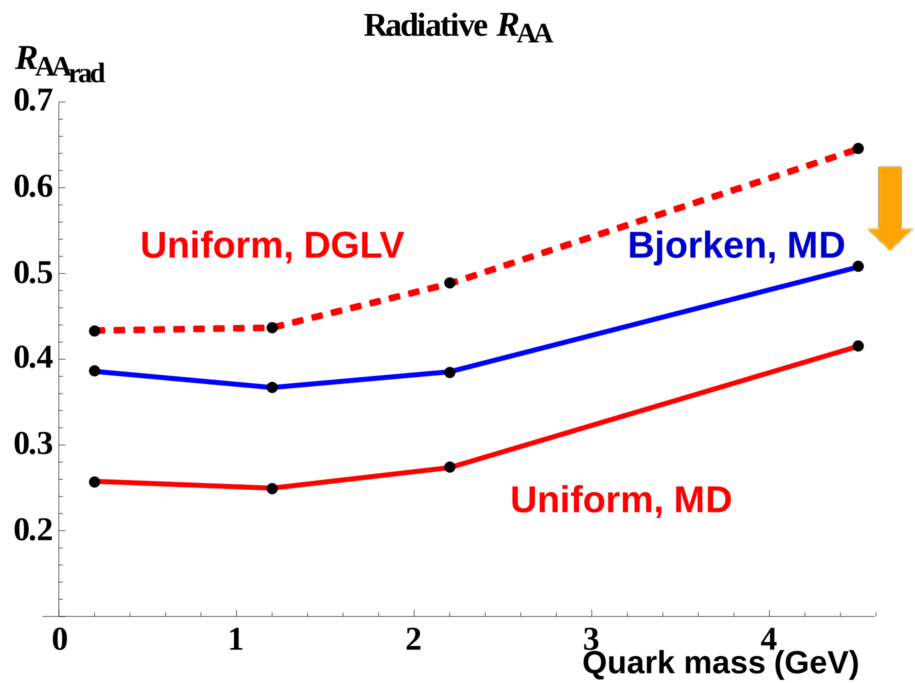


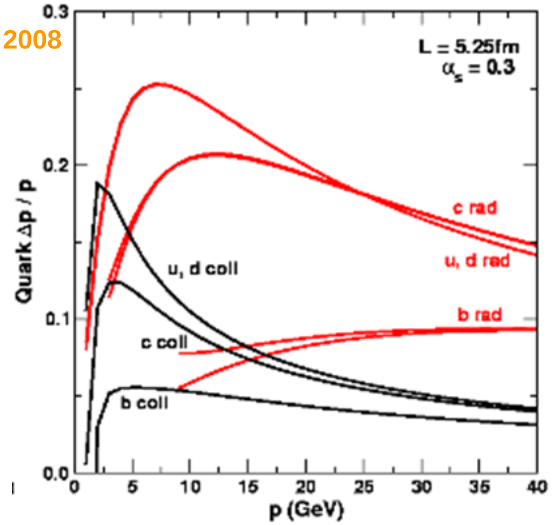


$$\left(\frac{\Delta E_u}{\Delta E_b}\right)^{DGLV}_{Uniform} \simeq 2.3 \longrightarrow \left(\frac{\Delta E_u}{\Delta E_b}\right)^{MD}_{Bjorken} \simeq 1.6$$

Elastic energy loss

- Elastic energy loss is included as:

Wicks / thesis 2008



1. $P_{el}(\varepsilon) = \delta(\varepsilon - \varepsilon_{el})$
2. $R_{AA}^{rad+el} = \int_0^1 d\varepsilon\, d\varepsilon' (1 - \varepsilon - \varepsilon')^n P_{rad}(\varepsilon) P_{el}(\varepsilon')$

$$\approx (1 - n\varepsilon_{el})\, R_{AA}^{rad}$$

- Pion R_{AA} has contribution from both light quarks and gluons

$$R_{AA}^{\pi} = \frac{1}{2}(R_{AA}^{l} + R_{AA}^{g})$$

  - Estimate for gluons is simply given by color factor

$$R_{AA}^{gluons} \approx \left(1 - \frac{9}{4}\varepsilon^{eff}\right)^n = \left(1 - \frac{9}{4}(1 - R_{AA}^{light})^{1/n}\right)^n$$

