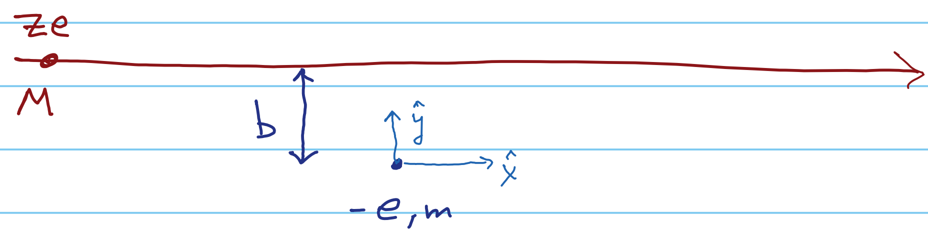
Survey of some topics in Chapter 13

Collisions, energy loss, Cerenkov & Transition Radiation

13.1 Coulomb collision between heavy particle, charge Ze, mass $M \gg$ electron mass m, charge $-e$ at rest

Assume an undeflected heavy (M) trajectory (nonrelativistic)



The position of Ze is $\vec{X}(t) = b\hat{y} + vt\,\hat{x}$

The e^- experiences a force $\vec{F} = \dfrac{(vt - x)\,\hat{x} + (b-y)\,\hat{y}}{\left[(vt-x)^2 + (b-y)^2\right]^{3/2}} e^2 Z$

when the e^- is at $\vec{x}(t) = x\hat{x} + y\hat{y}$

$\Rightarrow \dfrac{d\vec{p}}{dt} = \vec{F}$

$\dfrac{d\vec{x}}{dt} = \dfrac{\vec{p}}{m}$

If we neglect the small displacement of the e^- during the collision, then $\vec{x} \cong 0$

and $\Delta\vec{p} = \displaystyle\int_{-\infty}^{\infty} \vec{F}(t)\,dt = \frac{2Ze^2}{bv}\,\hat{y}$,

and energy transmitted to the e^- equal to

$$\Delta E = \frac{\Delta p^2}{2m} = \frac{2Z^2e^4}{mb^2v^2}$$

By momentum conservation the heavy particle is deflected, by small $\theta \simeq \dfrac{\Delta p}{p} = \dfrac{2Ze^2}{pvb}$

An exact nonrelativistic treatment (Rutherford cross section) gives

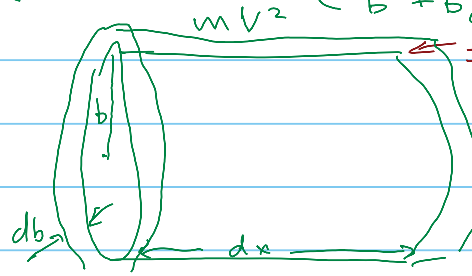
$$2\tan\frac{\theta}{2} = \frac{2Ze^2}{pvb}$$

(agrees at small θ with our approximation)

And a more accurate formula valid even as $b \to 0$ is readily derived (see problem 13.1):

$$\Delta E(b) = \frac{2Z^2e^4}{mv^2}\left(\frac{1}{b^2 + b_0^2}\right) \quad \text{where } b_0 = \frac{Ze^2}{\gamma m v^2}$$

How much energy is lost to the e's between b, $b + db$?



e's in this cylindrical shell is

$$n\,dx\,2\pi b\,db$$

where n = electron density $= N Z_1$ (if Z_1 e's per atom)

255

$$\Rightarrow \quad \frac{d^2E(b)}{dx\,db} = N\,2\pi b\,db\,\Delta E(b) \quad \text{or} \quad \frac{dE}{dx} = \int_{b_{min}}^{b_{max}} \frac{2z^2e^4}{mb^2v^2}\,2\pi n\,b\,db$$

giving a logarithmic dependence,

$$\boxed{\frac{dE}{dx} = 4\pi\frac{Nz^2e^4Z}{mv^2}\ln\frac{b_{max}}{b_{min}}}$$

Aside: Jackson argues that

$$b_{max} \simeq \left(\frac{Zz^2e^4}{mv^2\epsilon}\right)^{1/2}$$

$$b_{min} \simeq \frac{ze^2}{pv}$$

ϵ = electron binding energy

This has the correct basic structure, but a more careful and complete derivation by Bethe (1930) gives:

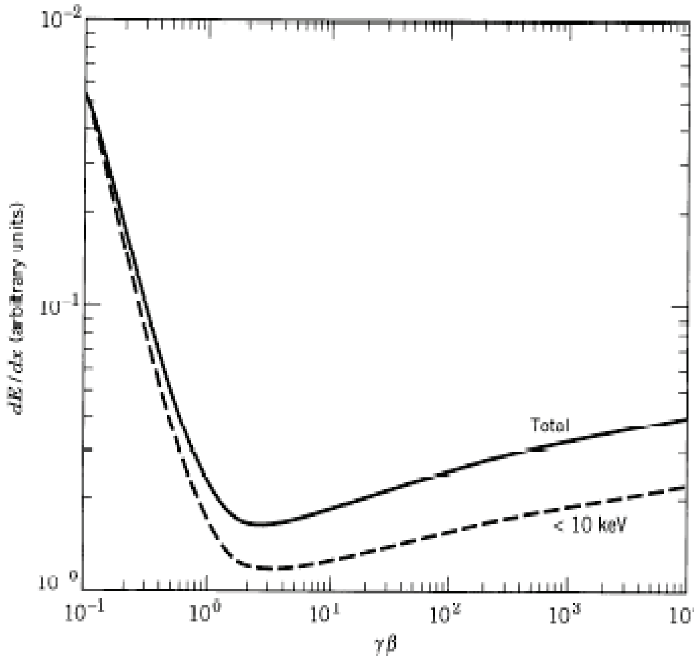
$$\frac{dE}{dx} = 4\pi N Z_1 \frac{z^2e^4}{mc^2\beta^2}\left\{\ln(B) - \beta^2\right\}, \quad \text{where } B = \frac{2\gamma\beta^2mc^2}{\hbar\langle\omega\rangle}$$

