Lab 8: Archimedes' Principle (M8)

Objectives

- Find the density of objects that have various shapes.
- Use Archimedes' principle to determine the percentage of brass in a solid cylinder made from two metals: brass and aluminum.

For our set-up:

To determine the volumes (and densities) of various objects, we will take two mass measurements using the two configurations shown as Figure A and Figure B.

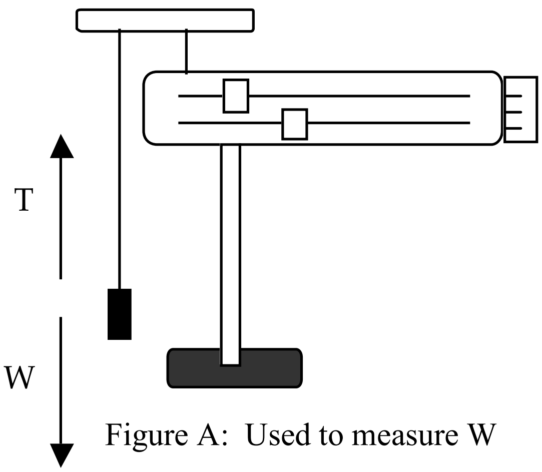


Figure A: Used to measure W

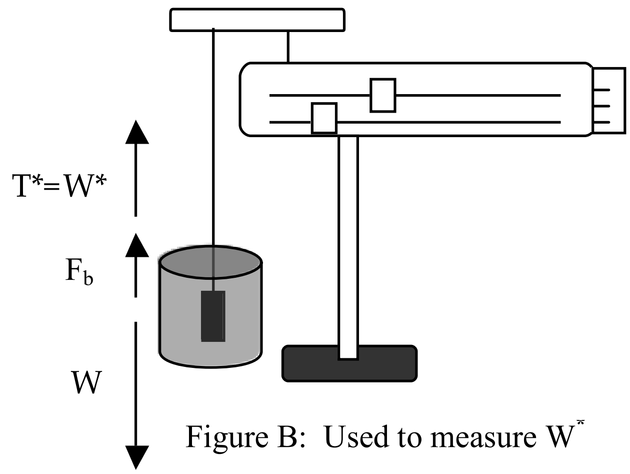


Figure B: Used to measure W^*

In Figure A, the tension (T) in the string is equal to the **weight (W)** of the object and is measured by adjusting the two sliding bars until the balance is centered. T = W = mg.

In Figure B, there are three forces acting on the submerged object. The object's weight, W = mg is acting down, the second tension (T^*) is acting upward and the buoyant force is also acting upward. Because this buoyant force is also acting upward, the measurement of the apparent weight W^* will be less than the normal weight W. The **apparent weight W^*** is equal to the normal (measured in air or vacuum) weight of an object minus the buoyant force ($F_b = \rho_w gV$):

$$T^* = W^* = W - F_b = W - \rho_w gV = mg - \rho_w gV$$

Remember that the density of water is equal to: $\rho_w = 1000\ kg/m^3$.

W and W^* are computed from the measured mass values (m) displayed on the scale. For a given m, W or W^* = mg, where g is the acceleration due to gravity and $g = 9.80\ m/s^2$.

The overall view of the apparatus for this experiment is shown below. You will not need computers for this experiment.

Theory

Density (often denoted by the Greek letter 'rho': ρ) is equal to $\frac{\text{mass}}{\text{volume}}$.

$$\rho = \frac{m}{V} \tag{1}$$

The unit of density is: kg/m^3 (= 0.001 g/cm^3).

A **buoyant force** is exerted <u>upward</u> on a fully or partially submerged object by the fluid.

Archimedes' principle states that the buoyant force has the same magnitude as the weight of the displaced fluid ($F_b = m_{fluid}\,g = \rho_w Vg$), but the opposite direction.

Derivations

It was shown in the set-up section (Figures A and B, previous page) using force diagrams that the weight when the object was hanging in air was W = mg and, when the object was submerged in water, the apparent weight was: $W^* = mg - \rho_w gV$.

