Corrections to Jackson's Sec. 1.10, p.38

Jackson writes his GF equation as $\nabla'^2 G(\vec{x},\vec{x}') = -4\pi\delta(\vec{x}-\vec{x}')$ WRONG

Why is this wrong and inconsistent with his equations 1.42 – 1.46?

We said that $G(\vec{x},\vec{x}')$ is the potential at $\vec{x}$ due to a point charge of magnitude "$q = 4\pi\epsilon_0$" at $\vec{x}'$ and Jackson's later equations 1.42 – 1.46 are consistent with this interpretation. Therefore, cross out his INCORRECT (1.39), (1.41) and replace them with my revised versions:

$$\nabla^2 G(\vec{x},\vec{x}') = -4\pi\,\delta(\vec{x}-\vec{x}') \qquad \text{CG 1.39}$$

$$\nabla^2 F(\vec{x},\vec{x}') = 0 \qquad \text{CG 1.41}$$

where $G(\vec{x},\vec{x}') = \frac{1}{|\vec{x}-\vec{x}'|} + F(\vec{x},\vec{x}')$ okay in J. 1.40

Now, for instance, if Neumann boundary conditions are given on one or more surfaces $\left(\frac{\partial\Phi}{\partial n}\Big|_S = \text{specified}\right)$ then the appropriate Neumann Green's function almost obeys as in J. 1.45,

$$\frac{\partial G_N(\vec{x},\vec{x}')}{\partial n} = -\frac{4\pi}{S} \quad \text{for } \vec{x} \text{ on } S$$

(area of surface = S)

Why demand this? because $-\int \frac{\partial\Phi}{\partial n}\,da = \frac{q}{\epsilon_0}$ by Gauss's Law and in our G_N, "$q = 4\pi\epsilon_0$"

Consistent with Gauss's Law!

Here J has another error and writes instead: $\frac{\partial G_N}{\partial n'}(\vec{x},\vec{x}') = -\frac{4\pi}{S}$ WRONG

Moreover, in J 3.26(b) you should interchange $r \leftrightarrow r'$ everywhere!