QM correction for soft collisions with energies "small" < ϵ

relativistically valid!

and $\langle\omega\rangle$ is defined in terms of the quantum atomic oscillator strengths f_j associated with transition of frequency ω_j by

$$Z_1\ln\langle\omega\rangle \equiv \sum_j f_j\ln\omega_j \qquad (13.14)$$

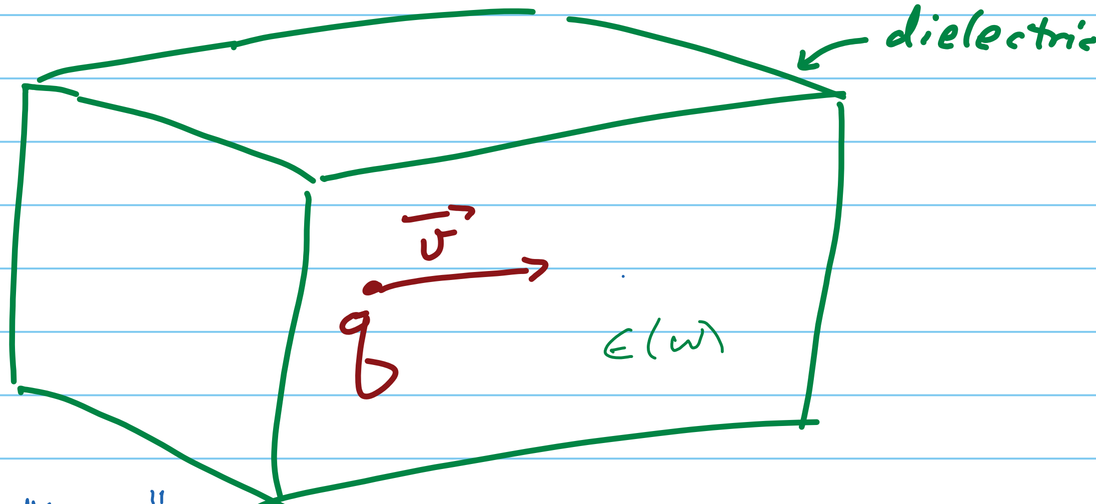


J: **Figure 13.1** Energy loss as a function of $\gamma\beta$ of the incident heavy particle. The solid curve is the total energy loss (13.14) with $\hbar\langle\omega\rangle$ = 160 eV (aluminum). The dashed curve is the energy loss in soft collisions (13.12) with ε = 10 keV. The ordinate scale corresponds to the curly-bracketed quantities in (13.12) and (13.14), multiplied by 0.15.

Section 13.3 Density effect in collisional energy loss

The ideas that led to Bethe's formula (13.14) are inaccurate for larger impact parameters that greatly exceed the average atom-atom separation, especially for a dense material. The required correction to 13.14 was derived by Fermi, Phys. Rev. 57 p. 485 (1940)

The reason is because the dielectric material reduces the electric field at larger distances from Ze



A charged particle going through a linear dielectric medium experiences energy loss in two ways:

i) short-range "hard" collisions

ii) distant coherent interactions with the medium (complex $\epsilon(\omega)$ causes dissipation)

Case i) is treated in Jackson Sec. 13.1

Here we will discuss case ii), i.e. Sec. 13.3; (see also problem 7.26)

Starting point: Fourier representation of potentials $A_\mu(x)$, sources $J_\mu(x)$; i.e. write for every quantity,

$$F(\vec{x},t) = (2\pi)^{-2}\int d^3k \int d\omega\, F(\vec{k},\omega)\, e^{i\vec{k}\cdot\vec{x}-\omega t}$$

Then the Fourier transformed wave equations are:

$$\left[k^2 - \frac{\omega^2}{c^2}\epsilon(\omega)\right]\Phi(\vec{k},\omega) = \frac{4\pi}{\epsilon(\omega)}\rho(\vec{k},\omega) \quad (1)$$

$$\left[k^2 - \frac{\omega^2}{c^2}\epsilon(\omega)\right]\vec{A}(\vec{k},\omega) = \frac{4\pi}{c}\vec{J}(\vec{k},\omega)$$

Consider charge Ze at velocity $\vec{v}$:

Then $\rho(\vec{x},t) = Ze\,\delta(\vec{x}-\vec{v}t)$, $\vec{J}(\vec{x},t) = \vec{v}\rho(\vec{x},t)$ and their Fourier transforms are simple:

$$\rho(\vec{k},\omega) = \frac{Ze}{2\pi}\delta(\omega - \vec{k}\cdot\vec{v}) \qquad \vec{J}(\vec{k},\omega) = \vec{v}\rho(\vec{k},\omega)$$

Plugging into Eq. (1) gives the potentials:

$$\Phi(\vec{k},\omega) = \frac{2Ze}{\epsilon(\omega)} \frac{\delta(\omega - \vec{k}\cdot\vec{v})}{k^2 - \frac{\omega^2}{c^2}\epsilon(\omega)} \quad , \quad \vec{A}(\vec{k},\omega) = \epsilon(\omega)\frac{\vec{v}}{c}\Phi(\vec{k},\omega)$$