- Electron R_{AA} has contribution from charm and bottom

$$R_{AA}^{e} = \frac{1}{2}(R_{AA}^{c} + R_{AA}^{b})$$

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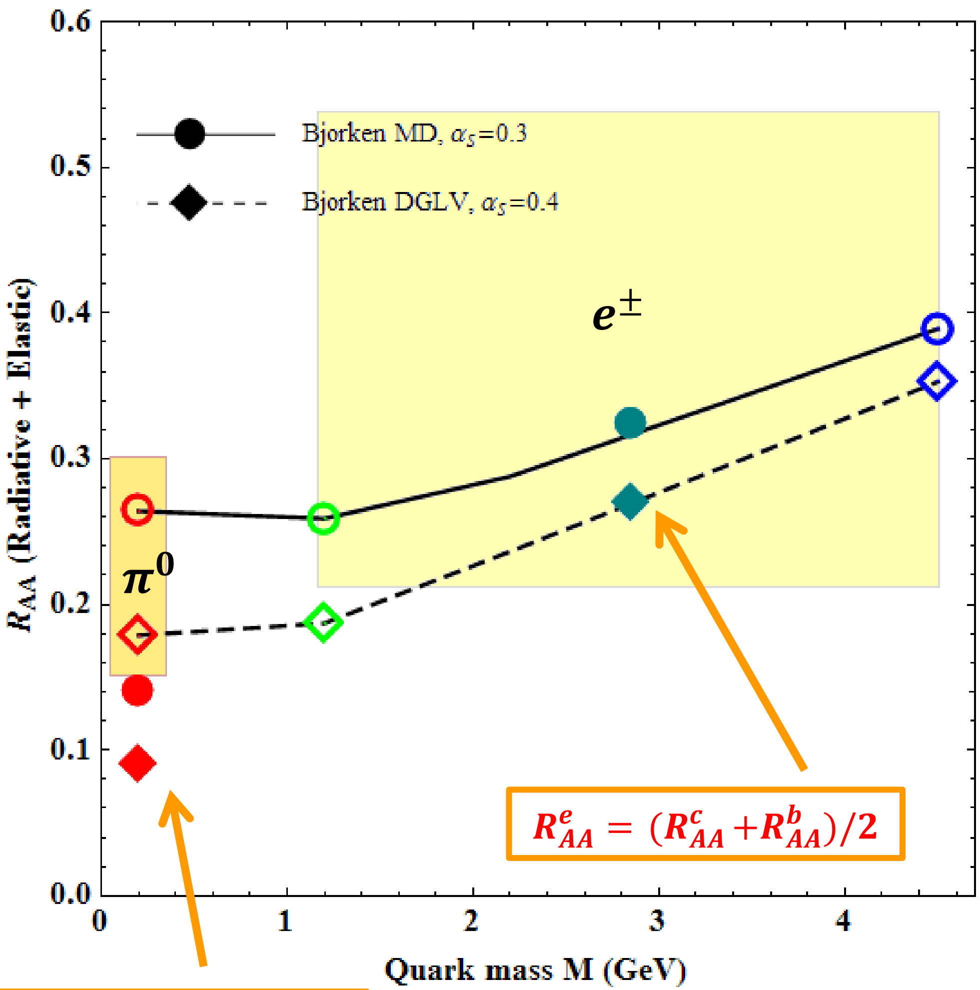
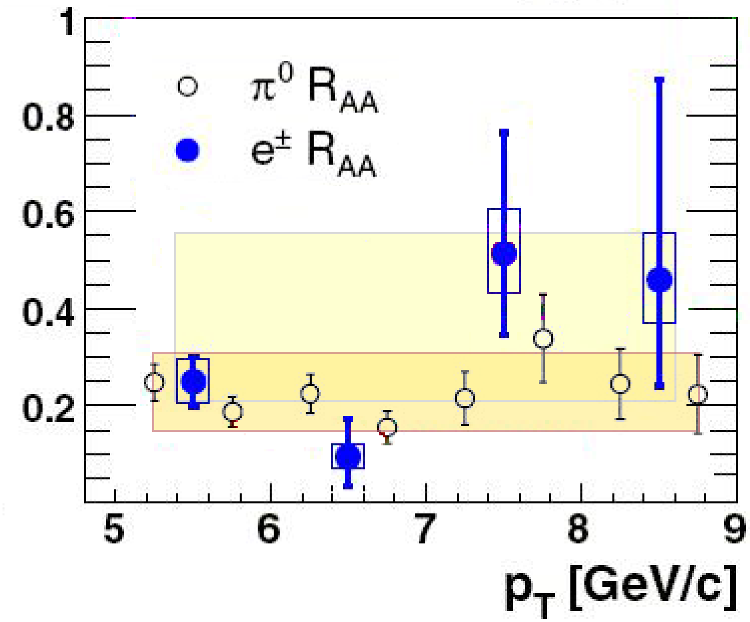
Schematic R_{AA}