Solving for V (the volume of the displaced fluid and therefore the volume of the object), we obtain:

$$V = \frac{W - W^*}{\rho_w g} \tag{2}$$

Therefore, the density ρ of an object is given by the following formula:

$$\rho = \frac{m}{V} = \frac{m}{(W - W^*)/\rho_w g} = \frac{mg\rho_w}{(W - W^*)} = \frac{W}{(W - W^*)}\rho_w$$

$$\rho = \frac{W}{(W - W^*)}\rho_w \tag{3}$$

In ***Activity 4***, you will be given an object that is composed of two metals, brass and aluminum. The volume of brass in the object may be determined from the two measured mass values by using the formula:

$$V_{\text{brass}} = W^*\left(\frac{\rho_2}{\rho_w(\rho_1 - \rho_2)g}\right) - W\left(\frac{\rho_2 - \rho_w}{\rho_w(\rho_1 - \rho_2)g}\right) \tag{4}$$

A short derivation of the above formula follows. If you are unsure about any of the steps, feel free to ask your TA to explain them.

Define the variables:

V_{brass} = the volume of brass in the cylinder

V_{Al} = the volume of aluminum in the cylinder

$\rho_1 = \frac{m_{brass}}{V_{brass}}$ = the density of brass = 8600 kg/m^3

$\rho_2 = \frac{m_{aluminum}}{V_{Al}}$ = the density of aluminum = 2700 kg/m^3

ρ_w = 1000 kg/m^3 = the density of water.

The cylinder is composed of brass and aluminum and its volume (V) may be written as the sum of the volume of brass and the volume of aluminum: $V = V_{brass} + V_{Al}$

Rearranging, the previous equation, we obtain: $V_{Al} = V - V_{brass.}$

For the measurement in air,

$$W = mg = (m_{brass} + m_{aluminum})g = (\rho_1 V_{brass} + \rho_2 V_{Al})g = (\rho_1 V_{brass} + \rho_2 (V - V_{brass}))g$$

When the object is in the water,

$$W^* = mg - \rho_w g V = \rho_1 g V_{brass} + \rho_2 g(V - V_{brass}) - \rho_w g V$$

In the above equation, group V and V_{brass} terms and then substitute using: $V = \dfrac{W - W^*}{\rho_W g}$

$$W^* = (\rho_1 - \rho_2) g V_{brass} + (\rho_2 - \rho_w) g V = (\rho_1 - \rho_2) g V_{brass} + (\rho_2 - \rho_w) g \left(\frac{W - W^*}{\rho_w g} \right)$$

Group W and W^* terms:

$$-W\left(\frac{\rho_2 - \rho_w}{\rho_w}\right) + W^*\left(\frac{\rho_2}{\rho_w}\right) = (\rho_1 - \rho_2) g V_{brass}$$

Solve for V_{brass}:

$$V_{brass} = W^*\left(\frac{\rho_2}{\rho_w(\rho_1 - \rho_2)g}\right) - W\left(\frac{\rho_2 - \rho_w}{\rho_w(\rho_1 - \rho_2)g}\right)$$

After you have made your W and W^* measurements, you will use this formula to solve for the volume of brass (V_{brass}) in the cylinder (*Activity 4*).

Procedure:

Make sure that the balance is **zeroed** when the chain is in place but no object is attached. Use the small knob on the left-hand side of the balance.

For accuracy, you should re-check the zero to make sure that the balance is zeroed at the beginning of each activity.

