Physics Graduate School Qualifying Examination

January 10, 1997

Part II

Instructions: Work all problems. This is a closed book examination. Calculators may not be used. Start each problem on a new pack of yellow paper and use only one side of each sheet. All problems carry the same weight. Write your student number on the upper right-hand corner of each answer sheet.

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1. A sphere with uniform density spins about a horizontal axis through its center with an angular velocity ω_0. It is placed on a rough horizontal surface (coefficient of kinetic friction μ).

   a How much time elapses before the sphere begins to roll without slipping?

   b How far does the sphere move during the time interval calculated above?

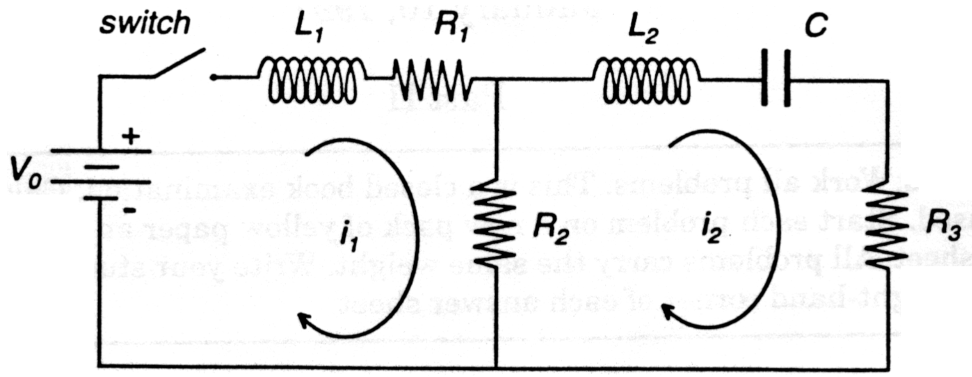
2. An asteroid is observed at a distance r_0 from the sun, and moving with a speed v_0 relative to it. As a result of a collision with another body, the asteroid's velocity is abruptly rotated by 45° in the plane of motion without change in its magnitude. After the collision, the asteroid moves in a circular orbit of radius r_0. Find the minimum and maximum distance from the sun for the asteroid's **original** orbit.

3. A parallel-plate capacitor has the region between its plates filled with a dielectric slab of dielectric constant K. The plates have width w and length l, and are separated by a distance d. The capacitor is charged while connected to a potential difference V_0, after which it is disconnected. The dielectric slab is then partially withdrawn along the l dimension until only a length x remains between the plates.

   a What is the potential difference across the capacitor?

   b What is the force tending to pull the dielectric slab back to its original position?

4. In the circuit shown below, the switch is closed at $t=0$.



a Determine the loop currents, i_1 and i_2, an instant after the switch is closed.

b Determine the first time derivatives of the loop currents an instant after the switch is closed.

c Determine the asymptotic values of the loop currents as $t \rightarrow \infty$.

5. In the Rutherford experiment, α-particles are scattered from various atomic species. The surprising result of this experiment was important in the development of the Bohr theory of the atom, and could be understood in terms of classical elastic scattering. Find the maximum scattering angle θ for the scattering of mass 4 particles from mass 1 particles which are initially at rest.

6. A particle of mass m is in a state represented by a wave function

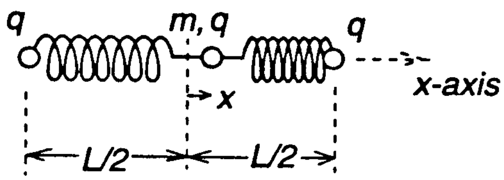
$$\Psi(x,t) = A\exp\{-a[(mx^2/\hbar)+it]\}$$

where A and a are real positive constants.

a For what potential energy $V(x)$ does Ψ satisfy the Schrödinger equation?

b Does the particle in this state have a definite energy? If it does, what is that energy value?

7. A small particle with mass m and electric charge q is connected to two identical linear massless springs with elastic constant k and unstretched length $L/2$ as shown. The springs lie along the x-axis. The far ends of the springs are fixed at points separated by a distance L. Identical charges q are attached to the far ends of the springs. The particle is constrained to move along the x-axis.



a Determine the equation of motion for the particle.

b Determine the frequency of small oscillations for the particle.

8. An electron with kinetic energy K is projected parallel to a horizontal grounded conducting plate. It starts from a point at a distance h above the plate. Calculate the horizontal distance traveled before it strikes the plate.
(Neglect magnetic, gravitational and relativistic effects!)