Now $\vec{E}(\vec{x},t) = -\nabla\Phi \rightarrow -\nabla\int \frac{\Phi(\vec{k},\omega)}{2\pi} e^{i\vec{k}\cdot\vec{x} - \omega t} = \int \frac{-i\vec{k}}{2\pi}\Phi(\vec{k},\omega) e^{i\vec{k}\cdot\vec{x} - i\omega t}$

$$-\frac{1}{c}\frac{\partial\vec{A}}{\partial t} \rightarrow -\int \frac{\epsilon(\omega)\vec{v}}{2\pi\, c}\Phi(\vec{k},\omega)(-i\omega)e^{i\vec{k}\cdot\vec{x} - i\omega t}$$

Thus

$$\Rightarrow \vec{E}(\vec{k},\omega) = \left(\frac{i\epsilon(\omega)\omega}{c}\frac{\vec{v}}{c} - i\vec{k}\right)\Phi(\vec{k},\omega) \; ; \; \vec{B}(\vec{k},\omega) = i\epsilon(\omega)\frac{\vec{k}\times\vec{v}}{c}\Phi(\vec{k},\omega)$$

Now compute the energy lost in collision with an e^- at impact param. b:

$$\Delta E = -e\int_{-\infty}^{\infty}\vec{v}\cdot\vec{E}\,dt = 2e\,\mathrm{Re}\int_0^{\infty} i\omega\vec{x}(\omega)\cdot\vec{E}^*(\omega)\,d\omega$$

where $\vec{E}(\omega) = (2\pi)^{-3/2}\int d^3k\,\vec{E}(\vec{k},\omega)e^{ibk_y}$ since we want E-field at $(0,b,0)$

e.g. let's first calculate the x-component: k_x is easy!

$$E_x(\omega) = \frac{2iZe}{\epsilon(\omega)(2\pi)^{3/2}}\int d^3k\, e^{ik_yb}\left(\frac{\omega\epsilon(\omega)v}{c^2} - k_x\right)\frac{\delta(\omega - vk_x)}{k^2 - \frac{\omega^2}{c^2}\epsilon(\omega)}$$

$$= \frac{-2iZe\omega}{(2\pi)^{3/2}v^2}\left(\frac{1}{\epsilon(\omega)} - \beta^2\right)\int_{-\infty}^{\infty}dk_y e^{ik_yb}\left[\int_{-\infty}^{\infty}\frac{dk_z}{k_y^2 + k_z^2 + \lambda^2}\right] = \frac{\pi}{\sqrt{\lambda^2 + k_y^2}}$$

where $\lambda^2 = \frac{\omega^2}{v^2} - \frac{\omega^2}{c^2}\epsilon(\omega) = \frac{\omega^2}{v^2}\left(1 - \beta^2\epsilon(\omega)\right)$

and the final integral here is

$$E_x = \frac{-iZe\omega}{(2\pi)^{1/2}v^2}\left(\frac{1}{\epsilon(\omega)} - \beta^2\right)\left[\int_{-\infty}^{\infty}\frac{e^{ibk_y}}{(\lambda^2 + k_y^2)^{1/2}}dk_y\right] = 2K_0(\lambda b)$$

modified Bessel function!

$\Rightarrow$ finally $E_x(\omega) = -\frac{iZe\omega}{v^2}\left(\frac{2}{\pi}\right)^{1/2}\left(\frac{1}{\epsilon(\omega)} - \beta^2\right)K_0(\lambda b)$ (*)

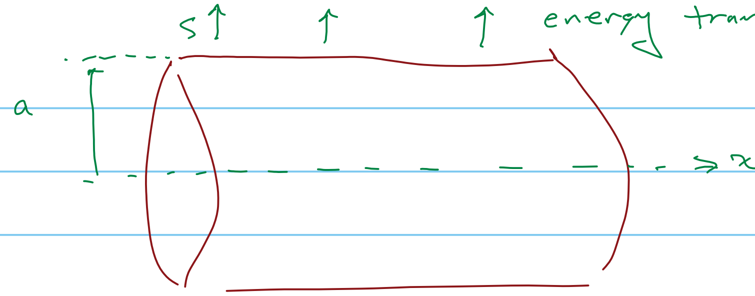
and similarly,

$$E_y(\omega) = \frac{Ze}{v}\left(\frac{2}{\pi}\right)^{1/2}\frac{\lambda}{\epsilon(\omega)}K_1(\lambda b)$$

and

$$B_z(\omega) = \epsilon(\omega)\beta E_y(\omega)$$

Now, the energy lost per dx by collisions happening at $b > a$ can either be computed by acting on all charges in a cylinder (like Bethe's), OR instead, by an energy conservation argument:



(and $dx = v\,dt$)

$\Rightarrow$ energy flow involves the radial component of the Poynting vector

i.e. $$\left(\frac{dE}{dx}\right)_{b>a} = \frac{1}{v}\frac{dE}{dt} = \frac{-c}{4\pi v}\underbrace{\int_{-\infty}^{\infty} 2\pi a \overbrace{dx}^{v\,dt} \left(B_z(t)E_x(t)\right)}_{2\pi a v \,\mathrm{Re}\left[\int_0^\infty B_z^*(\omega)E_x(\omega)\,d\omega\right]}$$