$R^e_{AA} = (R^c_{AA} + R^b_{AA})/2$

$R^{\pi}_{AA} = (R^l_{AA} + R^g_{AA})/2$

PHENIX : PRL98, 172301 (2007)



e± data is extremely poor resolution on c and b quenching separately

R_{AA} (p_T) – I

- **Proper inclusion of Elastic Energy loss**

$$P(\varepsilon) = \int dx\, P_{rad}(\varepsilon)\, P_{el}(x-\varepsilon)$$

The probability of losing a fractional energy ε is the convolution of Radiative and Elastic contributions

- $P_{el}(\varepsilon) = e^{-<N_c>}\delta(\varepsilon) + N e^{-\frac{(\varepsilon-\bar{\varepsilon})}{4T\bar{\varepsilon}}}$
  - We consider **gaussian fluctuations** of the Elastic Energy loss centered at $\bar{\varepsilon}$
  - Probability of **not losing energy** is determined by the average number of elastic collisions N_c
- $P_{rad}(\varepsilon) = e^{-<N_g>}\delta(\varepsilon) + \widetilde{P}(\varepsilon)$
  - As before

The leading logarithmic expression for the Elastic Energy loss is given by

$$\frac{dE}{dx} = -C_R \pi^2 \alpha^2 T^2 \log[\frac{4pT}{E-p+4T}/\mu]$$

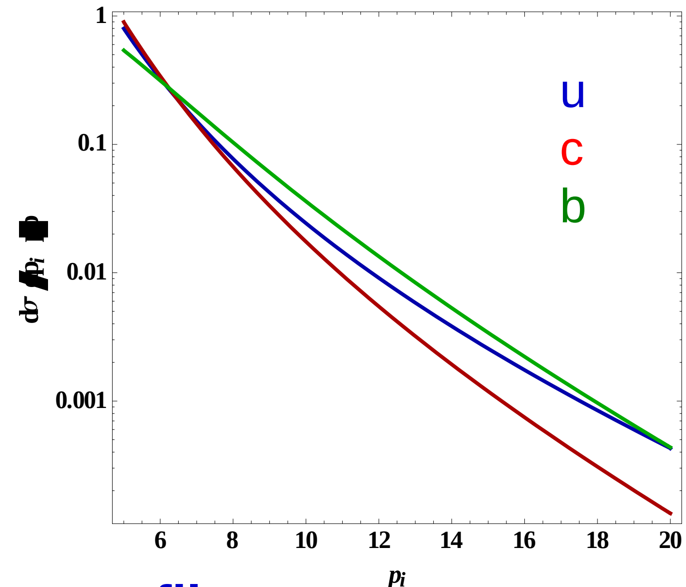
TG model (Thoma, Gyulassy / Nucl. Phys. B 351)

R_{AA} (p_T) – II

- Obtain $W(p_f, p_i; x_\perp, \varphi)$, the probability for a Jet produced at $x_\perp$ with direction φ and momentum p_i to exit the plasma with momentum p_f.

  Then average over the initial pp production spectrum:

$$\frac{d\sigma^{AA}}{dp_f}(p_f; x_\perp, \varphi) = \int dp_i \, \frac{d\sigma^{pp}}{dp_i}(p_i)\, W(p_f, p_i; x_\perp, \varphi)$$