Activity 1: Density

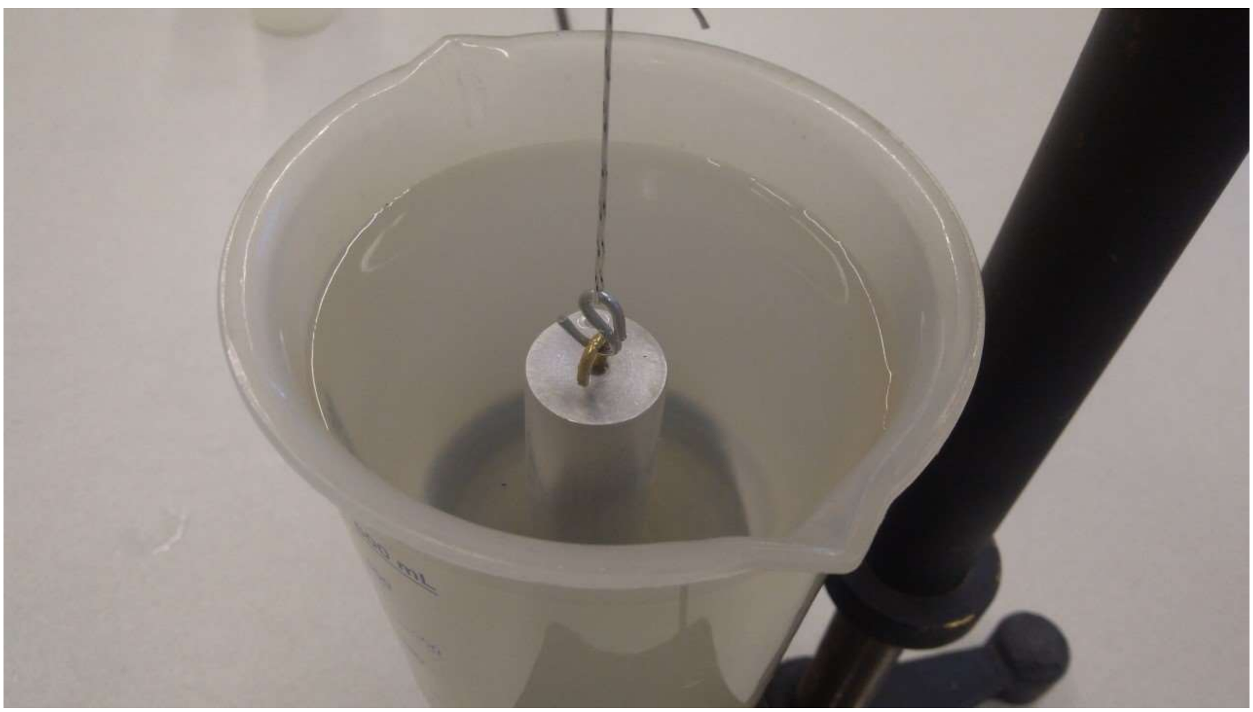
Measure and record the radius of the aluminum cylinder and its height using a ruler. Calculate the volume of the cylinder. This volume calculation is an estimate since it neglects the volume of the hook attached to the top of the cylinder.

Make sure that the <u>balance is zeroed</u>. There should be a thin string attached to the aluminum cylinder. Attach this string to the hook on the balance and allow the cylinder to be supported by the tension in the string (Figure A). Do **not** touch the cylinder, string, or hook while you are taking measurements.

Now that you have measured the mass and calculated the volume, estimate the density of the cylinder using the definition of density (Equation 1). Compare this result with the known value ($\rho_{aluminum} = 2700\ kg/m^3$). What is the percent difference?

Activity 2: Archimedes' Principle

Use Archimedes' Principle to measure the volume of the cylinder. Connect the cylinder's string to the hook and then submerge the aluminum cylinder into the container of water (Figure B). Be sure that there are <u>no air bubbles around or under the cylinder</u> and that water covers the top of the cylinder and the hook. Do not allow the cylinder to touch the bottom of the glass jar. Find the tension in the string by balancing the scale.



Using the Archimedes' principle, calculate the density (ρ) for the measured aluminum cylinder (Equation 3). Compare this result with the known value ($\rho_{aluminum} = 2700\ kg/m^3$). What is the percent difference?

Remember that the density of water is equal to: $\rho_w = 1000\ kg/m^3$

Do we need a correction for the density of air?

Strictly speaking, when we measure mass of the aluminum cylinder in air, we should also consider the buoyant force due to the air. Let us make an estimate to see how big is that correction. Consider an aluminum cylinder that has mass m = 0.10 kg (close to the mass of the cylinder that we are using in this experiment). The density of aluminum is 2700 kg/m^3 and the density of air at room temperature 20ºC is equal to ρ_{air} = 1.21 kg/m^3.