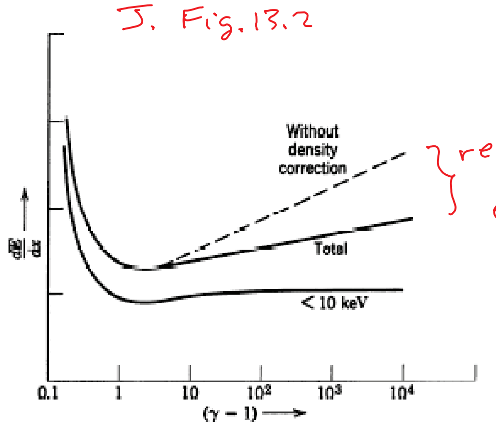
(13.35)

and giving finally Fermi's result:

$$\boxed{\left(\frac{dE}{dx}\right)_{b>a} = \frac{2}{\pi}\frac{(ze)^2}{v^2}\,\mathrm{Re}\left\{\int_0^\infty i\omega\lambda^* a\, K_1(\lambda^* a)K_0(\lambda a)\left(\frac{1}{\epsilon(\omega)} - \beta^2\right)d\omega\right\}}$$

Woohoo!

J. Fig. 13.2



reduction of energy loss due to the density effect

e.g. for silver bromide
$\frac{1}{N}\frac{dE}{dx} \simeq 1.02\ \mathrm{MeV\cdot cm^2/g}$
approx constant at high E

See also U. Fano, Ann. Rev. Nucl. Sci. 13, 1 (1963)

Sec. 13.4 Cerenkov radiation

General statement: When an object (e.g. a charge) moves through a medium at a speed v greater than the speed of waves that travel in the medium (EM waves here) then waves are generated as the object travels.

e.g. phonons excited if you drag a particle/object through a BEC faster than sound speed

e.g. aircraft faster than sound speed in air (sonic booms!)

In the case considered here, a charge moves at $v > c/n$ ← index of refraction

Back in Sec. 13.3 we considered the regime where $|\lambda a| \ll 1$, but now we look at the opposite limit where $|\lambda a| \gg 1$, and use the fields derived in Sec. 13.3 (see * above) except now in the asymptotic expansions of the Bessel functions:

$$\Rightarrow \quad E_x(\omega, b) \to i\,\frac{ze\omega}{c^2}\left(1 - \frac{1}{\beta^2\epsilon(\omega)}\right)\frac{e^{-\lambda b}}{\sqrt{\lambda b}} \qquad (**)$$

$$E_y(\omega, b) \to \frac{ze}{v\,\epsilon(\omega)}\left(\frac{\lambda}{b}\right)^{1/2} e^{-\lambda b}$$

$$B_z(\omega, b) \to \beta\,\epsilon(\omega)\,E_y(\omega, b)$$

And so the integral 13.37 becomes

$$\frac{dE}{dx} = -\frac{ca}{2}\,\mathrm{Re}\int_0^\infty B_z^*(\omega)\,E_x(\omega)\,d\omega$$

$$= \mathrm{Re}\int_0^\infty \frac{z^2e^2}{c^2}\left(-i\left(\frac{\lambda^*}{\lambda}\right)^{1/2}\right)\omega\left(1 - \frac{1}{\beta^2\epsilon(\omega)}\right)e^{-(\lambda+\lambda^*)a}$$

where, again, $\lambda = \frac{\omega}{v}\left(1 - \beta^2\epsilon(\omega)\right)^{1/2} = \frac{\omega}{v}\sqrt{1 - \frac{v^2\epsilon(\omega)}{c^2}}$

For simplicity in the following analysis, assume $\epsilon(\omega) = \text{real}$, meaning that λ will be purely IMAGINARY if $\boxed{v > \frac{c}{\sqrt{\epsilon(\omega)}}}$ and then there is no exponential decay in $e^{-(\lambda+\lambda^*)a}$

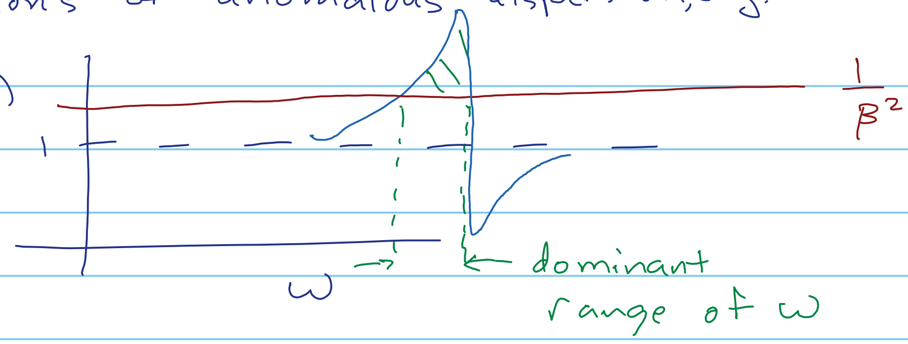
This is the case of Cerenkov radiation that escapes to ∞, i.e.

$$\frac{dE}{dx} = \frac{z^2e^2}{c^2}\int_{\substack{\omega \\ \text{where} \\ \epsilon(\omega) > \frac{1}{\beta^2}}}\left[\omega\left(1 - \frac{1}{\beta^2\epsilon(\omega)}\right)\right]d\omega$$

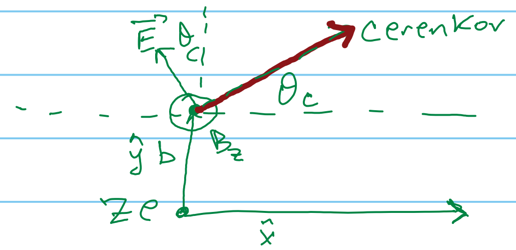
And the integrand [---] yields the distribution of Cerenkov radiation versus frequency ω

Recall from Chapter 7 that $\epsilon(\omega)$ can get quite large in regions of anomalous dispersion, e.g.

