PHYSICS GRADUATE SCHOOL QUALIFYING EXAMINATION

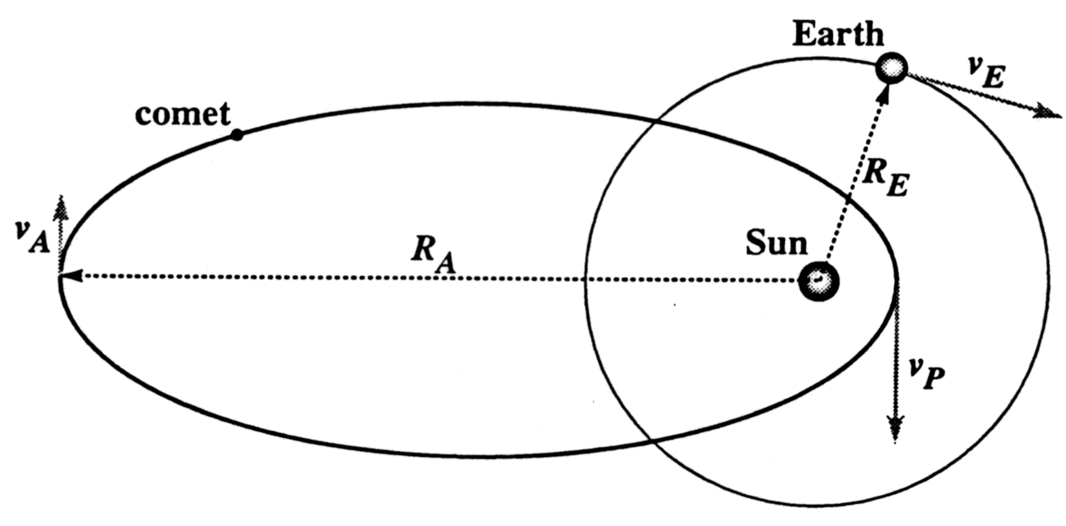
January 4, 1996

Part I

INSTRUCTIONS: Work all problems. This is a closed book examination. Start each problem on a new pack of yellow paper and use only one side of each sheet. All problems carry the same weight. *Write your student number* on the upper right-hand corner of each answer sheet.

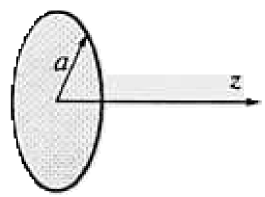
1. A rocket of mass M, while at rest, is filled with a mass m_0 of fuel. During burning, the gases are ejected at a fixed velocity v_0 relative to the rocket. Ignore the effects of air resistance and gravity during the burning period. Find the velocity V of the rocket at the instant when all of the fuel is consumed.

2. A comet in an orbit about the sun has a velocity of v_A at aphelion and v_P at perihelion. If the earth's velocity in a circular orbit is v_E, and the radius of its orbit is R_E, find the aphelion distance R_A for the comet in terms of the quantities given.