The volume of the cylinder $V = m/\rho_{aluminum} = 0.10$ kg/2700 kg/m^3 = $3.7*10^{-5}$ m^3

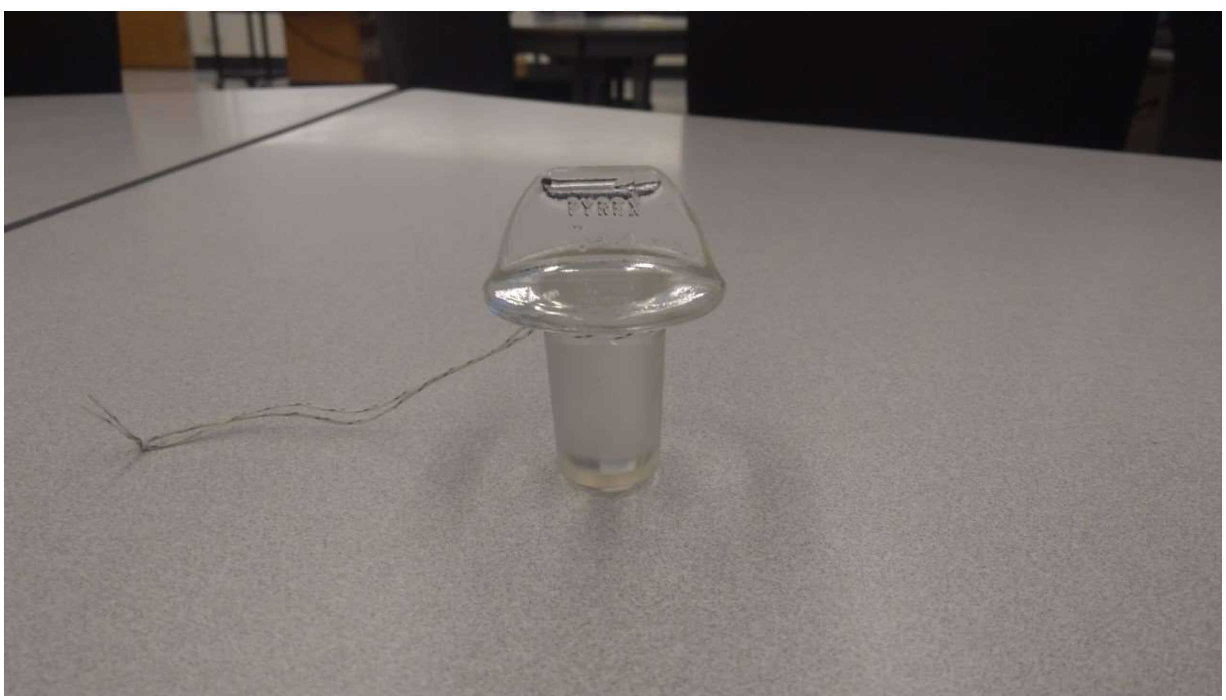
The buoyant force due to the air displaced by the cylinder is equal to:

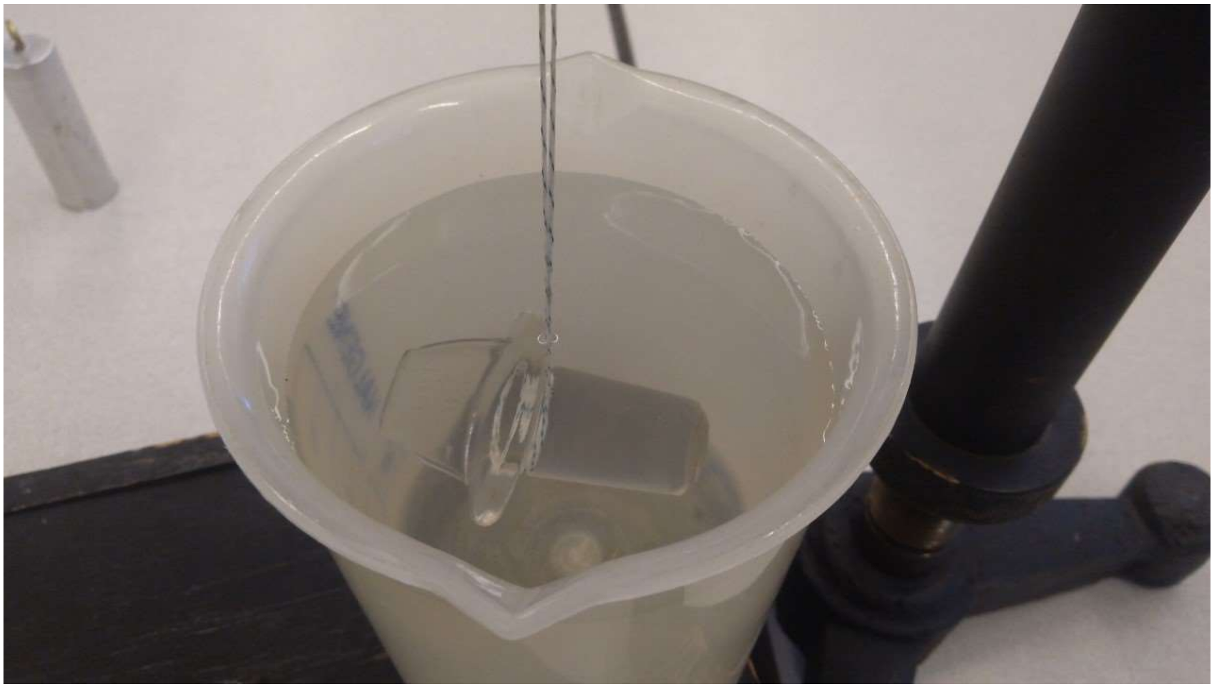
$$F_{b\ air} = \rho_{air} Vg = 1.21 \text{ kg/m}^3 * 3.7*10^{-5} \text{ m}^3 * 9.8 \text{ m/s}^2 = 4.4*10^{-4} \text{ N}$$

