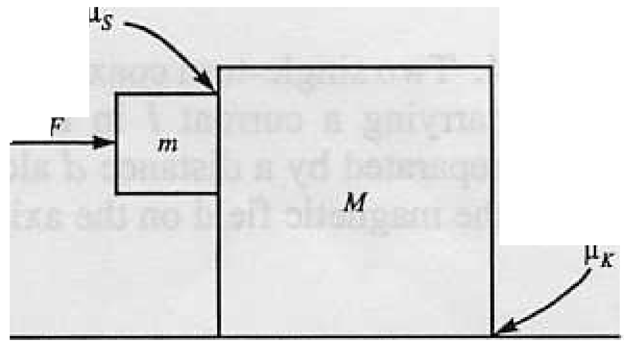
PHYSICS GRADUATE SCHOOL QUALIFYING EXAMINATION

January 5, 1995

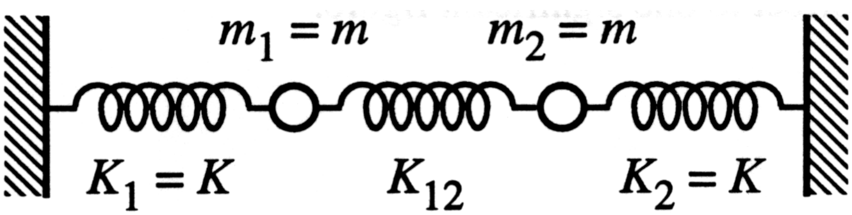
Part I

INSTRUCTIONS: Work all problems. This is a closed book examination. Start each problem on a new pack of yellow paper and use only one side of each sheet. All problems carry the same weight. Write your student number on the upper right-hand corner of each answer sheet.

1. Two blocks are positioned as shown in the accompanying figure. The masses of the blocks are m and M, as shown. The coefficient of static friction between the two blocks is μ_S, and the coefficient of sliding (kinetic) friction between the block of mass M and the horizontal surface is μ_K. What constant horizontal force must be applied just to keep the block of mass m from sliding down the interface between the blocks?



2. Two harmonic oscillators are coupled by a spring as in the figure.



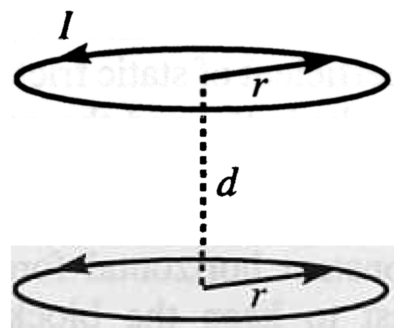
Calculate the two eigenfrequencies for the system in terms of K, m, K_{12}.

3. A spherical capacitor consists of an outer conducting sphere of fixed radius b and a concentric inner one of adjustable radius a. The space between the spheres is filled with air, which has a breakdown electric field strength E_0.

a. Determine the greatest possible potential difference between the spheres.

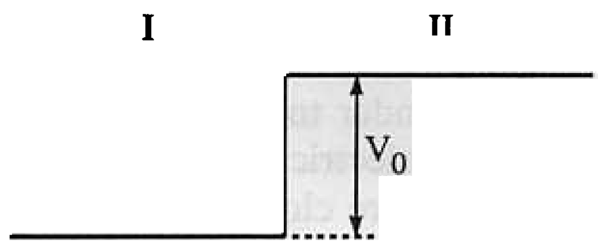
b. Determine the greatest possible electrostatic energy stored in the capacitor.

4. Two single-turn coaxial coils of radius r, each carrying a current I in the same direction are separated by a distance d along their axis. Find the magnetic field on the axis.

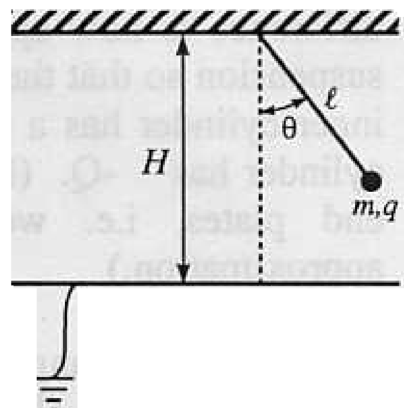


5. A sodium vapor lamp radiates light of wavelength $\lambda \approx 6 \times 10^{-7}$ m isotropically at a power rate P of 100 W. Determine at what distance from the lamp the photons have an average density ρ_p of $10^6/m^3$. Your answer need only be correct to one significant figure.