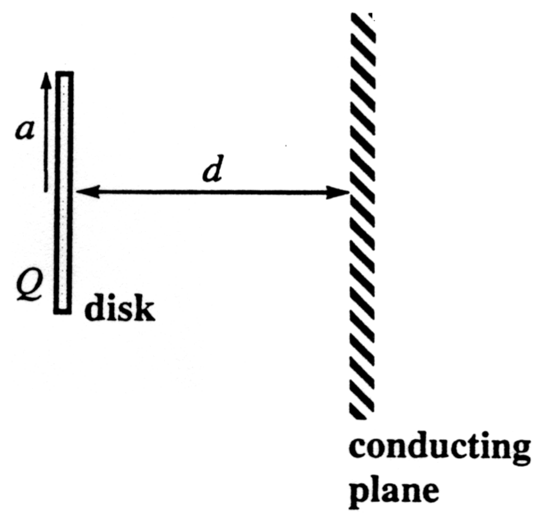


3. An insulating disk of a radius a has a uniformly distributed positive charge Q.

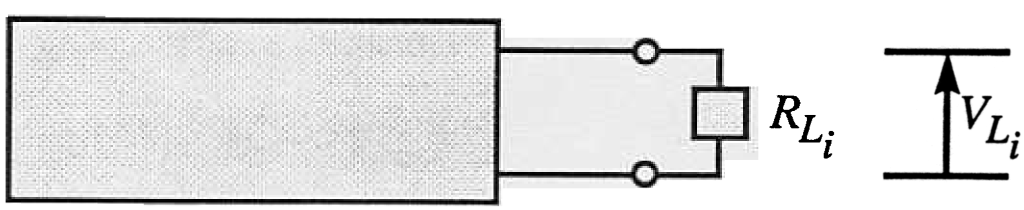
(a) Find the electric field along the z-axis through the center of the disk.



(b) The disk is placed parallel to a conducting plane with a separation d. Find the surface charge density on the conducting plane at the point on the axis of the disk.



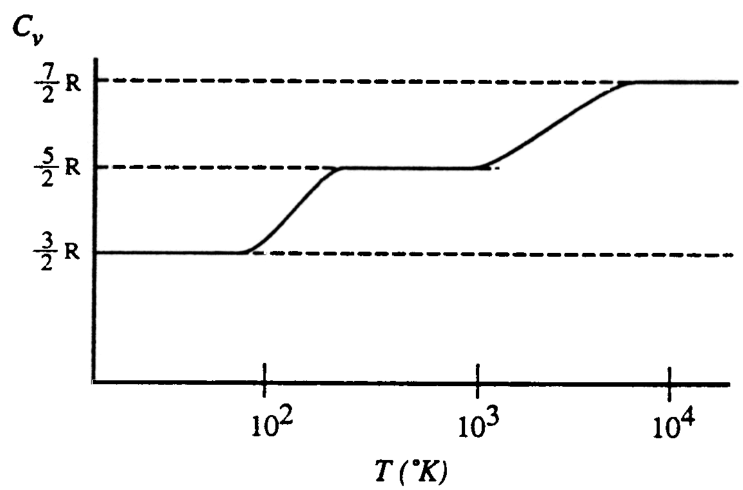
4. Consider a linear electrical device made up of voltage sources and resistors. The voltage drop across a load resistance R_{L_1} connected between its terminals is V_{L1}, and the voltage drop across a different load resistance R_{L_2} connected between its terminals is V_{L2}. Determine the voltage drop V_L across load resistance R_L connected between its terminals.



5. A photon of wavelength λ collides with a fast electron of energy E traveling in the opposite direction. Calculate the wavelength of the photon scattered in the direction opposite to its incident direction.

6. The molar specific heat of an ideal gas is $(3/2)R$ ($R = 2$ kcal / kg-mole-°C) and independent of temperature. The first figure shows the specific heat of molecular hydrogen gas.

(a) Explain why the specific heat depends on temperature and why the values of "step" increases are equal to one R.

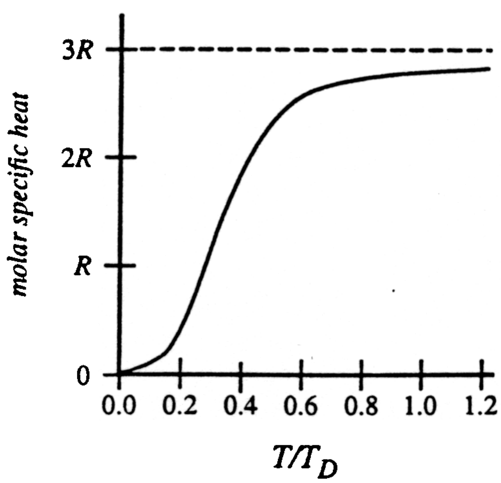


The second figure shows the specific heat of a typical solid.