Before Chap. 11 we called this $\epsilon_r(\omega)$



Direction of propagation

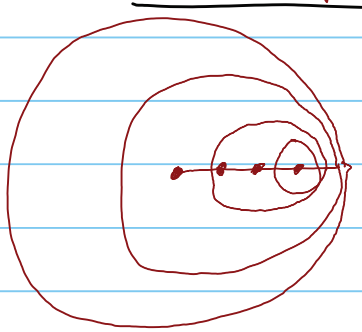


$$\boxed{\tan\theta_c = \frac{-E_x}{E_y}}$$

$$\Rightarrow \quad \cos\theta_c = \frac{1}{\beta\sqrt{\epsilon(\omega)}} \quad \text{using } (**)$$

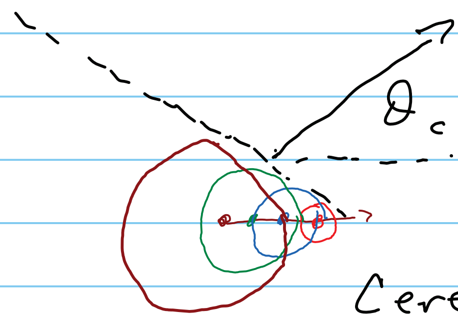
Qualitative idea

$v < c/\sqrt{\epsilon(\omega)}$



no Cerenkov

$v > c/\sqrt{\epsilon(\omega)}$

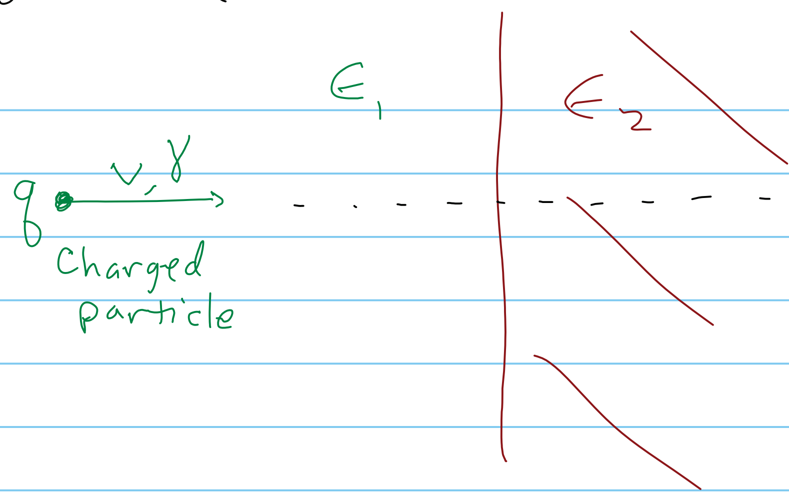


Cerenkov radiation "shock wave"

Transition Radiation, Sec. 13.7

I am showing this topic for general information without going through the detailed and rather complicated derivation (see Jackson pp 646–654 or Landau-Lifshitz, "Electrodynamics of Cont. Media", Sec. 116)

The basic idea: A charged particle in medium 1 (ϵ_1) strikes a boundary with a second medium (ϵ_2)



The disruption of the fields of q caused by the boundary produces radiation.

This is used in cosmic ray and particle physics experiments to detect very high energy particles.

Landau – Lifshitz (above Ref.) derives (for $\epsilon_1 = 1$, $\epsilon_2 = \epsilon$):

$$\frac{dI}{d\omega\, d\Omega} = \frac{e^2 \beta^2}{\pi^2 c (1-\beta^2\cos^2\theta)^2} \left| \frac{(\epsilon-1)\left[1-\beta^2+\beta(\epsilon-\sin^2\theta)^{1/2}\right]}{\left[1+\beta(\epsilon-\sin^2\theta)^{1/2}\right]\left[\epsilon\cos\theta+(\epsilon-\sin^2\theta)^{1/2}\right]} \right|^2$$

And the radiation is linearly polarized, with $\vec{E}$ in the $\vec{k}-\vec{v}$ plane

In the non-relativistic limit, this simplifies to

$$\frac{dI}{d\omega\, d\Omega} = \frac{e^2 v^2}{\pi^2 c^3} \frac{\sin^2\theta}{(1-\beta^2\cos^2\theta)^2}$$

But practical detectors are usually for $\gamma \gg 1$

e.g. for $\gamma \sim 10^3$ typical photon energies are about 2 – 20 keV

J: Fig. 13.10 Angle Distrib.