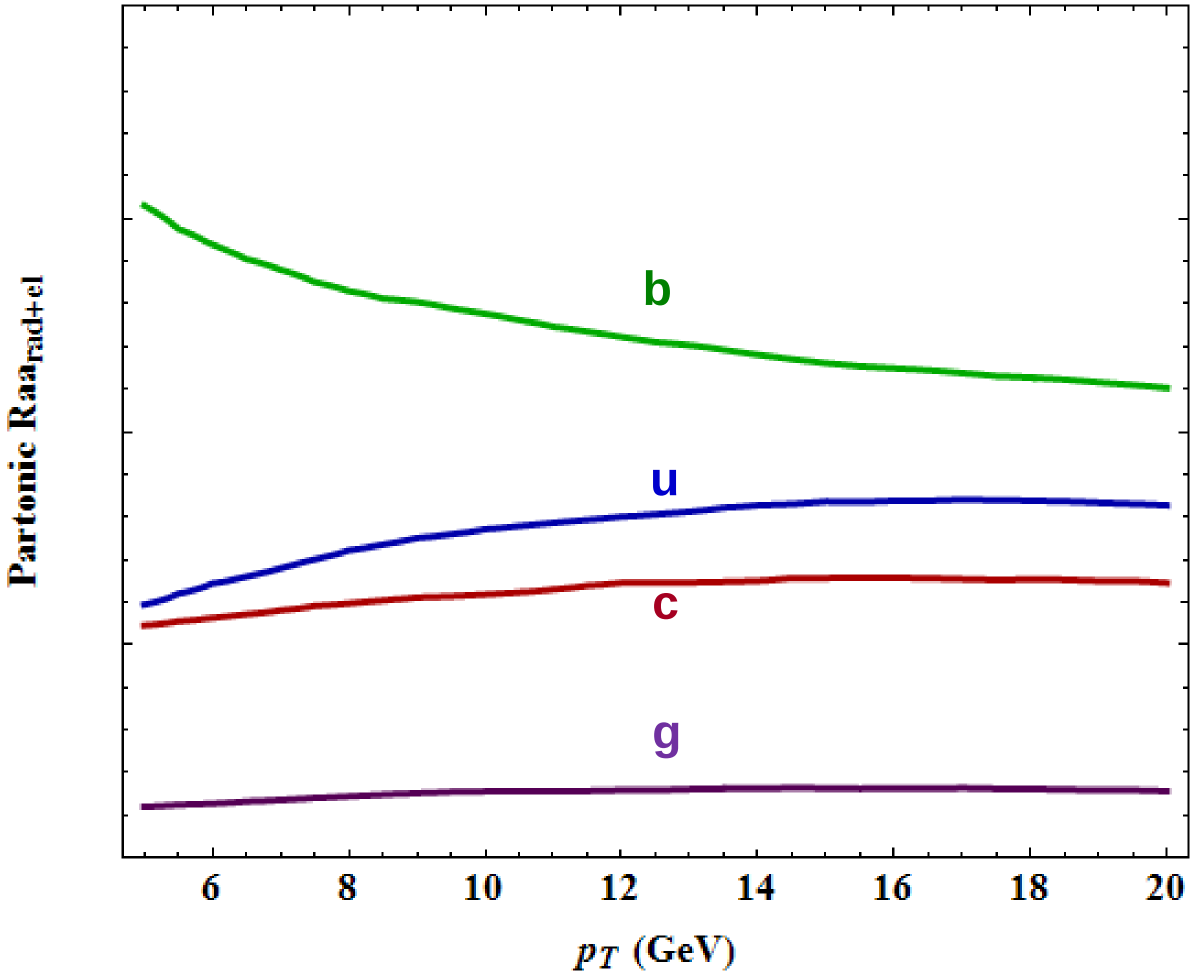


- **We get R_{AA} averaging over the Jet production profile:**

$$R_{AA}(p) = \frac{\int dx_\perp d\varphi\, \rho_{Jet}(x_\perp)\, \frac{d\sigma^{AA}}{dp}(p)}{\frac{d\sigma^{pp}}{dp}(p)}$$

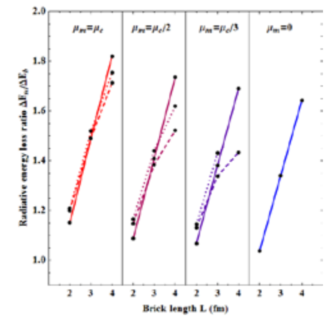
Partonic R_{AA} (p_T) – RHIC results

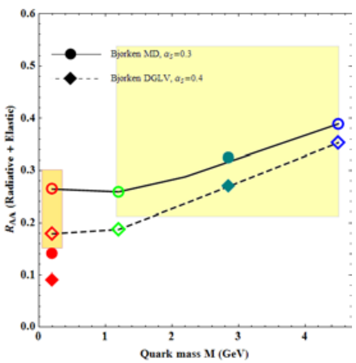
PRELIMINARY



Conclusions

- **The dynamical model helps reducing the difference between light and heavy quark energy loss, even though the effect is small.**
  - **Higher orders in opacity might further improve the results**





- **Adding the effects of a non uniform Bjorken expanding geometry we flatten the mass dependence of R_{AA}**
  - **Need to flavor tag heavy quark hadrons**
  - **Identifying c and b could put the last word on pQCD heavy quark problem**
- **Charm and light quark behavior is almost equal**
  - **Possibility to reveal gluon contribution to pion R_{AA}**