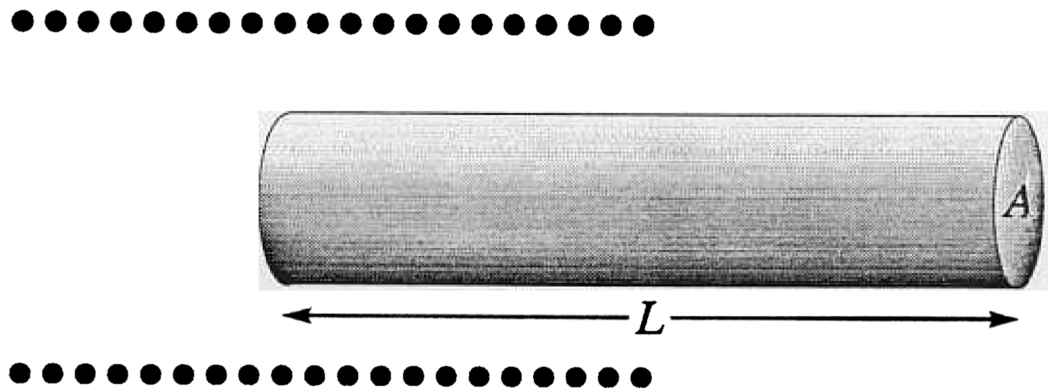
(b) Explain why the specific heat approaches $3R$ at high temperatures and why it falls at low temperatures.

This figure is typical of the specific heat of both insulators and metals. However, metals have free conduction electrons that act much like a free gas.

(c) Explain why the conduction electrons do not add another $(3/2)R$ to the specific heat of a metal.



7. Consider a long solenoid of N turns and length L carrying a constant current I. If an iron rod of constant permeability μ, circular cross sectional area A, and length L is inserted halfway into the solenoid, calculate the magnetic force on the rod.



8. Estimate at what temperature the RMS speed of $\mathbf{H_2}$ is equal to the escape speed from the earth's gravitational field ($G = 7\text{x}10^{-11}$ N/m^2/S^2, $k = 1.4\text{x}10^{-23}$ J/K, $M_E = 6\text{x}10^{24}$ kg, $m_H = 1.7\text{x}10^{-27}$ kg and $R_E = 6\text{x}10^{6}$ m). Write down an expression for the fraction of the $\mathbf{H_2}$ molecules moving outwards at the top of the atmosphere which escape (you need not solve or attempt to solve the equation).

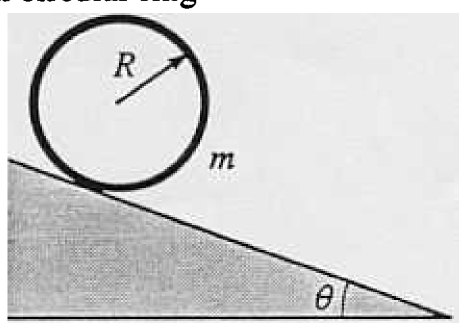
PHYSICS GRADUATE SCHOOL QUALIFYING EXAMINATION

January 5, 1996

Part II

INSTRUCTIONS: Work all problems. This is a closed book examination. Start each problem on a new pack of yellow paper and use only one side of each sheet. All problems carry the same weight. *Write your student number* on the upper right-hand corner of each answer sheet.

1. Calculate the minimum coefficient of friction necessary to keep a thin circular ring of mass m and radius R from sliding as it rolls down a plane inclined at an angle θ with respect to the horizontal plane.

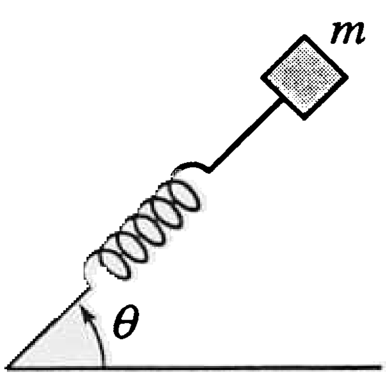


2. A mass m moves without friction on a horizontal plane and is connected to a fixed point by a spring of force constant k.

(a) Write the Lagrangian for the system in terms of θ, r and r_0, the value of r when the spring is unstretched. Use the Lagrangian to obtain the equations of motion.