13.11 Freq. Distrib.

Transition Radiation properties

$\nu = \dfrac{\omega}{\gamma\omega_p}$

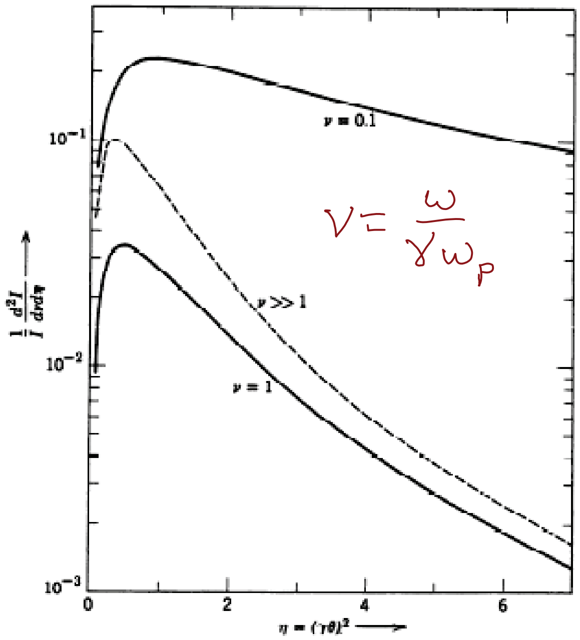


Figure 13.10 Angular distributions of transition radiation at $\nu = 0.1$, $\nu = 1$ and $\nu \gg 1$. The solid curves are the normalized angular distributions, that is, the ratio of (13.84) to (13.87). The dashed curve is ν^4 times that ratio in the limit $\nu \to \infty$.

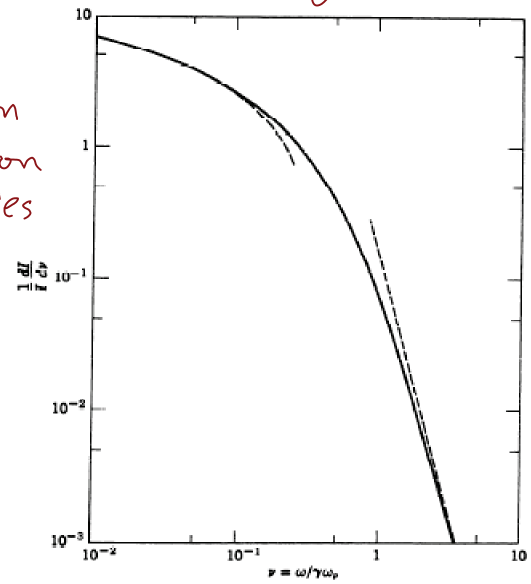


Figure 13.11 Normalized frequency distribution $(1/I)(dI/d\nu)$ of transition radiation as a function of $\nu = \omega/\gamma\omega_p$. The dashed curves are the two approximate expressions in (13.86).

Note: $\hbar\omega_p \simeq 20\,\text{eV}$

In practice, transition radiation detectors use very many parallel planes of different dielectrics.

Check out the link below, to see a transition radiation detector used in CERN's ATLAS detector!

https://www.youtube.com/watch?v=HH1dMz288KA

And here are a few graphics from an article on particle astrophysics experiments on balloons + satellites:

Nuclear Instruments and Methods in Physics Research A 522 (2004) 9–15. Dietrich Müller

Transition radiation detectors in particle astrophysics

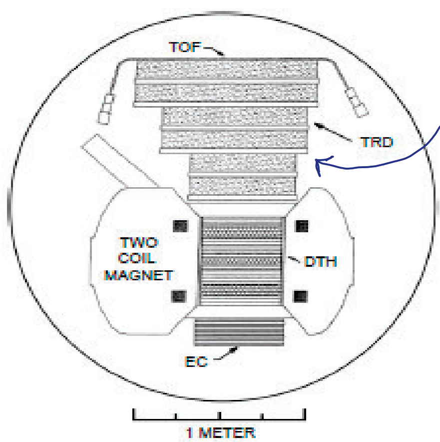


Fig. 1. Cross-section of the HEAT spectrometer for measurements of electrons and positrons. TOF: top time-of-flight scintillator; TRD: transition radiation detector; DTH: drift-tube hodoscope; EC: electromagnetic calorimeter.

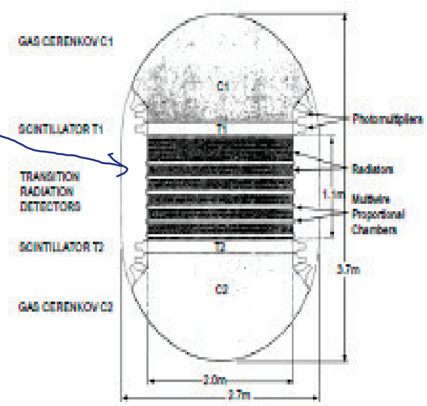


Fig. 5. Cross-section of the CRN detector.

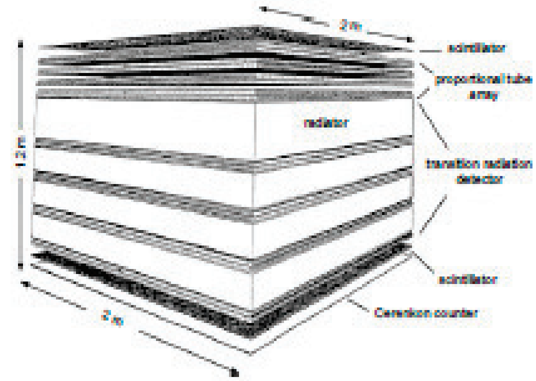


Fig. 6. Cross-section of the TRACER balloon instrument.