The error that we made during the mass measurement because we neglected buoyant force due to displaced air is equal to $F_{b\ air}/g = \rho_{air} V = 4.5*10^{-5}$ kg = 0.045 g. You cannot measure mass with this accuracy using the available scale. Therefore, the correction is small enough to be ignored.

Activity 3: Density of Glass

For irregularly shaped objects, it is hard to accurately measure their volume, which is necessary to find the average density. In this case, Archimedes' principle in combination with liquid that has known density (water in our case) provides an easier way to find the object's volume and then the object's density.





Take two tension measurements, one when the glass is in air and one while the glass is submerged in the water and use the formula to calculate the glass piece's volume.

Using the Archimedes' principle, calculate the density of the Pyrex glass piece (Equation 3) and compare this density with the known value (ρ_{glass} = 2230 kg/m^3). What is the percent difference?

Activity 4: Two-Metal Cylinder

The Legend[1]

During the rule of King Hieron, Archimedes (287 - 212 BC) was called upon to determine whether the king's crown was made of solid gold. Archimedes could not harm the crown, yet he needed to determine its composition. One evening while bathing, Archimedes observed the effect of the buoyant force. By submerging the crown in water, he could determine the crown's volume and the percentage of gold in it. Overcome with joy, a naked Archimedes leapt from his tub and ran down the street yelling "*Eureka!*" (*I have found it!*)

You have a similar problem - you have a cylinder composed of two known metals: brass and aluminum.