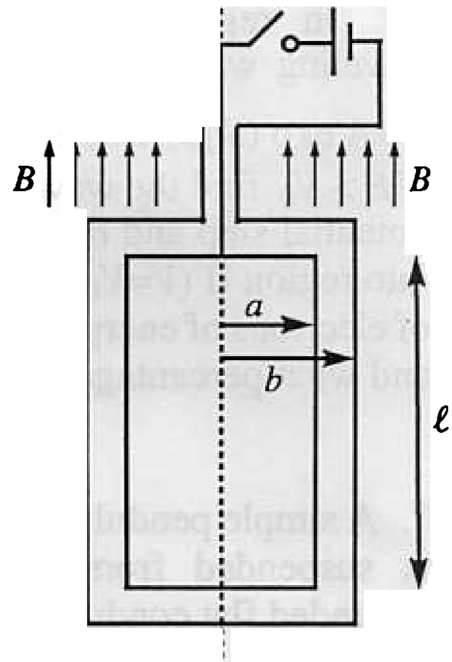
6. In region I (V=0) an electron traveling wave moving to the right is $\Psi = A \exp(ik_1 x)$ where $k_1 = (2mE/\hbar^2)^{1/2}$. If $E > V_0$ find the wave reflected by the potential step and the transmitted wave into region II ($V = V_0$). What percentage of electrons of energy $E > V_0$ is reflected and what percentage is transmitted?



7. A simple pendulum of length ℓ and bob mass m is suspended from a height H ($>\ell$) above a grounded flat conducting plate. The bob is given a net charge q. Determine the frequency of small oscillations of the pendulum.



8. Two long conducting cylindrical tubes of radii a and b have a coaxial suspension which permits each cylinder to rotate independently and also provides electrical connections. The top ends of the tubes are closed by conducting disks. The system is suspended in a uniform magnetic field B which is parallel to the axis. Initially the cylinders are at rest and uncharged. A potential difference is now applied through the coaxial suspension so that the cylindrical surface of the inner cylinder has a charge +Q and the outer cylinder has -Q. (Neglect any charge on the end plates, i.e. we use a long cylinder approximation.)



a. Calculate the angular momentum of each cylinder as a result of the charging process. (Hint: The current flows in the top disks are radial. Assume I_r between r=0 and a or b.)

b. Calculate the angular momentum of the combined electric and magnetic fields between the cylinders. Show that the total angular momentum of the entire system is still zero. (Hint: The linear momentum density of the combined $\bar{E}$ and $\bar{B}$ fields is $\bar{p} = \varepsilon_0 \bar{E} \times \bar{B}$.)

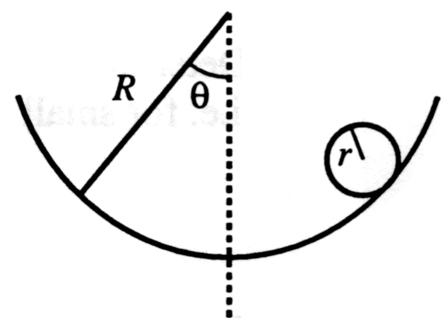
PHYSICS GRADUATE SCHOOL QUALIFYING EXAMINATION

January 6, 1995

Part II

INSTRUCTIONS: Work all problems. This is a closed book examination. Start each problem on a new pack of yellow paper and use only one side of each sheet. All problems carry the same weight. Write your student number on the upper right-hand corner of each answer sheet.

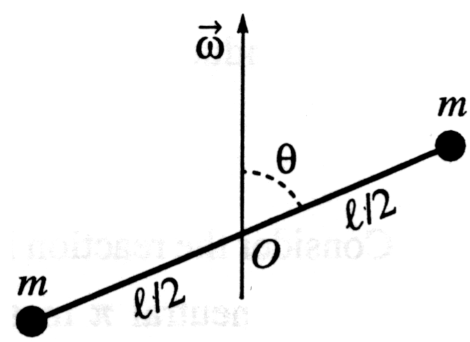
1. A solid sphere of radius r rolls without slipping in a cylindrical trough of radius $R > r$.



   a. Set up the Lagrangian, and derive the equation of motion.

   b. Show that for small displacements from equilibrium, the sphere executes simple harmonic motion. Find the period.

