Characterizing Porous Media

Porosity: $\phi = \dfrac{\text{void volume}}{\text{total volume}}$

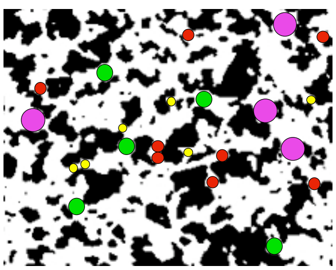
Effective Porosity: $\phi_{eff} = \dfrac{\text{connected void volume}}{\text{total volume}}$

Void Ratio: $e = \dfrac{\phi}{1-\phi}$

Specific Surface Area: $s = \dfrac{Internal\ Area}{Total\ Volume}$

Characterizing Porous Media

Pore Size Distribution:



Characterizing Porous Media

Pore Size Distribution:

$$\int_{\partial}^{\infty} \alpha(\delta) d\delta = 1$$

α is the fraction of total pore space that has a pore diameter of δ and δ+dδ

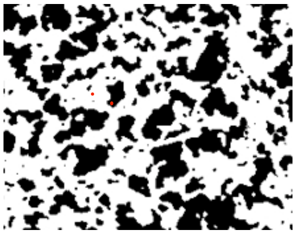
Cummulative Pore Size Distribution:

$$f(\delta) = \int_{\partial}^{\infty} \alpha(\delta) d\delta \qquad f(0) = 1$$

f(δ) is the fraction of that have a pore diameter larger than δ.

Statistical Description of Porous Media

One Point Correlation Function, S_1



$$S_1 = \langle f(x) \rangle = \phi$$

Use binary image as f .

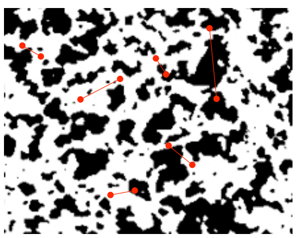
Indicator function:

f(x) = 1 if x is in a pore

f(x) = 0 if x is in a grain

Statistical Description of Porous Media

Two Point Correlation Function, S_2



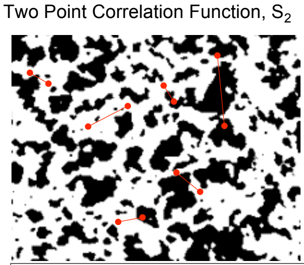
For isotropic medium:

$$S_2(r) = \langle f(x) f(x+r) \rangle$$

S_2 is the probability that two points separated by a distance r will be both in the pore space.

Statistical Description of Porous Media

Two Point Correlation Function, S_2



For random distribution of fully penetrable spheres having radius R:

$$S_2(r) = \phi^{\alpha}$$

where α = 2 for $r \geq 2R$ and

For r < 2R:

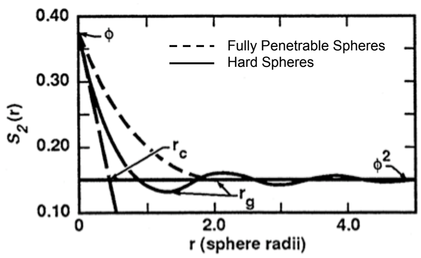
$$\alpha(r) = 1 + \frac{3r}{4R} - \frac{r^3}{16R^3}$$

For impenetrable spheres of radius R see Torquato & Stell, 1982, 1985; Berryman, 1983. For small r and small ϕ:

$$S_2(r) = 1 + \alpha \ln \phi \quad \text{where} \quad \alpha(r) = 1 + \frac{3r}{4R} - \frac{r^3}{16R^3}$$

Statistical Description of Porous Media

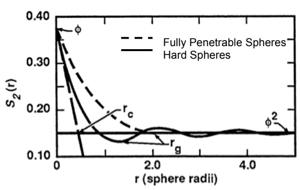
Two Point Correlation Function, S_2



(Berryman & Blair, 1986; Blair et al., 1993)

Statistical Description of Porous Media

Two Point Correlation Function, S_2



*Slope of $S_2(r)$ near origin is proportional to the specific surface area,s .

$$S_2'(0) = -\frac{s}{4}$$

*Asymptote $S_2(r)$ is ϕ^2.

*r_c is the effective pore diameter $$r_c = \frac{4\phi}{s}(1-\phi)$$

*r_g is an estimate of spherical particle size

Porous Media: Permeability

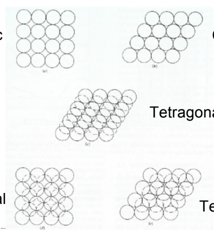
Permeability: ease with which a fluid may flow through a material by an applied pressure gradient.

From Darcy's Law (1856)

$$K = \frac{q\mu}{A}\frac{L}{\Delta P}$$

K - permeability (L^2)
q - flow rate per unit volume (L^3/T)
μ - viscosity of the fluid (L/MT)
ΔP - pressure difference across the length of the sample (M/LT^2)
L - length of the sample (L)
A - cross-sectional area (L^2)

Packing Theory to Model Porous Media



(from Mitchell, J., 1993, Fundamentals of Soil Behavior, 2nd Edition, John Wiley & Sons, Toronto)

Type of Packing	Coordination Number	Layer Spacing (R = radius)	Volume of Unit	Porosity (%)	Void Ratio
Simple Cubic	6	$2R$	$8R^3$	47.64	0.91
Cubical–Tetrahedral	8	$2R$	$4\sqrt{3R^3}$	39.54	0.65
Tetragonal–Sphenoidal	10	$R\sqrt{3}$	$6R^3$	30.19	0.43
Pyramidal	12	$R\sqrt{2}$	$4\sqrt{2R^3}$	25.95	0.34
Tetrahedral	12	$2R\sqrt{2/3}$	$4\sqrt{2R^3}$	25.95	0.34