(b) If the mass moves in a uniform circle of radius R with angular frequency ω, find an expression for R in terms of m, k, r_0 and ω

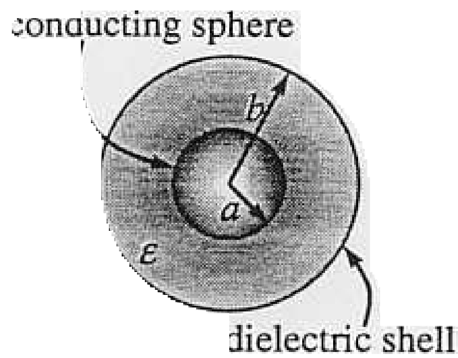
(c) For small radial displacements from circular motion ($r = R + \delta$) determine the frequency of oscillation, Ω, about the circular path.



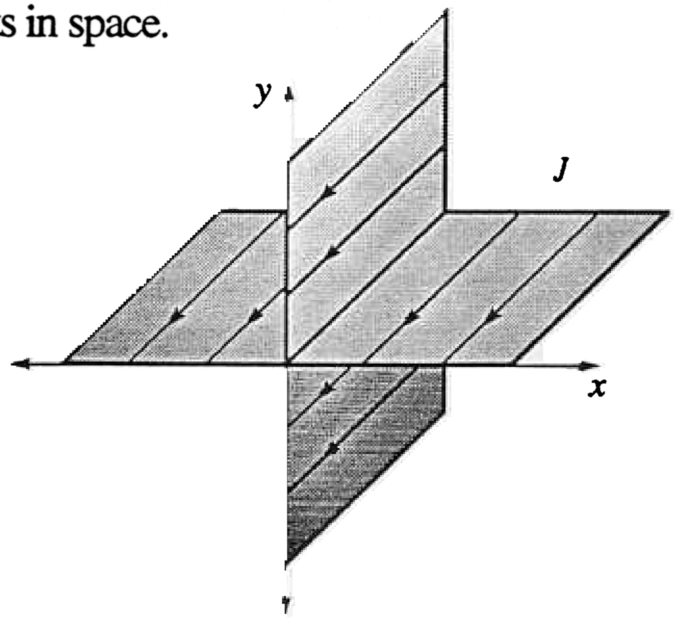
3. A conducting sphere of radius a carries a charge Q. It is surrounded by a spherical shell of linear dielectric material of permittivity ε out to a radius b.

(a) Find the potential of the conductor.

(b) Find the polarization of the dielectric.



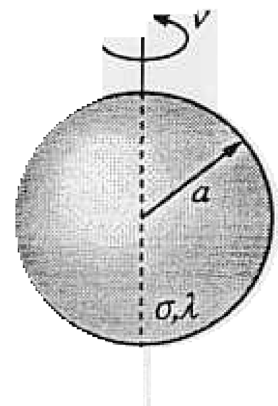
4. Two infinite, thin, conducting planes are perpendicular to each other. Each carries a uniformly distributed surface current density **J** amperes/meter parallel to the line of intersection of the plates. Calculate the resulting magnetic field **B** at all points in space.



5. Suppose that the Fermilab accelerator ring is used to collide μ^- and μ^+ particles. Suppose that the ring has a radius r, the muons have a rest mass m, and a mean life τ and that they are accelerated to an energy E. Find the fraction of the muons that decay in one revolution around the ring in terms of r, m, τ, and E.

6. A three dimensional harmonic oscillator has a potential energy given by $V(r) = 0.5\,Kr^2$. A particle of mass m is held by the potential. If the oscillator is in a state with quantum number N, what is the rms value for $<r^2>$?

7. A spherical non-conducting shell of radius a carries a uniform surface charge density of σ, and has a uniform surface mass density of λ. It spins with frequency ν on an axis through its center. Show explicitly that the ratio of its magnetic moment to its angular momentum is



$$\frac{\mu}{L} = \frac{q}{2m}$$

8. The individual plates of a vacuum-filled parallel plate capacitor have mass M and area A. The bottom plate is fixed and the movable top plate is connected to the fixed bottom one by an insulator Hooke spring of unstretched length D_o and elastic constant k. The capacitor is connected to an ideal battery of voltage V. Neglect fringing, magnetic and gravitational effects. The equilibrium value of the plate separation is $\tilde{D}$.

(a) Determine the value of the equilibrium charge on the capacitor in terms of D_o and other given quantities.

(b) Determine the frequency of small oscillations about the equilibrium position in terms of given quantities.