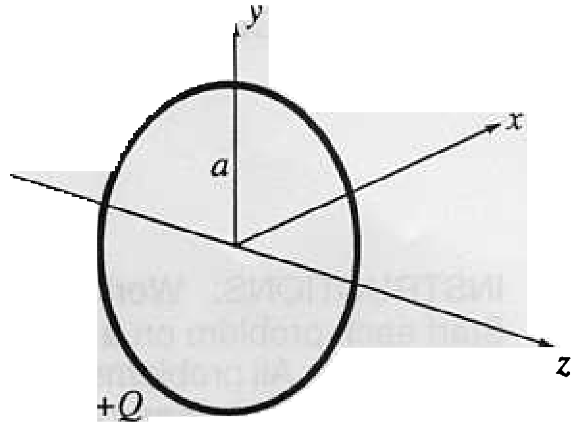
2. Two small balls of mass m are connected by a massless stiff rod of length ℓ. The center of the rod is rigidly attached to a vertical axle at an angle θ. The vertical axle spins with angular velocity ω. Calling the principal moments of inertia I_1, and I_2 with I_3=0,



   a. Find the instantaneous angular momentum of the system with respect to the origin O at the instant shown in the figure.

   b. Find the torque exerted by the axle on the rod with respect to O at the instant shown.

3. A ring of radius a has a uniformly distributed charge $+Q$ on its circumference.



a. Determine the electric potential, $V(z)$, along the z-axis which is perpendicular to the plane of the ring through the center.

b. Determine the electric field, E_z, for all points along the z-axis.

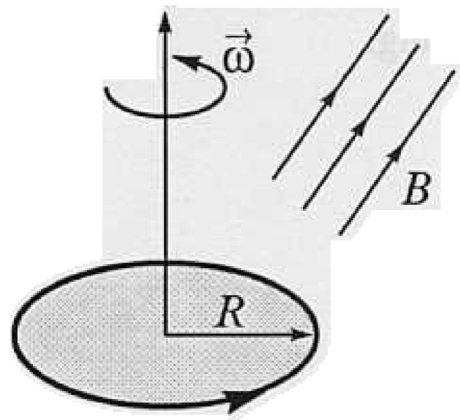
c. Determine the electric field in the plane of the ring near the center, i.e. for small r.

4. In a medium with a finite conductivity σ, an electric field is generated at time $t = 0$ such that $E(t = 0) = E_0$. Starting from Maxwell's equations, derive the time-evolution of this field. You may assume that $B = 0$. What is the dielectric relaxation time τ_{die} in terms of the dielectric permitivity ε and the conductivity σ?

5. Consider the reaction in which a photon collides with a proton at rest to produce a neutral π meson in addition to the proton in the final state. Letting M_0 be the proton rest mass and m_0 the pion rest mass find the minimum photon energy required for the reaction ($\gamma + p \rightarrow p + \pi_0$) to proceed.

6. A particle of mass m experiences an attractive potential of $V(x)=-a\delta(x)$ where $\delta(x)$ is a Dirac delta function. Starting from the one-dimensional Schrödinger equation, derive the energy of the bound state in terms of m, a and $\hbar$. Use the fact that the first derivative of the wavefunction is discontinuous at the origin because of the delta function there.

7. Consider a thin circular disk of radius R and uniform mass density σ_m that spins with angular velocity ω about an axis through its center normal to the surface. Its upper surface has a uniform charge density σ_c.



a. Find the magnetic moment M of the system.

b. Find the angular momentum L of the system and the ratio M/L.

c. Find the precession frequency of the magnetic moment about B in a uniform magnetic field (neglect gravity).

8. A solid copper ring (torus) has minor radius a, and major radius b, such that $a<<b$. The moment of inertia about a diameter is $Mb^2/2$ for the ring, and its volume is $2\pi^2 a^2 b$. It rotates about a diameter without friction. There is a uniform magnetic field B perpendicular to the rotation axis. Assume the conductivity of copper is σ, the density is ρ, and neglect self-inductance. Calculate the decay time for the rotational frequency to decrease to $1/e$ of its original value ω_0, assuming all loss is due to resistive heating.