263

more from the Müller reference above:

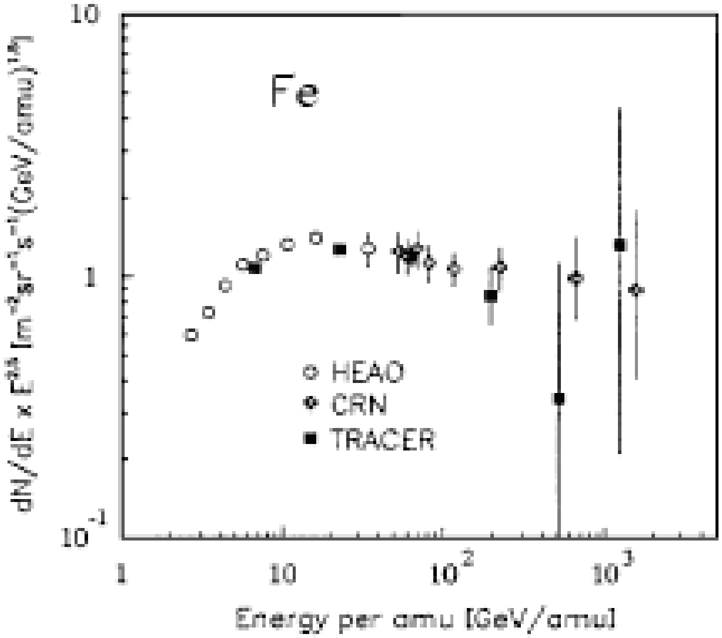


Fig. 7. Differential energy spectra of iron nuclei measured with HEAO-3, CRN and TRACER. Note that E is the energy per amu, and that the intensities are multiplied with $E^{2.5}$.

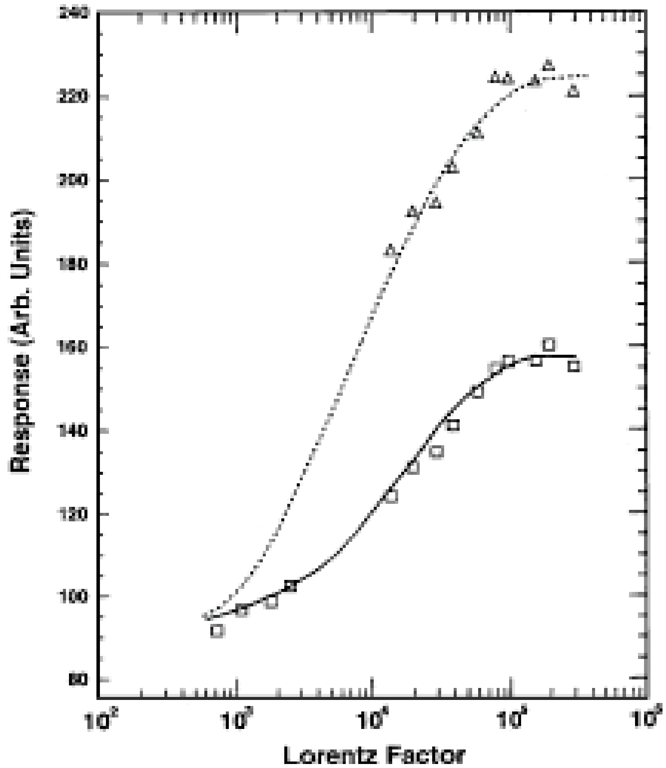


Fig. 8. Average signal versus Lorentz factor for a composite radiator/detector configuration consisting of plastic foils, foam, and fibers (triangles), and for a radiator of parallel Mylar foils of 76 μm thickness (squares). Note that the signal reaches saturation around $\gamma \approx 10^5$.

For some wacky fun, an intro to CERN is at:

https://www.youtube.com/watch?v=j50ZssEojtM

And for an audio summary of everything you learned this semester, check out

https://app.box.com/s/cq15ipcslwlnmcs93n1bhwaz1l1zhkt4

(call me nutty.mp3)

Final topic for this semester

- A simple introduction to EM field quantization

(in vacuum, no sources)

You will likely see a more systematic treatment of this subject in your advanced QM course in the future, but here we can see the basic idea in just a few pages

Note: I will return to SI units now

Reference: U. Fano and A.R.P. Rau, p. 26
Atomic Collisions & Spectra (1986)

Start from Maxwell's eqns in vacuum, no sources:

$$\nabla\cdot\vec{E}=0 \qquad \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \qquad \nabla\times\vec{B}=\underbrace{\mu_0\epsilon_0}_{\frac{1}{c^2}}\frac{\partial\vec{E}}{\partial t}$$

$$\nabla\cdot\vec{B}=0$$

And these have plane wave solutions,

$$\vec{E}(\vec{x},t)=\vec{E}_0 e^{i\vec{k}\cdot\vec{x}-i\omega t}$$

$$\vec{B}(\vec{x},t)=\vec{B}_0 e^{i\vec{k}\cdot\vec{x}-i\omega t}$$

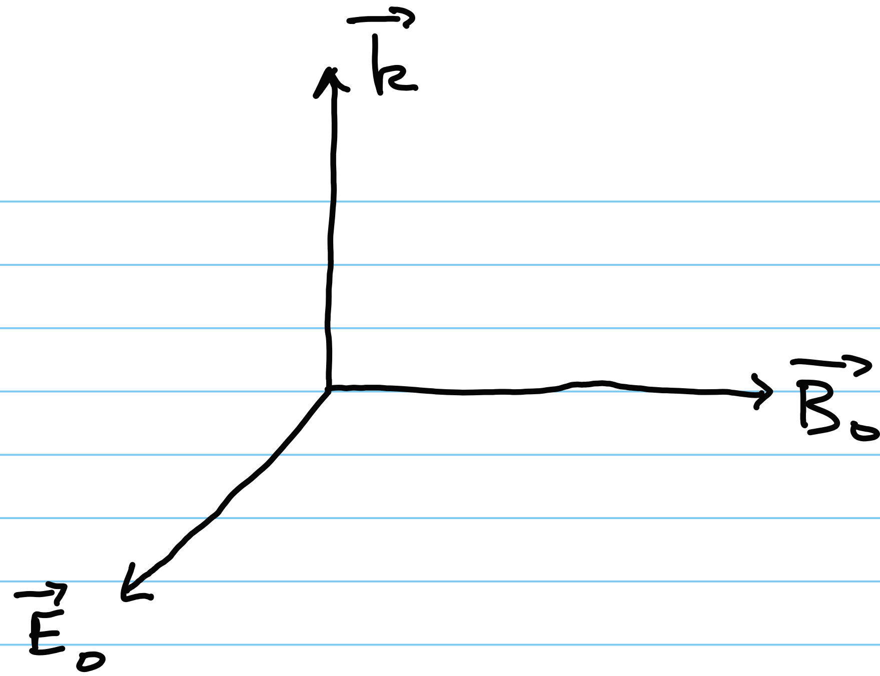
which only obey Maxwell's equations if also,