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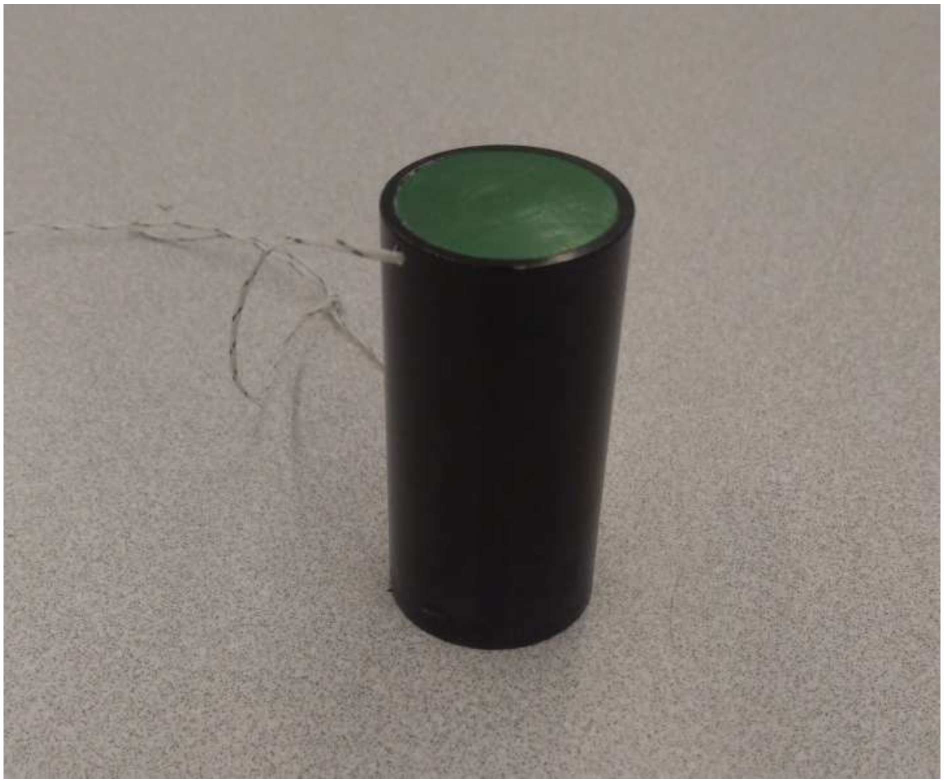
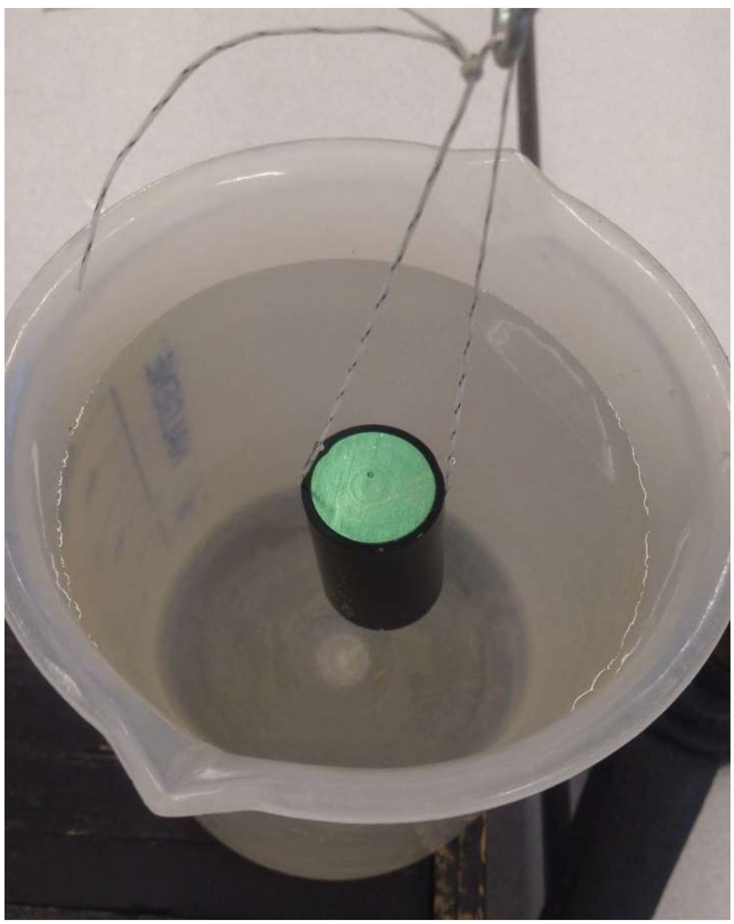
[1] Serway, Raymond A. and Jerry S. Faughn, College Physics, Fort Worth: Saunders College Publishing, 1992, p. 261.





Determine the percent composition of this cylinder by finding the weight of the two-metal cylinder while the cylinder is in air and then again when it is submerged in water.

The densities of brass and aluminum are:

$$\rho_{brass} = 8600 \text{ kg/m}^3 \quad ; \quad \rho_{aluminum} = 2700 \text{ kg/m}^3$$

Archimedes discovered that the king's crown was constructed from a metal alloy.

What percentage of the total volume of the cylinder is occupied by aluminum? What percentage of cylinder's mass is aluminum? Note that these two values are not equal!

Complete the lab report and return it to the lab TA.

Make sure to complete the following tasks:

You must submit the answers to the prelaboratory questions online. (3.5 points)

1. Your completed Data Sheets. (6.5 points)

2. Return the completed lab report to your lab TA.