$$i\vec{k}\cdot\vec{E}_0=0\ , \quad i\vec{k}\cdot\vec{B}_0=0 \quad \text{(transversality)}$$

AND if $\quad i\vec{k}\times\vec{E}_0=i\omega\vec{B}_0\ , \quad i\vec{k}\times\vec{B}_0=\frac{-i\omega}{c^2}\vec{E}_0$

which are satisfied only if $\frac{\omega}{c}=k$

and $\quad |\vec{B}_0|=\frac{1}{c}|\vec{E}_0|$



We will consider a specific, simple geometry with a wave traveling along $\hat{k} = +\hat{z}$ with

$$\vec{E}_0 = E_0\,\hat{x} \qquad \vec{B}_0 = B_0\,\hat{y} = \frac{E_0}{c}\,\hat{y}$$

The full EM wave then looks like

$$\boxed{\vec{E}(\vec{x},t) = \hat{x}\,\mathcal{E}(t)\,e^{ikz}} \quad \text{where} \quad \boxed{\mathcal{E}(t) \equiv E_0\,e^{-i\omega t}}$$

and $\vec{B}(\vec{x},t) = \hat{y}\left(\frac{E_0}{c}e^{-i\omega t}\right)e^{ikz}$

which we can recast as

$$\vec{B}(\vec{x},t) = \hat{y}\left(\frac{\omega E_0}{kc^2}e^{-i\omega t}\right)e^{ikz}$$

or $\boxed{\vec{B}(\vec{x},t) = \hat{y}\,\frac{i}{\omega c}\frac{\partial}{\partial t}\mathcal{E}(t)\,e^{ikz}}$

$\Rightarrow$ Now write out the Hamiltonian, as the energy in such a wave within a large but finite quantization volume V

i.e. energy density is $u = \frac{\epsilon_0 |\vec{E}|^2}{2} + \frac{1}{2\mu_0} |\vec{B}|^2$

$= \text{constant in space here}$

$\Rightarrow$ energy is

$$H = \frac{1}{2}\int_V u \, d^3x = \frac{V\epsilon_0}{4}\left(|\vec{E}|^2 + c^2 |\vec{B}|^2\right)$$

or $$\boxed{H = \frac{V\epsilon_0}{4}\left(\mathcal{E}(t)^2 + \frac{1}{\omega^2}\dot{\mathcal{E}}(t)^2\right)}$$

which is the classical Hamiltonian for the particular mode we chose

IDEA: This looks just like the Hamiltonian of a classical harmonic oscillator.

The analogy is clearest if we define a "generalized momentum" as $P \equiv \frac{V\epsilon_0}{\omega\sqrt{2}}\dot{\mathcal{E}}$

and a "generalized coordinate" as $Q \equiv \frac{V\epsilon_0}{\omega\sqrt{2}}\mathcal{E}$

which recasts our Hamiltonian as

$$\boxed{H = \frac{P^2}{2} + \frac{1}{2}\omega^2 Q^2}$$

which is the Hamiltonian of a unit mass oscillator!

So next, simply quantize it!

Assume the usual quantization rules,

e.g. $[Q, P] = i\hbar$

We all learn in elementary QM how to solve this oscillator problem with raising and lowering operators:

$$a^{\pm} = \mp \frac{i}{\sqrt{2\hbar\omega}} (P \pm i\omega Q)$$

and all the properties of quantum oscillators can be used immediately, such as the energy levels associated with this one mode (ω)

$$\Longrightarrow \quad E_n = (n+\tfrac{1}{2})\hbar\omega \quad , n = 0, 1, 2, \ldots$$

Observations

1. This field oscillator is present even in the GROUND STATE ($n=0$), i.e. all fields have at least the zero point energy $\frac{\hbar\omega}{2}$

2. We can interpret n = # photons in this EM mode of frequency ω, and each photon has energy equal to $\hbar\omega$

3. These quantized fields can be used to compute (using the "Fermi golden rule") the spontaneous radiative decay of an excited atomic state

   e.g. for atomic hydrogen, $|2p, n=0\rangle \to |1s, n=1\rangle$ in which a photon ($n=1$) appears

4. Weirdness: there are of course an infinite number of frequencies ω_s, and if you sum all the zero point energies $E^{rad}_{tot} = \sum_s \frac{\hbar\omega_s}{2} \to \infty$

   A puzzling result! Infinite vacuum energy!

WHAT COULD IT MEAN... ??

To summarize, there are still unsolved vistas in classical electrodynamics. And for descriptions of actual real-world phenomena, these puzzles lead one into the world of quantum electrodynamics, quantum optics, renormalization. You will hopefully learn these beautiful subjects in your future studies.